11-4 Inscribed Angles

Find each measure.

1. \( m \angle B \)

[Diagram of a circle with \( A \), \( B \), and \( C \) as points on the circle, with \( \angle B \) inscribed.

\[ \text{SOLUTION:} \]
If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore, \( m \angle B = \frac{1}{2}(m \overarc{AC}) = 30. \)

2. \( m \overarc{RT} \)

[Diagram of a circle with \( R \), \( T \), \( S \), and \( m \overarc{RT} \) as points on the circle, with \( m \overarc{RT} \) an inscribed angle.

\[ \text{SOLUTION:} \]
If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore, \( m \overarc{RT} = 2(m \angle S) = 126. \)

3. \( m \overarc{WX} \)

[Diagram of a circle with \( W \), \( X \), \( Y \), and \( m \overarc{WX} \) as points on the circle, with \( m \overarc{WX} \) an inscribed angle.

\[ \text{SOLUTION:} \]
Here, \( \overarc{XY} \) is a semi-circle. So, \( m \overarc{XY} = 180. \)
\( m \overarc{WX} + m \overarc{XY} = 180. \)
If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. So, \( m \overarc{XY} = 2(m \angle X) = 114. \)
Therefore, \( m \overarc{WX} = 180 - m \overarc{XY} = 66. \)

4. SCIENCE The diagram shows how light bends in a raindrop to make the colors of the rainbow. If \( m \overarc{ST} = 144 \) what is \( m \angle R \)?

[Diagram of a raindrop with \( S \), \( T \), \( R \), and \( m \overarc{ST} \) as points on the circle, with \( m \overarc{ST} \) an inscribed angle.

\[ \text{SOLUTION:} \]
If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore, \( m \angle R = \frac{1}{2}(m \overarc{ST}) = 72. \)

ALGEBRA Find each measure.

5. \( m \angle H \)

[Diagram of a circle with \( H \), \( J \), \( K \), and \( G \) as points on the circle, with \( m \angle H \) an inscribed angle.

\[ \text{SOLUTION:} \]
If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent. So, \( m \angle G = m \angle H. \)
\( 2x - 54 = x \)
\( x = 54 \)
Therefore, \( m \angle H = 54. \)
6. \( m \angle B \)

[Diagram showing two angles labeled \( x + 24 \) and \( 3x \) with points A, B, C, D, and circle]

**SOLUTION:**
If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent.
So, \( m \angle B = m \angle C \).
\[ 3x = x + 24 \]
\[ 2x = 24 \]
\[ x = 12 \]
Therefore, \( m \angle B = 12 + 24 = 36 \).

7. **PROOF** Write a two-column proof.

**Given:** \( RT \) bisects \( SU \).

**Prove:** \( \triangle RVS \cong \triangle UVT \)

[Diagram showing triangle RVS and triangle UVT with points R, V, S, U, T]

**SOLUTION:**
**Given:** \( RT \) bisects \( SU \).
**Prove:** \( \triangle RVS \cong \triangle UVT \)

**Proof:**
**Statements (Reasons)**
1. \( RT \) bisects \( SU \). (Given)
2. \( SV \equiv VU \) (Def. of segment bisector)
3. \( \angle SRT \) intercepts \( \overline{ST} \). \( \angle SUT \) intercepts \( \overline{ST} \). (Def. of intercepted arc)
4. \( \angle SRT \cong \angle SUT \) (Inscribed \( \angle s \) of same arc are \( \cong \)).
5. \( \angle RVS \cong \angle UVT \) (Vertical \( \angle s \) are \( \cong \)).
6. \( \triangle RVS \cong \triangle UVT \) (AAS)

8. \( m \angle R \)

[Diagram showing angles labeled \( 3x + 1 \) and \( 7x - 1 \) with points P, Q, R]

**SOLUTION:**
An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle.
So, \( m \angle Q = 90 \).
The sum of the measures of the angles of a triangle \( (3x + 1) + (7x - 1) + 90 = 180 \)
is 180. So, \( 10x = 90 \)
\[ x = 9 \]
Therefore, \( m \angle R = 7(9) - 1 = 62 \).

