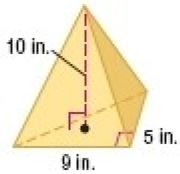
12-5 Volumes of Pyramids and Cones

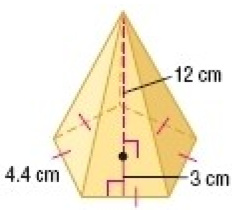
Find the volume of each pyramid.

1. 

SOLUTION:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the pyramid. The base of this pyramid is a right triangle with legs of 9 inches and 5 inches and the height of the pyramid is 10 inches.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}\left(\frac{1}{2}\times 9\times 5\right)10 \\ &= \frac{1}{3}\left(\frac{45}{2}\right)10 \\ &= 75\ \text{in}^3 \end{aligned}$$

2. 

SOLUTION:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the pyramid. The base of this pyramid is a regular pentagon with sides of 4.4 centimeters and an apothem of 3 centimeters. The height of the pyramid is 12 centimeters.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}\left(\frac{1}{2}aP\right)12 \\ &= \frac{1}{3}\left(\frac{1}{2}\times 3\times(5\times 4.4)\right)12 \\ &= 132\ \text{cm}^3 \end{aligned}$$

3. a rectangular pyramid with a height of 5.2 meters and a base 8 meters by 4.5 meters

SOLUTION:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the pyramid. The base of this pyramid is a rectangle with a length of 8 meters and a width of 4.5 meters. The height of the pyramid is 5.2 meters.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(8\times 4.5)5.2 \\ &= 62.4\ \text{m}^3 \end{aligned}$$

4. a square pyramid with a height of 14 meters and a base with 8-meter side lengths

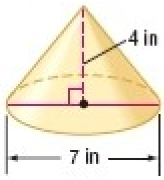
SOLUTION:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the pyramid. The base of this pyramid is a square with sides of 8 meters. The height of the pyramid is 14 meters.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(8\times 8)14 \\ &\approx 298.7\,\text{m}^3 \end{aligned}$$

12-5 Volumes of Pyramids and Cones

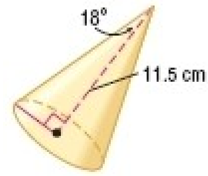
Find the volume of each cone. Round to the nearest tenth.

5. 

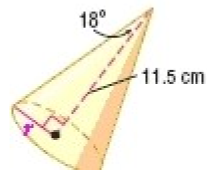
SOLUTION:

The volume of a circular cone is $V = \frac{1}{3}Bh$, or $V = \frac{1}{3}\pi r^2 h$, where *B* is the area of the base, *h* is the height of the cone, and *r* is the radius of the base. Since the diameter of this cone is 7 inches, the radius is $\frac{7}{2}$ or 3.5 inches. The height of the cone is 4 inches.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(3.5)^2(4) \\ &\approx 51.3 \text{ in}^3 \end{aligned}$$

6. 

SOLUTION:



Use trigonometry to find the radius *r*.

$$\begin{aligned} \tan 18 &= \frac{r}{11.5} \\ r &= 11.5 \tan 18 \end{aligned}$$

The volume of a circular cone is $V = \frac{1}{3}Bh$, or $V = \frac{1}{3}\pi r^2 h$, where *B* is the area of the base, *h* is the height of the cone, and *r* is the radius of the base. The height of the cone is 11.5 centimeters.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(11.5 \tan 18)^2(11.5) \\ &\approx 168.1 \text{ cm}^3 \end{aligned}$$

7. an oblique cone with a height of 10.5 millimeters and a radius of 1.6 millimeters

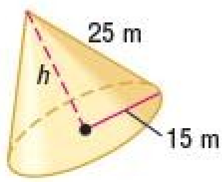
SOLUTION:

The volume of a circular cone is $V = \frac{1}{3}Bh$, or $V = \frac{1}{3}\pi r^2 h$, where *B* is the area of the base, *h* is the height of the cone, and *r* is the radius of the base. The radius of this cone is 1.6 millimeters and the height is 10.5 millimeters.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(1.6)^2(10.5) \\ &\approx 28.1 \text{mm}^3 \end{aligned}$$

8. a cone with a slant height of 25 meters and a radius of 15 meters

SOLUTION:



Use the Pythagorean Theorem to find the height *h* of the cone. Then find its volume.

$$\begin{aligned} h &= \sqrt{25^2 - 15^2} \\ &= \sqrt{625 - 225} \\ &= \sqrt{400} \\ &= 20 \end{aligned}$$

So, the height of the cone is 20 meters.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(15)^2\, 20 \\ &\approx 4712.4 \text{ m}^3 \end{aligned}$$

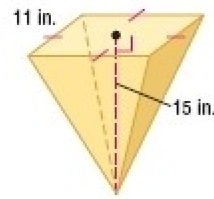
9. **MUSEUMS** The sky dome of the National Corvette Museum in Bowling Green, Kentucky, is a conical building. If the height is 100 feet and the area of the base is about 15,400 square feet, find the volume of air that the heating and cooling systems would have to accommodate. Round to the nearest tenth.

SOLUTION:

The volume of a circular cone is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the cone.
For this cone, the area of the base is 15,400 square feet and the height is 100 feet.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(15{,}400)100 \\ &\approx 513{,}333.3 \text{ ft}^3 \end{aligned}$$

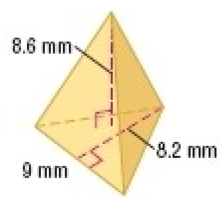
CCSS SENSE-MAKING Find the volume of each pyramid. Round to the nearest tenth if necessary.

