8.1 ANGLES OF POLYGONS

Find and use the sum of the measures of the interior and exterior angles of a polygon

Polygon Interior Angles Sum

A DIAGONAL of a polygon is a segment that connects any two nonconsecutive vertices



TriangleQuadrilateralPentagonHexagon 180° $2(180^{\circ}) = 360^{\circ}$ $3(180^{\circ}) = 540^{\circ}$ $4(180^{\circ}) = 720^{\circ}$

Recall

 A POLYGON is a closed figure formed by a finite number of coplanar segments called sides such that: the sides have a common endpoint that are not collinear and each side intersects exactly two other sides, but only at their vertices

Polygons

Sides	Name	Measure of Interior Angles
3	Triangle	$1 \cdot 180^\circ = 180^\circ$
4	Quadrilateral	$2 \cdot 180^\circ = 360^\circ$
5	Pentagon	$3 \cdot 180^\circ = 540^\circ$
6	Hexagon	$4 \cdot 180^\circ = 720^\circ$
7	Heptagon	$5 \cdot 180^\circ = 900^\circ$
8	Octagon	$6 \cdot 180^{\circ} = 1080^{\circ}$
9	Nonagon	$7 \cdot 180^{\circ} = 1260^{\circ}$
10	Decagon	$8 \cdot 180^{\circ} = 1440^{\circ}$
11	Hendecagon	$9 \cdot 180^{\circ} = 1620^{\circ}$
12	Dodecagon	$10 \cdot 180^{\circ} = 1800^{\circ}$
Ν	n-gon	$(n-2)\cdot 180^{\circ}$

Polygon Interior Angle Theorem

The sum of the interior angle measures of an n –sided convex polygon $(n - 2) \cdot 180$.

Example 1: Find the sum of the measures of the interior angles of a 13 - gon

13 sides total $(13 - 2) \cdot 180^{\circ}$ $11 \cdot 180^{\circ}$ The sum of the interior angles of a 13 - gon is 1980°. Example 2: Find the measure of each interior angle of the pentagon *HJKLM* shown



5 sides total $(5-2) \cdot 180^{\circ}$ The sum of the interior angles is 540° $m \angle H + m \angle J + m \angle K + m \angle L + m \angle M = 540^{\circ}$ 2x + 142 + 2x + 3x + 14 + 3x + 14 = 540° 10x + 170 = 540° 10x = 370° x = 37

 $m \angle J = 142^{\circ}$ $m \angle H = m \angle K = 2(37) = 74^{\circ}$ $m \angle M = m \angle L = (3(37) + 14) = 125^{\circ}$

Recall:

A **REGULAR POLYGON** is a polygon in which all of the sides are congruent and all the angles are congruent

Example 3: Find the measure of each interior angle of the regular hendecagon that appears on the face of a Susan B. Anthony one-dollar coin.

11 sides

 $(11 - 2) \cdot 180^{\circ}$

1620° is the sum of the interior angles for a hendecagon

 $1620 \div 11$

The measure of each interior angle of a regular hendecagon is about 147.27.

Example 4: The measure of each interior angle of a polygon is 150. Find the number of sides in the polygon

Let n be the number of sides. Since all angles of a regular polygon are congruent, the sum of the interior angles can be expressed as 150n

 $150n = (n - 2) \cdot 180$ 150n = 180n - 360 -30n = -360n = 12

There are 12 sides.

Polygon Exterior Angle Sum

 The sum of the exterior angle measures of a convex polygon, one angle at each vertex is 360.

Example 5: Find the value of *x* in the diagram

 $(5x + 5) + (5x) + (4x - 6) + (5x - 5) + (4x + 3) + (6x - 12) + (2x + 3) = 360^{\circ}$ 31x - 12 = 360 31x = 372x = 12

5x - 5

Example 6: Find the measure of each exterior angle of a regular dodecagon

12 sides

Let *n* represent the measure of each exterior angle 12n = 360n = 30

Each exterior angle of a regular dodecagon is 30°.