9. \( x \)

[Diagram showing angles labeled \( 2x - 5 \) with point N]

**SOLUTION:**
An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle.
So, \( m \angle Q = 90 \). Since \( LM = NM \), \( m \angle L = m \angle N \).
The sum of the measures of the angles of a triangle \( (2x - 5) + (2x - 5) + 90 = 180 \).
is 180. So, \( 4x - 10 = 90 \)
\[ x = 25 \]
10. \( m \angle C \) and \( m \angle D \)

\[
\text{SOLUTION:} \\
\text{If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.} \\
58 + 2x = 180 \quad \text{and} \quad (3y + 4) + (2y + 16) = 180 \\
\text{Solve the two equations to find the values of } x \text{ and } y. \\
2x = 122 \quad \quad 5y = 160 \\
x = 61 \quad \quad y = 32 \\
\text{Use the values of the variables to find } m \angle C \text{ and } m \angle D. \\
m \angle C = 2(61) = 122 \\
m \angle D = 2(32) + 16 = 80
\]

**ALGEBRA** Find each measure.

11. \( m \overline{DH} \)

\[
\text{SOLUTION:} \\
\text{If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore, } m \overline{DH} = 2(m \angle F) = 162.
\]

12. \( m \angle K \)

\[
\text{SOLUTION:} \\
\text{If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore, } m \angle K = \frac{1}{2}(m \overline{KL}) = 46.
\]

13. \( m \angle P \)

\[
\text{SOLUTION:} \\
The sum of the measures of the central angles of a circle with no interior points in common is 360. So, \( 120 + 100 + m \overline{QN} = 360 \). \\
m \overline{QN} = 140 \\
\text{If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore, } m \angle P = \frac{1}{2}(m \overline{QN}) = 70.
\]

14. \( m \overline{AC} \)

\[
\text{SOLUTION:} \\
\text{If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore, } m \overline{AC} = 2(m \angle B) = 48.
\]

15. \( m \overline{GH} \)

\[
\text{SOLUTION:} \\
\text{Here, } m \overline{GH} = m \overline{HJ} - m \overline{HJ}. \\
The arc \overline{GHJ} \text{ is a semicircle. So, } m \overline{GHJ} = 180. \\
\text{If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. So, } m \overline{HJ} = 2(m \angle G) = 40. \\
\text{Therefore, } m \overline{GH} = 140.
\]
11-4 Inscribed Angles

16. \( m\angle S \)

\[
\begin{array}{c}
\text{SOLUTION:} \\
\text{Here, } \widehat{RT} = \widehat{RTS} - \widehat{TS}.
\end{array}
\]

The arc \( \widehat{RTS} \) is a semicircle. So, \( \widehat{RTS} = 180 \). Then, \( \widehat{RT} = 180 - 48 = 132 \).

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore, \( m\angle S = \frac{1}{2}(\widehat{RT}) = 66 \).

17. **ALGEBRA** Find each measure.

\[
\begin{array}{c}
m\angle R
\end{array}
\]

\[
\begin{array}{c}
\text{SOLUTION:} \\
\text{If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent.} \\
\text{So, } m\angle R = m\angle Q = 32.
\end{array}
\]

18. **ALGEBRA** Find each measure.

\[
\begin{array}{c}
m\angle S
\end{array}
\]

\[
\begin{array}{c}
\text{SOLUTION:} \\
\text{If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent.} \\
\text{So, } m\angle T = m\angle S. \\
6x - 2 = 5x + 4 \\
x = 6
\end{array}
\]

Therefore, \( m\angle S = 5(6) + 4 = 34 \).

19. **ALGEBRA** Find each measure.

\[
\begin{array}{c}
m\angle A
\end{array}
\]

\[
\begin{array}{c}
\text{SOLUTION:} \\
\text{If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent.} \\
\text{Since } \angle A \text{ and } \angle B \text{ both intercept arc } CD, m\angle A = m\angle B. \\
5x = 7x - 8 \\
4 = x
\end{array}
\]

Therefore, \( m\angle A = 5(4) \) or 20.
20. **ALGEBRA** Find each measure.

![Diagram](Image)

**SOLUTION:**
If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent. Since ∠C and ∠D both intercept arc AB, m∠C = m∠D.

\[ 5y - 3 = 4y + 7 \]
\[ y = 10 \]
Therefore, \( m\angle C = 5(10) - 3 \) or 47.

**PROOF** Write the specified type of proof.
21. paragraph proof

**Given:** \( m\angle T = \frac{1}{2} m\angle S \)

**Prove:** \( m\overline{UR} = 2m\overline{RS} \)

![Diagram](Image)

**SOLUTION:**
Proof: Given \( m\angle T = \frac{1}{2} m\angle S \) means that \( m\angle S = 2m\angle T \). Since \( m\angle S = \frac{1}{2} m\overline{UR} \) and \( m\angle T = \frac{1}{2} m\overline{RS} \), the equation becomes \( \frac{1}{2} m\overline{UR} = 2 \left( \frac{1}{2} m\overline{RS} \right) \). Multiplying each side of the equation by 2 results in \( m\overline{UR} = 2m\overline{RS} \).