10. 

SOLUTION:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the pyramid.

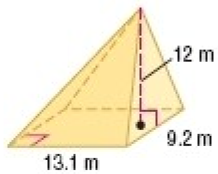
$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(11 \times 11)15 \\ &= 605 \text{ in}^3 \end{aligned}$$

11. 

SOLUTION:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the pyramid.

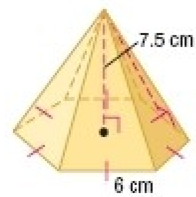
$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}\left(\frac{1}{2} \times 9 \times 8.2\right)8.6 \\ &= 105.8 \text{ mm}^3 \end{aligned}$$

12. 

SOLUTION:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the pyramid.

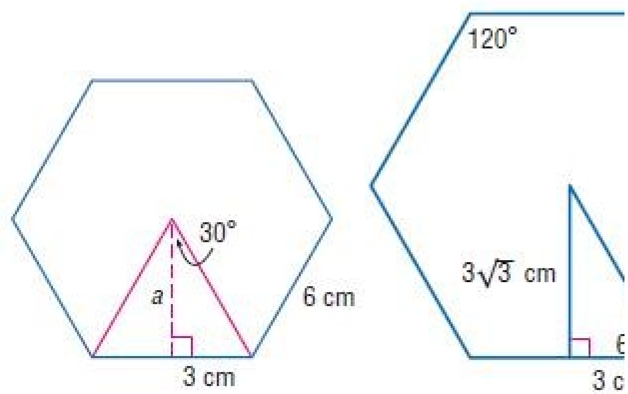
$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(13.1 \times 9.2)12 \\ &= 482.1 \text{ m}^3 \end{aligned}$$

13. 

SOLUTION:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where B is the base and h is the height of the pyramid.

The base is a hexagon, so we need to make a right tri
determine the apothem. The interior angles of the he
120°. The radius bisects the angle, so the right triangl
90° triangle.



The apothem is $3\sqrt{3}$.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}\left(\frac{1}{2}aP\right)12 \\ &= \frac{1}{3}\left(\frac{1}{2} \times 3\sqrt{3} \times (6 \times 6)\right)7.5 \\ &\approx 233.8 \text{ cm}^3 \end{aligned}$$

14. a pentagonal pyramid with a base area of 590 square feet and an altitude of 7 feet

SOLUTION:

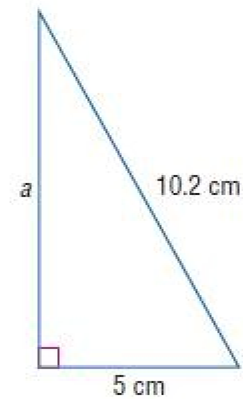
The volume of a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the pyramid.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(590)7 \\ &= 1376.7 \text{ ft}^3 \end{aligned}$$

15. a triangular pyramid with a height of 4.8 centimeters and a right triangle base with a leg 5 centimeters and hypotenuse 10.2 centimeters

SOLUTION:

Find the height of the right triangle.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 5^2 &= 10.2^2 \\ a^2 &= 10.2^2 - 5^2 \\ a &= \sqrt{10.2^2 - 5^2} \\ a &\approx 8.9 \end{aligned}$$

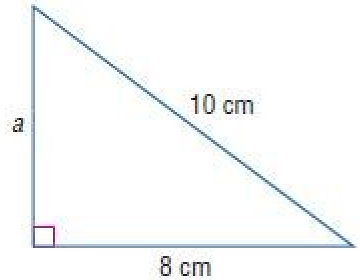
The volume of a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the pyramid.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &\approx \frac{1}{3}\left(\frac{1}{2} \times 5 \times 8.9\right)4.8 \\ &= 35.6 \text{ cm}^3 \end{aligned}$$

16. A triangular pyramid with a right triangle base with a leg 8 centimeters and hypotenuse 10 centimeters has a volume of 144 cubic centimeters. Find the height.

SOLUTION:

The base of the pyramid is a right triangle with a leg of 8 centimeters and a hypotenuse of 10 centimeters. Use the Pythagorean Theorem to find the other leg *a* of the right triangle and then find the area of the triangle.



$$a^2 + b^2 = c^2$$
$$a^2 + 8^2 = 10^2$$
$$a^2 = 100 - 64$$
$$a = \sqrt{36} \text{ or } 6$$

The length of the other leg of the right triangle is 6 cm.

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(8)(6)$$
$$= 24$$

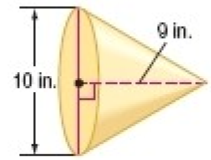
So, the area of the base *B* is 24 cm^2.

Replace *V* with 144 and *B* with 24 in the formula for the volume of a pyramid and solve for the height *h*.

$$V = \frac{1}{3}Bh$$
$$144 = \frac{1}{3}(24)h$$
$$144 = 8h$$
$$18 = h$$

Therefore, the height of the triangular pyramid is 18 cm.

Find the volume of each cone. Round to the nearest tenth.

17. 

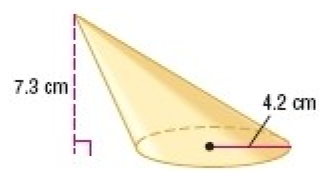
SOLUTION:

The volume of a circular cone is $V = \frac{1}{3}\pi r^2 h$, where *r* is the radius of the base and *h* is the height of the cone.

