51. **Picture the Problem**: Water flows through a hose at a constant volume flow rate. When the diameter of the hose is decreased by partially blocking the end, the speed through the end increases so the same volume flows in the same time.

**Strategy**: We wish to find the speed through the partially blocked end. Write the continuity equation (15-12) in terms of the diameters of the hose and solve for the speed at the end.

**Solution**: 1. Write the continuity equation in terms of the diameters of the hose:
\[
A_v v_1 = A_v v_2
\]
\[
\left(\pi d_1^2 / 4\right) v_1 = \left(\pi d_2^2 / 4\right) v_2
\]

2. Solve for the end velocity:
\[
v_2 = \left(\frac{d_1}{d_2}\right)^2 v_1 = \left(\frac{3.4 \text{ cm}}{0.57 \text{ cm}}\right)^2 (1.1 \text{ m/s}) = 39 \text{ m/s}
\]

**Insight**: The ratio of the speeds is inversely proportional to the square of the diameters. If the diameter were to be cut in half, the speed would increase by a factor of four.

52. **Picture the Problem**: Water flows through a pipe of given diameter at a given speed. We wish to calculate the mass flow rate, or the mass of water that passes a given point in the pipe per second.

**Strategy**: Multiply the density of the water by the cross-sectional area of the pipe and the velocity to calculate the mass flow rate.

**Solution**: 1. Write the mass flow rate in terms of the density, diameter, and velocity:
\[
\frac{\Delta m}{\Delta t} = \rho A v = \rho \left(\pi d^2 / 4\right) v
\]

2. Insert the given values:
\[
\frac{\Delta m}{\Delta t} = (1000 \text{ kg/m}^3) \left(\pi (0.038 \text{ m})^2 / 4\right) (2.1 \text{ m/s}) = 2.4 \text{ kg/s}
\]

**Insight**: This flow rate is slightly over a \( \frac{1}{2} \) gallon per second.

53. **Picture the Problem**: A child’s pool is filled from a garden hose. The volume flow rate through the hose is equal to the rate at which the pool fills.

**Strategy**: In this problem we wish to calculate the time necessary to fill the pool. Use the volume continuity equation \( A_v v = A_v v \) (equation 15-11) to calculate the speed at which the water rises in the pool. Divide the depth of the pool by the speed of the rising water to determine the time necessary to fill the pool.

**Solution**: 1. Solve the continuity equation for the speed at which the pool fills:
\[
v_p = \frac{A_v v}{A_p v} = \frac{4\pi d_h^2}{\pi d_p^2} v = \left(\frac{d_h}{d_p}\right)^2 v
\]
\[
\frac{0.029 \text{ m}^2}{2.0 \text{ m}} (1.3 \text{ m/s}) = 2.7 \times 10^{-4} \text{ m/s}
\]

2. Divide the height of the pool by the fill speed to find the required fill time:
\[
t = \frac{h}{v_{pool}} = \frac{0.26 \text{ m}}{2.7 \times 10^{-4} \text{ m/s}} = 960 \times \frac{1 \text{ min}}{60 \text{ sec}} = 16 \text{ min}
\]

**Insight**: To decrease the time necessary to fill the pool, either the diameter of the hose or the speed of the water should be increased.
54. **Picture the Problem**: The volume rate of blood flow through the heart per minute is given and we are asked to calculate the volume of blood that flows in a day.

**Strategy**: Multiply the given volume flow rate by the number of minutes in a day to calculate the volume of blood that passes though the heart each day. Multiply the volume of blood by the density to calculate the mass of blood that flows through the heart each day.

**Solution**: 1. Convert the flow rate from per minute to per day:

\[ \frac{\Delta V}{\Delta t} = \left( \frac{5.00 \text{ L}}{\text{min}} \right) \left( \frac{60 \text{ min}}{1 \text{ h}} \right) \left( \frac{24 \text{ h}}{1 \text{ day}} \right) = 7200 \text{ L/day} \]

2. Multiply the volume flow rate by the density of blood:

\[ \frac{\Delta m}{\Delta t} = \rho \left( \frac{\Delta V}{\Delta t} \right) = \left( 1060 \text{ kg/m}^3 \right) \left( 7200 \text{ L} \right) \left( \frac{m^3}{10^3 \text{ L}} \right) = 7630 \text{ kg/day} \]

**Insight**: The average human heart pumps over 7½ tons of blood per day!

55. **Picture the Problem**: This solution contains an art image.

**Strategy**: The volume flow rate is equal to the cross-sectional area of the blood vessel times the velocity of the blood through the blood vessel. Divide the flow rate by the cross-sectional area to calculate the speed. In the capillaries the flow rate through each capillary will be equal to the total flow rate divided by the number of capillaries.

**Solution**: 1. (a) Divide the volume flow rate in the arteriole branch by the branch’s cross-sectional area:

\[ \frac{\Delta V}{\Delta t} = A \nu \Rightarrow \nu = \frac{\Delta V}{A} = \frac{5.5 \times 10^{-6} \text{ cm}^3/\text{s}}{\pi \left( \frac{1}{2} \cdot 0.003 \text{ cm} \right)^2} = 0.78 \text{ cm/s} \]

2. (b) Divide the volume flow rate in the capillaries (1/340th of the arteriole branch flow rate) by the cross-sectional area of the capillary:

\[ \nu = \frac{\Delta V}{A} = \frac{3 \times 10^{-6} \text{ cm}^3/\text{s}}{340 \pi \left( \frac{1}{2} \cdot 4 \times 10^{-4} \text{ cm} \right)^2} = 0.13 \text{ cm/s} \]

**Insight**: The blood speed in the capillaries is much slower than in the other blood vessels.

56. **Picture the Problem**: In our sketch we label the speed of the water in the hose with \( v_1 \) and the speed of the water coming out the nozzle as \( v_2 \). We are given the mass flow rate through the hose and the diameters of the hose and nozzle. We need to calculate the speed of the water in the hose and in the nozzle.