22. two-column proof

**Given:** \( \bigcirc C \)

**Prove:** \( \triangle KML \sim \triangle JMH \)

![Diagram](Image)

**SOLUTION:**

**Statements (Reasons):**
1. \( \bigcirc C \) (Given)
2. \( \angle H \equiv \angle L \) (Inscribed \( \angle \)s intercepting same arc are \( \equiv \).)
3. \( \angle KML \equiv \angle JMH \) (Vertical \( \angle \)s are \( \equiv \).)
4. \( \triangle KML \sim \triangle JMH \) (AA Similarity)

**ALGEBRA** Find each value.

![Diagram](Image)

23. \( x \)

**SOLUTION:**
An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle. So, \( m\angle S = 90 \).
The sum of the measures of the angles of a triangle \( x + 2x + 90 = 180 \).

is 180. So, \( 3x = 90 \)
\( x = 30 \)

24. \( m\angle T \)

**SOLUTION:**
An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle. So, \( m\angle S = 90 \).
The sum of the measures of the angles of a triangle \( x + 2x + 90 = 180 \).

is 180. So, \( 3x = 90 \)
\( x = 30 \)
Therefore, \( m\angle T = 2(30) = 60 \).
25. $x$

**SOLUTION:**
An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle. So, $m\angle D = 90$.
The sum of the measures of the angles of a triangle $(5x - 12) + (3x) + 90 = 180$.
is 180. So, $8x = 102$
$x = 12.75$
Therefore, $m\angle C = 5(12.75) - 12 = 51.75$.

**CCSS STRUCTURE**

**Find each measure.**

29. $m\angle H$

**SOLUTION:**
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.
$2x + x + 21 = 180$.
$3x = 159$
$x = 53$
Therefore, $m\angle H = 2(53) = 106$.

30. $m\angle G$

**SOLUTION:**
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.
$3y + 9 + 4y - 11 = 180$.
$7x = 182$
$x = 26$
Therefore, $m\angle G = 4(26) - 11 = 93$. 

27. $m\angle T$

**SOLUTION:**
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.
$45 + m\angle T = 180$.
Therefore, $m\angle T = 135$.

28. $m\angle Z$

**SOLUTION:**
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.
$4x + 2x + 30 = 180$.
$6x = 150$
$x = 25$
Therefore, $m\angle Z = 2(25) + 30 = 80$. 

---

11-4 Inscribed Angles

---
31. **PROOF** Write a paragraph proof for Theorem 11.9.

**SOLUTION:**

Given: Quadrilateral \( ABCD \) is inscribed in \( O \).

Prove: \( \angle A \) and \( \angle C \) are supplementary. \( \angle B \) and \( \angle D \) are supplementary.

![Diagram of a quadrilateral inscribed in a circle]

Proof: By arc addition and the definitions of arc measure and the sum of central angles,

\[
m\overarc{DCB} + m\overarc{DAB} = 360.\]

Since by Theorem 10.6 \( m\angle C = \frac{1}{2}m\overarc{DAB} \) and

\[
m\angle A = \frac{1}{2}m\overarc{DCB}, \quad m\angle C + m\angle A = \frac{1}{2}(m\overarc{DCB} + m\overarc{DAB}), \quad \text{but} \]

\[
m\overarc{DCB} + m\overarc{DAB} = 360, \quad \text{so} \quad m\angle C + m\angle A = \frac{1}{2}(360) \text{ or } 180.\]

This makes \( \angle C \) and \( \angle A \) supplementary. Because the sum of the measures of the interior angles of a quadrilateral is 360,

\[
m\angle A + m\angle C + m\angle B + m\angle D = 360. \]

But

\[
m\angle A + m\angle C = 180, \quad \text{so} \quad m\angle B + m\angle D = 180, \quad \text{making them supplementary also.}\]

32. \( m\overarc{NPQ} \)

**SOLUTION:**

Since all the sides of the stop sign are congruent chords of the circle, the corresponding arcs are congruent. There are eight adjacent arcs that make up the circle, so their sum is 360. Thus, the measure of each arc joining consecutive vertices is \( \frac{1}{8} \) of 360 or 45.

\[
m(\overarc{NPQ}) = m(\overarc{NO}) + m(\overarc{OP}) + m(\overarc{PQ}) = 45 + 45 + 45 = 135\]

Therefore, measure of arc \( NPQ \) is 135.