Since the diameter of this cone is 10 inches, the radius is $\frac{10}{2}$ or 5 inches. The height of the cone is 9 inches.

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(5)^2(9)$$
$$\approx 235.6$$

Therefore, the volume of the cone is about 235.6 in^3.

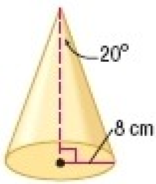
18. 

SOLUTION:

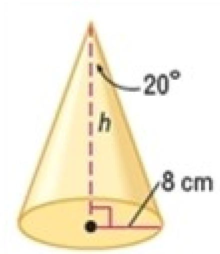
The volume of a circular cone is $V = \frac{1}{3}\pi r^2 h$, where *r* is the radius of the base and *h* is the height of the cone. The radius of this cone is 4.2 centimeters and the height is 7.3 centimeters.

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(4.2)^2(7.3)$$
$$\approx 134.8$$

Therefore, the volume of the cone is about 134.8 cm^3.

19. 

SOLUTION:



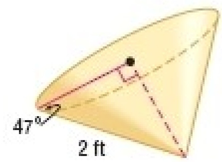
Use a trigonometric ratio to find the height h of the cone.

$$\tan 20 = \frac{8}{h}$$
$$h = \frac{8}{\tan 20}$$

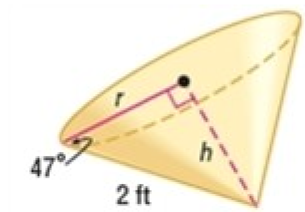
The volume of a circular cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height of the cone. The radius of this cone is 8 centimeters.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi (8)^2 \left(\frac{8}{\tan 20}\right) \\ &\approx 1473.1 \end{aligned}$$

Therefore, the volume of the cone is about 1473.1 cm^3.

20. 

SOLUTION:



Use trigonometric ratios to find the height h and the radius r of the cone.

$$\sin 47 = \frac{h}{2}$$
$$h = 2 \sin 47$$
$$\cos 47 = \frac{r}{2}$$
$$r = 2 \cos 47$$

The volume of a circular cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height of the cone.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi (2\cos 47)^2 (2\sin 47) \\ &\approx 2.8 \end{aligned}$$

Therefore, the volume of the cone is about 2.8 ft^3.

21. an oblique cone with a diameter of 16 inches and an altitude of 16 inches

SOLUTION:

The volume of a circular cone is $V = \frac{1}{3}\pi r^2 h$, where *r* is the radius of the base and *h* is the height of the cone. Since the diameter of this cone is 16 inches, the radius is $\frac{16}{2}$ or 8 inches.

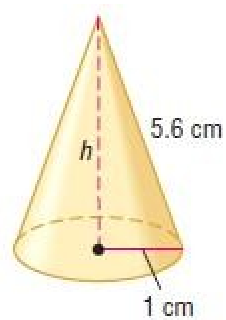
$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(8)^2(16) \\ &\approx 1072.3 \end{aligned}$$

Therefore, the volume of the cone is about 1072.3 in^3.

22. a right cone with a slant height of 5.6 centimeters and a radius of 1 centimeter

SOLUTION:

The cone has a radius *r* of 1 centimeter and a slant height of 5.6 centimeters. Use the Pythagorean Theorem to find the height *h* of the cone.



$$\begin{aligned} h^2 + 1^2 &= 5.6^2 \\ h^2 &= 31.36 - 1 \\ h &= \sqrt{30.36} \end{aligned}$$

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(1)^2\left(\sqrt{30.36}\right) \\ &\approx 5.8 \end{aligned}$$

Therefore, the volume of the cone is about 5.8 cm^3.

23. **SNACKS** Approximately how many cubic centimeters of roasted peanuts will completely fill a paper cone that is 14 centimeters high and has a base diameter of 8 centimeters? Round to the nearest tenth.

SOLUTION:

The volume of a circular cone is $V = \frac{1}{3}\pi r^2 h$, where *r* is the radius of the base and *h* is the height of the cone. Since the diameter of the cone is 8 centimeters, the radius is $\frac{8}{2}$ or 4 centimeters. The height of the cone is 14 centimeters.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(4)^2(14) \\ &\approx 234.6 \end{aligned}$$

Therefore, the paper cone will hold about 234.6 cm^3 of roasted peanuts.

24. **CCSS MODELING** The Pyramid Arena in Memphis, Tennessee, is the third largest pyramid in the world. It is approximately 350 feet tall, and its square base is 600 feet wide. Find the volume of this pyramid.

SOLUTION:

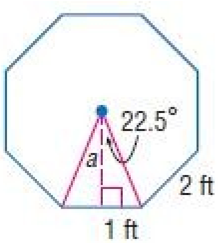
The volume of a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the pyramid.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(600 \times 600)350 \\ &= 42{,}000{,}000 \text{ ft}^3 \end{aligned}$$

25. **GARDENING** The greenhouse is a regular octagonal pyramid with a height of 5 feet. The base has side lengths of 2 feet. What is the volume of the greenhouse?

SOLUTION:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the pyramid. The base of the pyramid is a regular octagon with sides of 2 feet. A central angle of the octagon is $\frac{360°}{8}$ or 45°, so the angle formed in the triangle below is 22.5°.



Use a trigonometric ratio to find the apothem a.