**Strategy**: Divide the mass flow rate by the density and cross-sectional area to calculate the velocity \( v \) of the water in the hose and in the nozzle. Calculate the cross-sectional areas from the diameters.

**Solution**: 1. Solve the mass flow rate equation for \( v \):

\[ \frac{\Delta m}{\Delta t} = \rho A v \Rightarrow v = \frac{\Delta m}{\Delta t} \frac{1}{\rho A} = \frac{\Delta m}{\Delta t} \frac{4}{\rho \pi d^2} \]

2. (a) Insert the data for the flow in the hose:

\[ v_1 = \frac{4}{(1000 \text{ kg/m}^3) \pi (0.0322 \text{ m})^2} = 3.82 \text{ m/s} \]

3. (b) Insert the data for the flow in the nozzle:

\[ v_2 = \frac{4}{(1000 \text{ kg/m}^3) \pi (0.00732 \text{ m})^2} = 73.9 \text{ m/s} \]

4. (c) The mass flow rates in the nozzle and hose are equal because for incompressible fluids, conservation of mass requires that “what goes in equals what comes out.”

**Insight**: Because the nozzle has a much smaller diameter, the speed of the water in the nozzle is much greater than the speed in the hose.
Picture the Problem: Water flows through a river at a constant volume flow rate. We are given the widths and depths of the river at two points and the water speed at the initial point before the rapids. We wish to calculate the speed of the water in the rapids.

Strategy: Set the volume flow rate in the rapids equal to the flow rate before the rapids using equation 15-12. Solve the equation for the speed in the rapids. The cross-sectional areas are given by the depth of the water times the width of the river.

Solution: 1. Solve equation 15-12 for the speed in the rapids:

\[ A_1 v_1 = A_2 v_2 \]

\[ w_1 d_1 v_1 = w_2 d_2 v_2 \quad \Rightarrow \quad v_2 = \frac{w_1 d_1 v_1}{w_2 d_2} \]

2. Insert the given data:

\[ v_2 = \frac{(12 \text{ m})(2.7 \text{ m})(1.2 \text{ m/s})}{(5.8 \text{ m})(0.85 \text{ m})} = 7.9 \text{ m/s} \]

Insight: For the river to have the same volume flow rate at two different points, the region with a higher water velocity must have a smaller cross-sectional area than the region with a slower velocity.

Picture the Problem: The volume flow rate throughout the circulatory system must be equal. We are given the diameter and velocity of blood in the aorta. We are also given the diameter and velocity of the blood in the capillaries. Using this information we need to calculate the number of capillaries.

Strategy: Use equation 15-11 to set the volume flow rate in the aorta equal to the volume flow rate in the capillaries, where the flow rate in the capillaries is equal to the flow rate in one capillary times the number of capillaries, \( n \). Solve the resulting equation for the number of capillaries.

Solution: 1. Solve equation 15-11 for the number of capillaries:

\[ A_4 v_4 = n A_5 v_5 \quad \Rightarrow \quad n = \frac{A_4 v_4}{A_5 v_5} \]

2. Write the cross-sectional areas in terms of the diameters:

\[ n = \frac{\pi d_4^2 v_4}{\pi d_5^2 v_5} = \left( \frac{d_4}{d_5} \right)^2 \frac{v_4}{v_5} \]

3. Insert the given data:

\[ n = \left( \frac{0.0050 \text{ m}}{1.0 \times 10^{-5} \text{ m}} \right)^2 \left( \frac{1.0 \text{ m/s}}{0.01 \text{ m/s}} \right) = 2.5 \times 10^7 \]

Insight: The 25 million capillaries are crammed into the human body, which has a volume of about 0.5 m³. That comes to approximately 50 capillaries in every cubic centimeter!
59. **Picture the Problem**: The image shows two regions of a blood vessel. The initial region has no plaque buildup and has a diameter $d_1 = 1.1$ cm and pressure $P_1$. The blood flows through this region at a rate of 15 cm/s. The second region has plaque buildup so its diameter is only $d_2 = 0.75$ cm and its pressure is $P_2$.

**Strategy**: We wish to find the speed of the blood at the plaque buildup and the difference in pressures between the two regions. Solve the volume continuity equation (equation 15-11) for the speed in the narrower region. Use the speeds through the two areas and the density of blood (1060 kg/m$^3$) to solve equation 15-14 for the pressure difference $\Delta P$.

**Solution**:  
(a) Solve the continuity equation for $v_2$:

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi d_1^2}{\pi d_2^2} v_1 = \left(\frac{d_1}{d_2}\right)^2 v_1$$

2. Insert the given values:

$$v_2 = \left(\frac{1.1 \text{ cm}}{0.75 \text{ cm}}\right)^2 (15 \text{ cm/s}) = 32 \text{ cm/s}$$

3. (b) Solve equation 15-14 for $\Delta P$:

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$\Delta P = \frac{1}{2} (1060 \text{ kg/m}^3) \left[(0.323 \text{ m/s})^2 - (0.15 \text{ m/s})^2\right] = 43 \text{ Pa}$$

**Insight**: The drop in pressure at the plaque means that the heart must increase its output pressure by 40 Pa in order to maintain the same flow rate as there was in the plaque-free vessel.