33. \( m\angle RLQ \)

**SOLUTION:**

Since all the sides of the stop sign are congruent chords of the circle, all the corresponding arcs are congruent. There are eight adjacent arcs that make up the circle, so their sum is 360. Thus, the measure of each arc joining consecutive vertices is \( \frac{1}{8} \) of 360 or 45. Since \( \angle RLQ \) is inscribed in the circle, its measure equals one half the measure of its intercepted arc \( QR \).

\[
m\angle RLQ = \frac{1}{2}m(\overarc{QR})
\]

\[
= \frac{1}{2}(45)
\]

\[
= 22.5
\]

Therefore, the measure of \( \angle RLQ \) is 22.5.

---

**11-4 Inscribed Angles**

**problem:**

Find each measure.

1. 

**solution:**

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore, the correct choice is C.

2. 

**solution:**

Since B is the midpoint of \( AB \), \( AC \), and \( CD \) is a semicircle, then intercepted same arc and \( m \angle 11-4 = 45 \). An inscribed angle of a triangle intercepts a diameter. The sum of the measures of the angles of a triangle is 180. This makes \( \angle C \) and \( \angle A \) supplementary. Because the sum of the measures of the interior angles of a quadrilateral is 360, \( m\angle A + m\angle C + m\angle B + m\angle D = 360 \). But \( m\angle A + m\angle C = 180 \), so \( m\angle B + m\angle D = 180 \), making them supplementary also.

3. 

**solution:**

Since all the sides of the stop sign are congruent chords of the circle, the corresponding arcs are congruent. There are eight adjacent arcs that make up the circle, so their sum is 360. Thus, the measure of each arc joining consecutive vertices is \( \frac{1}{8} \) of 360 or 45. Since \( \angle RLQ \) is inscribed in the circle, its measure equals one half the measure of its intercepted arc \( QR \).

\[
m\angle RLQ = \frac{1}{2}m(\overarc{QR})
\]

\[
= \frac{1}{2}(45)
\]

\[
= 22.5
\]

Therefore, the measure of \( \angle RLQ \) is 22.5.
34. \( m \angle LRQ \)

**SOLUTION:**

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. So, \( m \angle LRQ = \frac{1}{2} (m \overline{LNQ}) \).

The sum of the measures of the central angles of a circle with no interior points in common is 360. Here, 360 is divided into 8 equal arcs and each arc measures \( \frac{360}{8} = 45 \).

Then, \( m \overline{LNQ} = m \overline{LM} + m \overline{MO} + m \overline{NP} + m \overline{PO} = 5(45) = 225 \).

Therefore, \( m \angle LRQ = \frac{1}{2} (225) = 112.5 \).

35. \( m \angle LSR \)

**SOLUTION:**

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. So, \( m \angle LSR = \frac{1}{2} (m \overline{LSR}) \).

The sum of the measures of the central angles of a circle with no interior points in common is 360. Here, 360 is divided into 8 equal arcs and each arc measures \( \frac{360}{8} = 45 \).

Then, \( m \overline{LSR} = 6 \left( m \overline{LM} \right) = 6(45) = 270 \).

Therefore, \( m \angle LSR = \frac{1}{2} (270) = 135 \).

36. **ART** Four different string art star patterns are shown. If all of the inscribed angles of each star shown are congruent, find the measure of each inscribed angle.

a. 

b. 

c. 

d. 

**SOLUTION:**

a. The sum of the measures of the central angles of a circle with no interior points in common is 360. Here, 360 is divided into 5 equal arcs and each arc measures \( \frac{360}{5} = 72 \).

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. So, the measure of each inscribed angle is 36.

b. Here, 360 is divided into 6 equal arcs and each arc measures \( \frac{360}{6} = 60 \). Each inscribed angle is formed by intercepting alternate vertices of the star. So, each intercepted arc measures 120. If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. So, the measure of each inscribed angle is 60.

c. Here, 360 is divided into 7 equal arcs and each arc measures \( \frac{360}{7} \approx 51.43 \). If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. So, the measure of each inscribed angle is about 25.7.

d. The sum of the measures of the central angles of a circle with no interior points in common is 360. Here, 360 is divided into 8 equal arcs and each arc measures \( \frac{360}{8} = 45 \). Each inscribed angle is formed by intercepting alternate vertices of the star. So, each intercepted arc measures 90. If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. So, the measure of each inscribed angle is 45.
11-4 Inscribed Angles

PROOF Write a two-column proof for each case of Theorem 11.6.
37. Case 2

Given: \( P \) lies inside \( \angle ABC \). \( \overline{BD} \) is a diameter.