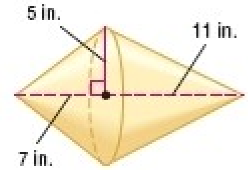
$$\tan 22.5 = \frac{1}{a}$$
$$a = \frac{1}{\tan 22.5}$$

The height of this pyramid is 5 feet.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}\left(\frac{1}{2}aP\right)h \\ &= \frac{1}{3}\left[\frac{1}{2}\left(\frac{1}{\tan 22.5}\right)(8 \times 2)\right](5) \\ &\approx 32.2 \end{aligned}$$

Therefore, the volume of the greenhouse is about 32.2 ft^3.

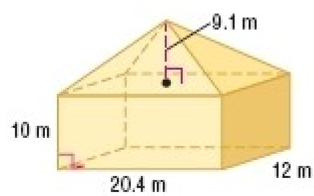
Find the volume of each solid. Round to the nearest tenth.

26. 

SOLUTION:

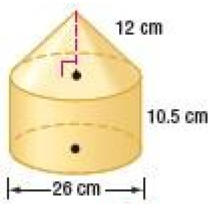
Volume of the solid given = Volume of the small cone + Volume of the large cone

$$\begin{aligned} V(\text{solid}) &= V(\text{small}) + V(\text{large}) \\ &= \frac{1}{3}\pi(5)^2 7 + \frac{1}{3}\pi(5)^2 11 \\ &= \frac{25}{3}\pi(7 + 11) \\ &= \frac{25}{3}\pi(18) \\ &= 150\pi \\ &\approx 471.2 \text{ in}^3 \end{aligned}$$

27. 

SOLUTION:

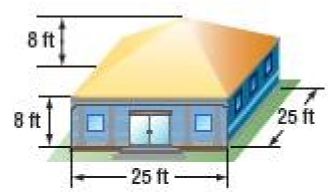
$$\begin{aligned} \text{V(solid)} &= \text{V(prism)} + \text{V(pyramid)} \\ &= Bh + \frac{1}{3}Bh_1 \\ &= 20.4 \times 12 \times 10 + \frac{1}{3}(20.4 \times 12)9.1 \\ &= (20.4 \times 12)\left(10 + \frac{9.1}{3}\right) \\ &\approx 3190.6 \text{ m}^3 \end{aligned}$$

28. 

SOLUTION:

$$\begin{aligned} V(\text{solid}) &= V(\text{cylinder}) + V(\text{cone}) \\ &= \pi r^2 h + \tfrac{1}{3}\pi r^2 h \\ &= \pi(13)^2(10.5) + \tfrac{1}{3}\pi(13)^2(12) \\ &= \pi(169)(10.5) + 4\pi(169) \\ &\approx 7698.5 \end{aligned}$$

29. **HEATING** Sam is building an art studio in her backyard. To buy a heating unit for the space, she needs to determine the BTUs (British Thermal Units) required to heat the building. For new construction with good insulation, there should be 2 BTUs per cubic foot. What size unit does Sam need to purchase?



SOLUTION:

The building can be broken down into the rectangular base and the pyramid ceiling. The volume of the base is

$$\begin{aligned} V &= l \cdot w \cdot h \\ &= (25)(25)(8) \\ &= 5000 \text{ ft}^3 \end{aligned}$$

The volume of the ceiling is

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(25)(25)(8) \\ &= 1666.67 \text{ ft}^3 \end{aligned}$$

The total volume is therefore 5000 + 1666.67 = 6666.67 ft^3. Two BTU's are needed for every cubic foot, so the size of the heating unit Sam should buy is 6666.67 × 2 = 13,333 BTUs.

30. **SCIENCE** Refer to page 825. Determine the volume of the model. Explain why knowing the volume is helpful in this situation.

SOLUTION:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the pyramid.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}\left(1 \times 1\frac{1}{2}\right)4 \\ &= 2 \text{ in}^3 \end{aligned}$$

It tells Marta how much clay is needed to make the model.

31. **CHANGING DIMENSIONS** A cone has a radius of 4 centimeters and a height of 9 centimeters. Describe how each change affects the volume of the cone.

a. The height is doubled.

b. The radius is doubled.

c. Both the radius and the height are doubled.

SOLUTION:

Find the volume of the original cone. Then alter the values.

$$\begin{aligned} V &= \tfrac{1}{3}\pi r^2 h \\ &= \tfrac{1}{3}\left[\pi(4)^2(9)\right] \\ &= \frac{144\pi}{3} \\ &= 48\pi \end{aligned}$$

a. Double h.

$$\begin{aligned} V &= \tfrac{1}{3}\pi r^2 h \\ &= \tfrac{1}{3}\left[\pi(4)^2(18)\right] \\ &= \frac{288\pi}{3} \\ &= 96\pi \end{aligned}$$

The volume is doubled.

b. Double r.

$$\begin{aligned} V &= \tfrac{1}{3}\pi r^2 h \\ &= \tfrac{1}{3}\left[\pi(8)^2(9)\right] \\ &= \frac{576\pi}{3} \\ &= 192\pi \end{aligned}$$

The volume is multiplied by 2^2 or 4.

c. Double r and h.

$$\begin{aligned} V &= \tfrac{1}{3}\pi r^2 h \\ &= \tfrac{1}{3}\left[\pi(8)^2(18)\right] \\ &= \frac{1152\pi}{3} \\ &= 384\pi \end{aligned}$$

volume is multiplied by 2^3 or 8.

Find each measure. Round to the nearest tenth if necessary.

32. A square pyramid has a volume of 862.5 cubic centimeters and a height of 11.5 centimeters. Find the side length of the base.