Prove: \( m\angle ABC = \frac{1}{2} m\overline{AC} \)

\[
\begin{align*}
&\text{PROOF:} \\
&\text{Proof:} \\
&\text{Statements (Reasons)} \\
&1. \ m\angle ABC = m\angle ABD + m\angle DBC \quad (\angle \text{ Addition Postulate}) \\
&2. \ m\angle ABD = \frac{1}{2} m(\text{arc } AD) \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad (\text{The measure of an inscribed } \angle \text{ whose side is a diameter is half the measure of the intercepted arc (Case 1)).} \\
&3. \ m\angle ABC = \frac{1}{2} m(\text{arc } AD) + \frac{1}{2} m(\text{arc } DC) \quad (\text{Substitution}) \\
&4. \ m\angle ABC = \frac{1}{2} [m(\text{arc } AD) + m(\text{arc } DC)] \quad (\text{Factor}) \\
&5. \ m(\text{arc } AD) + m(\text{arc } DC) = m(\text{arc } AC) \quad (\text{Arc Addition Postulate}) \\
&6. \ m\angle ABC = \frac{1}{2} m(\text{arc } AC) \quad (\text{Substitution}) \\
\end{align*}
\]

38. Case 3

Given: \( P \) lies outside \( \angle ABC \). \( \overline{BD} \) is a diameter.

Prove: \( m\angle ABC = \frac{1}{2} m\overline{AC} \)

\[
\begin{align*}
&\text{SOLUTION:} \\
&\text{Proof:} \\
&\text{Statements (Reasons)} \\
&1. \ m\angle ABC = m\angle DBC - m\angle DBA \quad (\angle \text{ Addition Postulate, Subtraction Property of Equality}) \\
&2. \ m\angle DBC = \frac{1}{2} m(\text{arc } DC) \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad (\text{The measure of an inscribed } \angle \text{ whose side is a diameter is half the measure of the intercepted arc (Case 1)).} \\
&3. \ m\angle ABC = \frac{1}{2} m(\text{arc } DC) - \frac{1}{2} m(\text{arc } DA) \quad (\text{Substitution}) \\
&4. \ m\angle ABC = \frac{1}{2} [m(\text{arc } DC) - m(\text{arc } DA)] \quad (\text{Factor}) \\
&5. \ m(\text{arc } DA) + m(\text{arc } AC) = m(\text{arc } DC) \quad (\text{Arc Addition Postulate}) \\
&6. \ m(\text{arc } AC) = m(\text{arc } DC) - m(\text{arc } DA) \quad (\text{Subtraction Property of Equality}) \\
&7. \ m\angle ABC = \frac{1}{2} m(\text{arc } AC) \quad (\text{Substitution}) \\
\end{align*}
\]
11-4 Inscribed Angles

PROOF Write the specified proof for each theorem.
39. Theorem 11.7, two-column proof

SOLUTION:

Given: \( \angle FAE \) and \( \angle CBD \) are inscribed; \( EF \cong DC \)
Prove: \( \angle FAE \cong \angle CBD \)
Proof:

Statements (Reasons)
1. \( \angle FAE \) and \( \angle CBD \) are inscribed; \( EF \cong DC \) (Given)
2. \( m\angle FAE = \frac{1}{2} mEF; \) \( m\angle CBD = \frac{1}{2} mDC \) (Measure of an inscribed \( \angle = \) half measure of intercepted arc.)
3. \( mEF = mDC \) (Def. of \( \cong \) arcs)
4. \( \frac{1}{2} mEF = \frac{1}{2} mDC \) (Mult. Prop. of Equality)
5. \( m\angle FAE = m\angle CBD \) (Substitution)
6. \( \angle FAE \cong \angle CBD \) (Def. of \( \cong \) \( \angle \)s)

40. Theorem 11.8, paragraph proof

SOLUTION:

Part I: Given: \( \overarc{ADC} \) is a semicircle.
Prove: \( \angle ABC \) is a right angle.
Proof: Since \( \overarc{ADC} \) is a semicircle, then \( m\overarc{ADC} = 180 \). Since \( \angle ABC \) is an inscribed angle, then \( m\angle ABC = \frac{1}{2} m\overarc{ADC} \) or 90. So, by definition, \( \angle ABC \) is a right angle.

Part II: Given: \( \angle ABC \) is a right angle.
Prove: \( \overarc{ADC} \) is a semicircle.
Proof: Since \( \angle ABC \) is an inscribed angle, then \( m\angle ABC = \frac{1}{2} m\overarc{ADC} \) and by the Multiplication Property of Equality, \( m\overarc{ADC} = 2m\angle ABC \). Because \( \angle ABC \) is a right angle, \( m\angle ABC = 90 \). Then \( m\overarc{ADC} = 2(90) \) or 180. So by definition, \( \overarc{ADC} \) is a semicircle.

41. MULTIPLE REPRESENTATIONS In this problem, you will investigate the relationship between the arcs of a circle that are cut by two parallel chords.

a. GEOMETRIC Use a compass to draw a circle with parallel chords \( AB \) and \( CD \). Connect points \( A \) and \( D \) by drawing segment \( AD \).

b. NUMERICAL Use a protractor to find \( m\angle A \) and \( m\angle D \). Then determine \( m\overarc{AC} \) and \( m\overarc{BD} \). What is true about these arcs? Explain.

c. VERBAL Draw another circle and repeat parts a and b. Make a conjecture about arcs of a circle that are cut by two parallel chords.

d. ANALYTICAL Use your conjecture to find \( m\overarc{PR} \) and \( m\overarc{QS} \) in the figure. Verify by using inscribed angles to find the measures of the arcs.
11-4 Inscribed Angles

SOLUTION:

a.

b. Sample answer: \( m_\angle A = 30 \), \( m_\angle D = 30 \);
\( m\overarc{AC} = 60 \), \( m\overarc{BD} = 60 \); The arcs are congruent
because they have equal measures.

c. Sample answer:

\[ m_\angle A = 15, m_\angle D = 15, m\overarc{BD} = 30, \text{ and } m\overarc{AC} = 30. \]
The arcs are congruent.
In a circle, two parallel chords cut congruent arcs.

40. triangle

SOLUTION:

The opposite angles of a triangle intercept a diameter, so each pair of opposite angles will be
 supplementary and inscribed in a circle. Therefore, the statement is always true.

41. parallelogram

SOLUTION:

The opposite angles of a parallelogram are always congruent. They will only be supplementary
when they are right angles. So, a parallelogram can be inscribed in a circle as long as it is a rectangle.
Therefore, the statement is sometimes true.

42. square

SOLUTION:

Squares have right angles at each vertex, therefore each pair of opposite angles will be supplementary
and inscribed in a circle. Therefore, the statement is always true.

43. rectangle

SOLUTION:

Rectangles have right angles at each vertex, therefore each pair of opposite angles will be
 supplementary and inscribed in a circle. Therefore, the statement is always true.

44. parallelogram

SOLUTION:

The opposite angles of a parallelogram are always congruent. They will only be supplementary when
they are right angles. So, a parallelogram can be inscribed in a circle as long as it is a rectangle.
Therefore, the statement is sometimes true.

45. rhombus

SOLUTION:

The opposite angles of a rhombus are always congruent. They will only be supplementary when
they are right angles. So, a rhombus can be inscribed in a circle as long as it is a square. Since the opposite
angles of rhombi that are not squares are not supplementary, they can not be inscribed in a circle.
Therefore, the statement is sometimes true.

46. kite

SOLUTION:

Exactly one pair of opposite angles of a kite are congruent. To be supplementary, they must each be
a right angle. If one pair of opposite angles for a quadrilateral is supplementary, the other pair must
also be supplementary. So, as long as the angles that compose the pair of congruent opposite angles are
right angles, a kite can be inscribed in a circle.
Therefore, the statement is sometimes true.
47. **CHALLENGE** A square is inscribed in a circle.

What is the ratio of the area of the circle to the area of the square?

**SOLUTION:**
A square with side \( s \) is inscribed in a circle with radius \( r \).

Using the Pythagorean Theorem,
\[
s^2 = r^2 + r^2
\]
\[
s^2 = 2r^2
\]
\[
s = r\sqrt{2}
\]
The area of a square of side \( s \) is \( A = s^2 \) and the area of a circle of radius \( r \) is \( A = \pi r^2 \).

\[
\frac{A_{\text{circle}}}{A_{\text{square}}} = \frac{\pi r^2}{s^2}
\]
\[
= \frac{\pi r^2}{(r\sqrt{2})^2}
\]
\[
= \frac{\pi}{2}
\]
Therefore, the ratio of the area of the circle to the area of the inscribed square is \( \frac{\pi}{2} \).