SOLUTION:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the pyramid. Let *s* be the side length of the base.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ 862.5 &= \frac{1}{3}(s \times s)11.5 \\ \frac{862.5}{11.5} &= \frac{1}{3}s^2 \\ 75 &= \frac{1}{3}s^2 \\ 225 &= s^2 \\ 15 &= s \end{aligned}$$

The side length of the base is 15 cm.

33. The volume of a cone is 196π cubic inches and the height is 12 inches. What is the diameter?

SOLUTION:

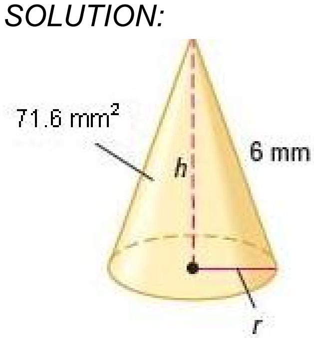
The volume of a circular cone is $V = \frac{1}{3}Bh$, or $V = \frac{1}{3}\pi r^2 h$, where *B* is the area of the base, *h* is the height of the cone, and *r* is the radius of the base. Since the diameter is 8 centimeters, the radius is 4 centimeters.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ 196\pi &= \frac{1}{3}\pi (r)^2 12 \\ 196\pi &= \pi (r)^2 4 \\ 196 &= 4r^2 \\ 49 &= r^2 \\ 7 &= r \end{aligned}$$

The diameter is 2(7) or 14 inches.

34. The lateral area of a cone is 71.6 square millimeters and the slant height is 6 millimeters. What is the volume of the cone?

SOLUTION:



The lateral area of a cone is $L = \pi r \ell$, where *r* is the radius and ℓ is the slant height of the cone. Replace *L* with 71.6 and ℓ with 6, then solve for the radius *r*.

$$\begin{aligned} 71.6 &= \pi r(6) \\ \frac{71.6}{6\pi} &= r \\ 3.8 &\approx r \end{aligned}$$

So, the radius is about 3.8 millimeters.

Use the Pythagorean Theorem to find the height of the cone.

$$
\begin{aligned}
h^2 + (3.8)^2 &= 6^2 \\
h^2 &= 36 - 14.44 \\
h &= \sqrt{21.56} \\
h &\approx 4.64
\end{aligned}
$$

So, the height of the cone is about 4.64 millimeters.

The volume of a circular cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height of the cone.

$$
\begin{aligned}
V &= \frac{1}{3}\pi r^2 h \\
&\approx \frac{1}{3}\pi(3.8)^2(4.64) \\
&\approx 70.2
\end{aligned}
$$

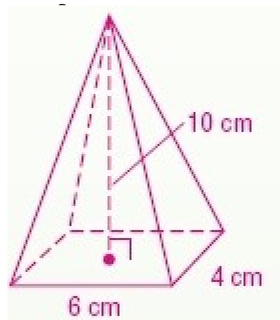
Therefore, the volume of the cone is about 70.2 mm^3.

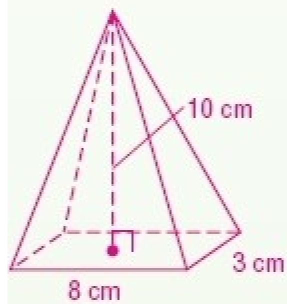
35. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate rectangular pyramids.
a. GEOMETRIC Draw two pyramids with different bases that have a height of 10 centimeters and a base area of 24 square centimeters.
b. VERBAL What is true about the volumes of the two pyramids that you drew? Explain.
c. ANALYTICAL Explain how multiplying the base area and/or the height of the pyramid by 5 affects the volume of the pyramid.

SOLUTION:

a. Use rectangular bases and pick values that multiply to make 24.

Sample answer:





b. The volumes are the same. The volume of a pyramid equals one third times the base area times the height. So, if the base areas of two pyramids are equal and their heights are equal, then their volumes are equal.

$$
\begin{aligned}
V &= \frac{1}{3}Bh \\
&= \frac{1}{3}(6)(4)(10) \\
&= 80
\end{aligned}
$$

$$
\begin{aligned}
V &= \frac{1}{3}Bh \\
&= \frac{1}{3}(3)(8)(10) \\
&= 80
\end{aligned}
$$

c. If the base area is multiplied by 5, the volume is multiplied by 5. If the height is multiplied by 5, the volume is multiplied by 5. If both the base area and the height are multiplied by 5, the volume is multiplied by $5 \cdot 5$ or 25.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(5[(3)(8)])(10) \\ &= \frac{1}{3}(120)(10) \\ &= 400 \\ &= 80 \times 5 \end{aligned}$$

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(3)(8)[5(10)] \\ &= \frac{1}{3}(24)(50) \\ &= 400 \\ &= 80 \times 5 \end{aligned}$$

$$\begin{aligned} V &= \frac{1}{3}Bh \\ &= \frac{1}{3}(5[(3)(8)])[5(10)] \\ &= \frac{1}{3}(120)(50) \\ &= 2000 \\ &= 80 \times 25 \end{aligned}$$

36. **CCSS ARGUMENTS** Determine whether the following statement is sometimes, always, or never true. Justify your reasoning.
The volume of a cone with radius r and height h equals the volume of a prism with height h.