48. **WRITING IN MATH** A \( 45^\circ - 45^\circ - 90^\circ \) right triangle is inscribed in a circle. If the radius of the circle is given, explain how to find the lengths of the right triangle’s legs.

**SOLUTION:**
Sample answer: A \( 45^\circ - 45^\circ - 90^\circ \) triangle will have two inscribed angles of 45 and one of 90. The hypotenuse is across from the 90, or right angle. According to theorem 10.8, an inscribed angle of a triangle intercepts a diameter if the angle is a right angle. Therefore, the hypotenuse is a diameter and has a length of \( 2r \). Use trigonometry to find the length of the equal legs.

\[
\sin 45 = \frac{\text{leg}}{\text{hyp}} \quad \sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \theta = 45, \text{hyp} = 2r
\]

\[
\frac{\sqrt{2}}{2} = \frac{\text{leg}}{2r}
\]

\[
\frac{\sqrt{2}(2r)}{2} = \text{leg} \quad \text{Multiply each side by 2r}.
\]

\[
\sqrt{2}r = \text{leg} \quad \text{Simplify}.
\]

Therefore, the length of each leg of the \( 45^\circ - 45^\circ - 90^\circ \) triangle can be found by multiplying the radius of the circle in which it is inscribed by \( \sqrt{2} \).

49. **OPEN ENDED** Find and sketch a real-world logo with an inscribed polygon.

**SOLUTION:**
See students’ work.

50. **WRITING IN MATH** Compare and contrast inscribed angles and central angles of a circle. If they intercept the same arc, how are they related?

**SOLUTION:**
An inscribed angle has its vertex on the circle. A central angle has its vertex at the center of the circle. If a central angle intercepts \( \text{arc } AB \), then the measure of the central angle is equal to \( m(\text{arc } AB) \). If an inscribed angle also intercepts \( \text{arc } AB \), then the measure of the inscribed angle is equal to \( \frac{1}{2} m(\text{arc } AB) \). So, if an inscribed angle and a central angle intercept the same arc, then the measure of the inscribed angle is one-half the measure of the central angle.
11-4 Inscribed Angles

51. In the circle below, \( m\overline{AC} = 160 \) and \( m\angle BEC = 38 \). What is \( m\angle AEB \)?

\[
\text{A}\ 42 \\
\text{B}\ 61 \\
\text{C}\ 80 \\
\text{D}\ 84
\]

**SOLUTION:**

If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc. Therefore,

\[ m\angle AEC = \frac{1}{2}(m\overline{AC}) = 80. \]

But, \( m\angle AEC = m\angle BEC + m\angle AEB \).

Therefore, \( m\angle AEB = 80 - 38 = 42 \).

The correct choice is A.

52. **ALGEBRA** Simplify

\[ 4(3x - 2)(2x + 4) + 3x^2 + 5x - 6. \]

\[
\begin{align*}
\text{F}\ & 9x^2 + 3x - 14 \\
\text{G}\ & 9x^2 + 13x - 14 \\
\text{H}\ & 27x^2 + 37x - 38 \\
\text{J}\ & 27x^2 + 27x - 26
\end{align*}
\]

**SOLUTION:**

Use the Distributive Property to simplify the first term.

\[
4(3x - 2)(2x + 4) + 3x^2 + 5x - 6
\]

\[
= 4\left(6x^2 + 12x - 4x - 8\right) + 3x^2 + 5x - 6
\]

\[
= 24x^2 + 32x - 32 + 3x^2 + 5x - 6
\]

\[
= 27x^2 + 37x - 38
\]

Therefore, the correct choice is H.

53. **SHORT RESPONSE** In the circle below, \( \overline{AB} \) is a diameter, \( AC = 8 \) inches, and \( BC = 15 \) inches. Find the diameter, the radius, and the circumference of the circle.

\[
\text{A}
\]

\[
\text{B}
\]

\[
\text{C}
\]

\[
\text{D}
\]

**SOLUTION:**

Use the Pythagorean Theorem to find the hypotenuse of the triangle which is also the diameter of the circle.

\[ AB = \sqrt{8^2 + 15^2} = \sqrt{289} \approx 17 \text{ in.} \]

The radius is half the diameter. So, the radius of the circle is about 8.5 in.

The circumference \( C \) of a circle of a circle of diameter \( d \) is given by \( C = \pi d \).

Therefore, the circumference of the circle is \( C = \pi(17) \approx 53.4 \text{ in.} \)

54. **SAT/ACT** The sum of three consecutive integers is 48. What is the least of the three integers?

\[
\text{A} -15 \\
\text{B} -16 \\
\text{C} -17 \\
\text{D} -18 \\
\text{E} -19
\]

**SOLUTION:**

Let the three consecutive integers be \( x \), \( x + 1 \), and \( x + 2 \). Then,

\[
\begin{align*}
\text{Sum of numbers} & = 48 \\
3x + 3 & = 48 \\
3x & = 45 \\
x & = 15
\end{align*}
\]

So, the three integers are \(-17\), \(-16\), and \(-15\) and the least of these is \(-17\).

Therefore, the correct choice is \( \text{C} \).
11-4 Inscribed Angles

In \( \odot M \), \( FL = 24 \), \( HJ = 48 \), and \( m\widehat{HP} = 65 \). Find each measure.

55. \( FG \)

**SOLUTION:**
If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc. So, \( ML \) bisects \( FG \). Therefore, \( FG = 2(FL) = 48 \) units.

56. \( m\widehat{PJ} \)

**SOLUTION:**
If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc. So, \( MN \) bisects \( HPJ \). Therefore, \( m\widehat{PJ} = m\widehat{HP} = 65 \).

57. \( NJ \)

**SOLUTION:**
If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc. So, \( MN \) bisects \( HJ \). Therefore, \( NJ = \frac{1}{2}(HJ) = 24 \) units.

58. \( m\widehat{HJ} \)

**SOLUTION:**
If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc. So, \( MN \) bisects \( HPJ \). Therefore, \( m\widehat{HJ} = 2(\widehat{HP}) = 130 \).

Find \( x \).

59.

**SOLUTION:**
The sum of the measures of the central angles of a circle with no interior points in common is 360. So, \( 135 + 118 + x = 360 \).

\[ 235 + x = 360 \]
\[ x = 107 \]

60.

**SOLUTION:**
The sum of the measures of the central angles of a circle with no interior points in common is 360. So, \( 24 + 84 + 90 + x = 360 \).

\[ 198 + x = 360 \]
\[ x = 162 \]

61.

**SOLUTION:**
The sum of the measures of the central angles of a circle with no interior points in common is 360. So, \( x + x + 36 + 36 = 360 \).

\[ 2x + 72 = 360 \]
\[ 2x = 288 \]
\[ x = 144 \]
62. PHOTOGRAPHY In one of the first cameras invented, light entered an opening in the front. An image was reflected in the back of the camera, upside down, forming similar triangles. Suppose the image of the person on the back of the camera is 12 inches, the distance from the opening to the person is 7 feet, and the camera itself is 15 inches long. How tall is the person being photographed?

**SOLUTION:**
The height of the person being photographed is the length of the base of the larger triangle. The two triangles are similar. So, their corresponding sides will be proportional. One foot is equivalent to 12 in. Then, the height of the larger triangle is \(7(12) = 84\) in. Let \(x\) be the length of the base of the larger triangle.

\[
\frac{x}{84} = \frac{12}{15}
\]

\[15x = 1008\]

\[x = 67.2\text{ in.} \text{ or } 5.6\text{ ft.}\]

Therefore, the person being photographed is 5.6 ft tall.

**ALGEBRA** Suppose \(B\) is the midpoint of \(AC\).

Use the given information to find the missing measure.

63. \(AB = 4x - 5, \ BC = 11 + 2x, \ AC = ?\)

**SOLUTION:**
Since \(B\) is the midpoint of \(AC, AB = BC\).

\[4x - 5 = 11 + 2x\]

\[2x = 16\]

\[x = 8\]

\[AC = 2(4(8) - 5)\]

\[= 2(27)\]

\[= 54\]

64. \(AB = 6y - 14, BC = 10 - 2y, \ AC = ?\)

**SOLUTION:**
Since \(B\) is the midpoint of \(AC, AB = BC\).

\[6y - 14 = 10 - 2y\]

\[8y = 24\]

\[y = 3\]

\[AC = 2(6(3) - 14)\]

\[= 8\]

65. \(BC = 6 - 4m, \ AC = 8, \ m = ?\)

**SOLUTION:**
Since \(B\) is the midpoint of \(AC, AC = 2(BC)\).

\[8 = 2(6 - 4m)\]

\[-4 = -8m\]

\[\frac{1}{2} = m\]

66. \(AB = 10s + 2, \ AC = 40, \ s = ?\)

**SOLUTION:**
Since \(B\) is the midpoint of \(AC, AC = 2(AC)\).

\[40 = 2(10s + 2)\]

\[36 = 20s\]

\[1.8 = s\]