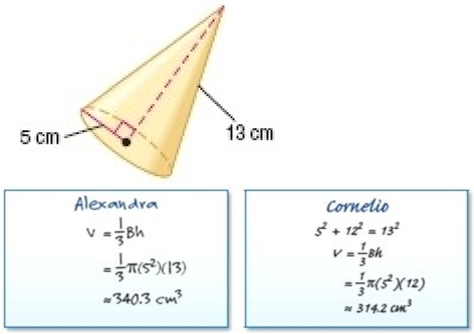
SOLUTION:

The volume of a cone with a radius *r* and height *h* is $V = \frac{1}{3}\pi r^2 h$. The volume of a prism with a height of *h* is $V = Bh$ where *B* is the area of the base of the prism. Set the volumes equal.

$$\begin{aligned} V_{\text{cone}} &= V_{\text{prism}} \\ \frac{1}{3}\pi r^2 h &= Bh \\ \frac{1}{3}\pi r^2 &= B \\ \pi r^2 &= 3B \\ r^2 &= \frac{3B}{\pi} \\ r &= \sqrt{\frac{3B}{\pi}} \end{aligned}$$

The volumes will only be equal when the radius of the cone is equal to $\sqrt{\frac{3B}{\pi}}$ or when $\pi r^2 = 3B$. Therefore, the statement is true *sometimes* if the base area of the cone is 3 times as great as the base area of the prism. For example, if the base of the prism has an area of 10 square units, then its volume is 10*h* cubic units. So, the cone must have a base area of 30 square units so that its volume is $\frac{1}{3}(30)h$ or 10*h* cubic units.

37. **ERROR ANALYSIS** Alexandra and Cornelio are calculating the volume of the cone below. Is either of them correct? Explain your answer.



SOLUTION:

The slant height is used for surface area, but the height is used for volume. For this cone, the slant height of 13 is provided, and we need to calculate the height before we can calculate the volume.

Alexandra incorrectly used the slant height.

38. **REASONING** A cone has a volume of 568 cubic centimeters. What is the volume of a cylinder that has the same radius and height as the cone? Explain your reasoning.

SOLUTION:

1704 cm^3; The formula for the volume of a cylinder is $V = Bh$, while the formula for the volume of a cone is $V = \frac{1}{3}Bh$. The volume of a cylinder is three times as much as the volume of a cone with the same radius and height.

39. **OPEN ENDED** Give an example of a pyramid and a prism that have the same base and the same volume. Explain your reasoning.

SOLUTION:

The formula for volume of a prism is $V = Bh$ and the formula for the volume of a pyramid is one-third of that. So, if a pyramid and prism have the same base, then in order to have the same volume, the height of the pyramid must be 3 times as great as the height of the prism.

$$V_{prism} = V_{pyramid}$$

$$Bh_{prism} = \frac{1}{3}Bh_{pyramid}$$

$$h_{prism} = \frac{1}{3}h_{pyramid}$$

Set the base areas of the prism and pyramid, and make the height of the pyramid equal to 3 times the height of the prism.

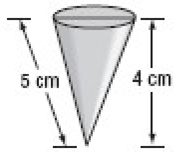
Sample answer:
A square pyramid with a base area of 16 and a height of 12, a prism with a square base area of 16 and a height of 4.

40. **WRITING IN MATH** Compare and contrast finding volumes of pyramids and cones with finding volumes of prisms and cylinders.

SOLUTION:

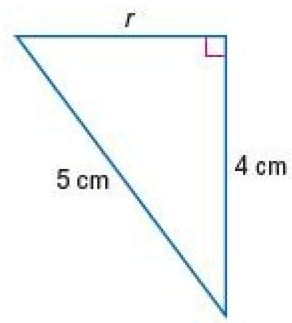
To find the volume of each solid, you must know the area of the base and the height. The volume of a pyramid is one third the volume of a prism that has the same height and base area. The volume of a cone is one third the volume of a cylinder that has the same height and base area.

41. A conical sand toy has the dimensions as shown below. How many cubic centimeters of sand will it hold when it is filled to the top?



A 12π
B 15π
C $\frac{80}{3}\pi$
D $\frac{100}{3}\pi$

SOLUTION:
Use the Pythagorean Theorem to find the radius *r* of the cone.



$$r^2 + 4^2 = 5^2$$
$$r^2 = 25 - 16$$
$$r = \sqrt{9} \text{ or } 3$$

So, the radius of the cone is 3 centimeters.

The volume of a circular cone is $V = \frac{1}{3}\pi r^2 h$, where *r* is the radius of the base and *h* is the height of the cone.

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(3)^2(4)$$
$$= 12\pi$$

Therefore, the correct choice is A.

42. **SHORT RESPONSE** Brooke is buying a tent that is in the shape of a rectangular pyramid. The base is 6 feet by 8 feet. If the tent holds 88 cubic feet of air, how tall is the tent's center pole?

SOLUTION:

The volume of a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the pyramid.

$$V = \frac{1}{3}Bh$$
$$88 = \frac{1}{3}(6 \times 8)h$$
$$88 = 16h$$
$$\frac{88}{16} = h$$
$$5.5 = h$$

43. **PROBABILITY** A spinner has sections colored red, blue, orange, and green. The table below shows the results of several spins. What is the experimental probability of the spinner landing on orange?

Color	Frequency
red	6
blue	4
orange	5
green	10

F $\frac{1}{5}$

G $\frac{1}{4}$

H $\frac{9}{25}$

J $\frac{1}{2}$

SOLUTION:

Possible outcomes: {6 red, 4 blue, 5 orange, 10 green}
Number of possible outcomes : 25
Favorable outcomes: {5 orange}
Number of favorable outcomes: 5

$$\begin{aligned} P(\text{orange}) &= \frac{\#\text{favorable}}{\#\text{possible}} \\ &= \frac{5}{25} \\ &= \frac{1}{5} \end{aligned}$$

So, the correct choice is F.

44. **SAT/ACT** For all $x \neq -2$ or $0, \frac{x^2 - 2x - 8}{x^2 + 2x} = ?$

A –8

B $x – 4$

C $\frac{-x-4}{x}$

D $\frac{-8}{x+2}$

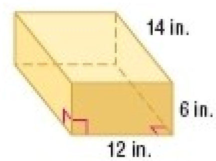
E $\frac{x-4}{x}$

SOLUTION:

$$\begin{aligned} \frac{x^2 - 2x - 8}{x^2 + 2x} &= \frac{(x-4)(x+2)}{x(x+2)} \\ &= \frac{x-4}{x} \end{aligned}$$

So, the correct choice is E.

Find the volume of each prism.

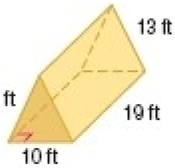
45. 

SOLUTION:

The volume of a prism is $V = Bh$, where B is the area of the base and h is the height of the prism. The base of this prism is a rectangle with a length of 14 inches and a width of 12 inches. The height h of the prism is 6 inches.

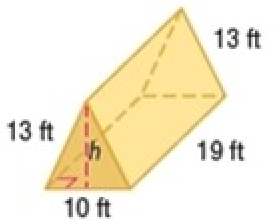
$$\begin{aligned} V &= Bh \\ &= (14 \times 12)(6) \\ &= (168)(6) \\ &= 1008 \end{aligned}$$

Therefore, the volume of the prism is 1008 in^3.

46. 

SOLUTION:

The volume of a prism is $V = Bh$, where B is the area of the base and h is the height of the prism. The base of this prism is an isosceles triangle with a base of 10 feet and two legs of 13 feet. The height h will bisect the base. Use the Pythagorean Theorem to find the height of the triangle.



$$h^2 + 5^2 = 13^2$$
$$h^2 = 169 - 25$$
$$h = \sqrt{144} \text{ or } 12$$

So, the height of the triangle is 12 feet. Find the area of the triangle.

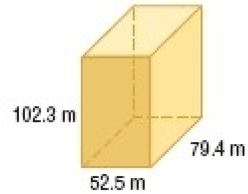
$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(10)(12)$$
$$= 60$$

So, the area of the base B is 60 ft^2.

The height h of the prism is 19 feet.

$$V = Bh$$
$$= (60)(19)$$
$$= 1140$$

Therefore, the volume of the prism is 1140 ft^3.

47. 

SOLUTION:

The volume of a prism is $V = Bh$, where B is the area of the base and h is the height of the prism. The base of this prism is a rectangle with a length of 79.4 meters and a width of 52.5 meters. The height of the prism is 102.3 meters.

$$V = Bh$$
$$= (79.4 \times 52.5)(102.3)$$
$$\approx 426,437.6$$

Therefore, the volume of the prism is about 426,437.6 m^3.

48. **FARMING** The picture shows a combination hopper cone and bin used by farmers to store grain after harvest. The cone at the bottom of the bin allows the grain to be emptied more easily. Use the dimensions in the diagram to find the entire surface area of the bin with a conical top and bottom. Write the exact answer and the answer rounded to the nearest square foot.
Refer to the photo on Page 847.

SOLUTION:

To find the entire surface area of the bin, find the surface area of the conical top and bottom and find the surface area of the cylinder and add them.
The formula for finding the surface area of a cone is πrl, where is r is the radius and l is the slant height of the cone.

Find the slant height of the conical top.

$$l^2 = 9^2 + 5^2$$
$$l = \sqrt{9^2 + 5^2}$$
$$= \sqrt{81 + 25}$$
$$= \sqrt{106}$$
$$SA(\text{conical top}) = \pi(9)\sqrt{106}$$
$$= 9\pi\sqrt{106}$$

Find the slant height of the conical bottom.
The height of the conical bottom is 28 – (5 + 12 + 2) or 9 ft.

$$l^2 = 9^2 + 9^2$$
$$l = \sqrt{9^2 + 9^2}$$
$$= \sqrt{81 + 81}$$
$$= 9\sqrt{2}$$
$$SA(\text{conical bottom}) = \pi(9)(9)\sqrt{2}$$
$$= 81\pi\sqrt{2}$$

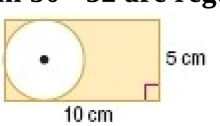
The formula for finding the surface area of a cylinder is $2\pi rh$, where is *r* is the radius and *h* is the slant height of the cylinder.

$$SA(\text{cylinder}) = 2\pi(9)(12)$$
$$= 216\pi$$

Surface area of the bin = Surface area of the cylinder + Surface area of the conical top + surface area of the conical bottom.

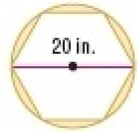
$$SA(\text{bin}) = 216\pi + 9\pi\sqrt{106} + 81\pi\sqrt{2}$$
$$= \pi\left(216 + 9\sqrt{106} + 81\sqrt{2}\right)$$
$$\approx 1330 \text{ ft}^2$$

Find the area of each shaded region. Polygons in 50 - 52 are regular.

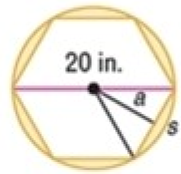
49. 

SOLUTION:
Area of the shaded region = Area of the rectangle – Area of the circle
Area of the rectangle = 10(5)
= 50

$$\text{Area of the circle} = \pi(2.5)^2$$
$$= 6.25\pi$$
$$\text{Area of the shaded region} = 50 - 6.25\pi$$
$$\approx 50 - 19.6$$
$$= 30.4 \text{ cm}^2$$

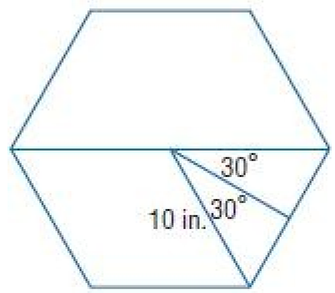
50. 

SOLUTION:
Area of the shaded region = Area of the circle – Area of the hexagon



$$A = \pi r^2$$
$$A = \pi(10)^2$$
$$A = 100\pi \text{ in}^2$$

A regular hexagon has 6 sides, so the measure of the interior angle is $\frac{360}{6} = 60$. The apothem bisects the angle, so we use 30° when using trig to find the length of the apothem.



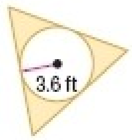
$$\cos 30 = \frac{adjacent}{hypotenuse}$$
$$\cos 30 = \frac{a}{10}$$
$$10\cos 30 = a$$

$$\sin 30 = \frac{opposite}{hypotenuse}$$
$$\sin 30 = \frac{\left(\frac{s}{2}\right)}{10}$$
$$10\sin 30 = \frac{s}{2}$$
$$20\sin 30 = s$$

Now find the area of the hexagon.

$$\text{Area (hexagon)} = \frac{1}{2}aP$$
$$= \frac{1}{2}(10\cos 30)(6 \times s)$$
$$= \frac{1}{2}(10\cos 30)(6 \times 20\sin 30)$$
$$\approx 259.8$$

$$\text{Area (shaded)} = 100\pi - 259.8$$
$$\approx 54.4 \text{ in}^2$$

51. 

SOLUTION:

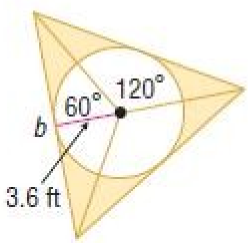
The shaded area is the area of the equilateral triangle less the area of the inscribed circle.

Find the area of the circle with a radius of 3.6 feet.

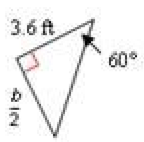
$$A(circle) = \pi r^2$$
$$= \pi(3.6)^2$$
$$\approx 40.7$$

So, the area of the circle is about 40.7 square feet.

Next, find the area of the equilateral triangle.



The equilateral triangle can be divided into three isosceles triangles with central angles of $\frac{360°}{3}$ or 120°, so the angle in the triangle created by the height of the isosceles triangle is 60° and the base is half the length *b* of the side of the equilateral triangle.



Use a trigonometric ratio to find the value of *b*.

$$\tan 60 = \frac{\left(\frac{b}{2}\right)}{3.6}$$
$$b = 7.2\tan 60$$

Now use the area of the isosceles triangles to find the area of the equilateral triangle.

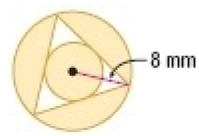
$$A(\text{big triangle}) = 3\left(\frac{1}{2}bh\right)$$
$$= \frac{3}{2}(7.2\tan 60)(3.6)$$
$$\approx 67.3$$

So, the area of the equilateral triangle is about 67.3 square feet.

Subtract to find the area of shaded region.

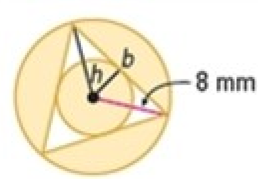
$$A(\text{shaded}) \approx 67.3 - 40.7$$
$$\approx 26.6$$

Therefore, the area of the shaded region is about 26.6 ft^2.

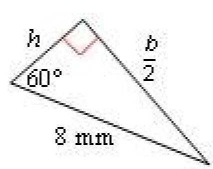
52. 

SOLUTION:

The area of the shaded region is the area of the circumscribed circle minus the area of the equilateral triangle plus the area of the inscribed circle. Let *b* represent the length of each side of the equilateral triangle and *h* represent the radius of the inscribed circle.



Divide the equilateral triangle into three isosceles triangles with central angles of $\frac{360°}{3}$ or 120°. The angle in the triangle formed by the height.*h* of the isosceles triangle is 60° and the base is half the length of the side of the equilateral triangle as shown below.



Use trigonometric ratios to find the values of *b* and *h*.

$$\sin 60 = \frac{\left(\frac{b}{2}\right)}{8}$$
$$b = 16 \sin 60$$

$$\cos 60 = \frac{h}{8}$$
$$h = 8 \cos 60$$

The area of the equilateral triangle will equal three times the area of any of the isosceles triangles. Find the area of the shaded region.

A(shaded region) = *A*(circumscribed circle) - *A*(equilateral triangle) + *A*(inscribed circle)

$$\begin{aligned} A(\text{shadedregion}) &= \pi r^2 - 3\left(\frac{1}{2}bh\right) + \pi h^2 \\ &= \pi(8)^2 - \frac{3}{2}(16\sin 60)(8\cos 60) \\ &\approx 201.06 - 83.14 + 50.27 \\ &\approx 168.2 \end{aligned}$$

Therefore, the area of the shaded region is about 168.2 mm^2.