Logarithms Review

The following properties serve to expand or condense a logarithm or logarithmic expression so it can be worked with.

Properties of logarithms	***Example***
$\log_a mn = \log_a m + \log_a n$	$\log_4 3x = \log_4 3 + \log_4 x$
$\log_a \frac{m}{n} = \log_a m - \log_a n$	$\log_2 \frac{x+1}{5} = \log_2(x+1) - \log_2 5$
$\log_a m^n = n\log_a m$	$\log_3(2x+1)^3 = 3\log_3(2x+1)$

Properties of Natural Logarithms	***Example***
$\ln mn = \ln m + \ln n$	$\ln(x+1)(x-5) = \ln(x+1) + \ln(x-5)$
$\ln \frac{m}{n} = \ln m - \ln n$	$\ln \frac{x}{2} = \ln x - \ln 2$
$\ln m^n = n\ln m$	$\ln 7^3 = 3\ln 7$

Log Form: $\log_a b = c$ Exp Form: $a^c = b$

Sample Questions

1) Write the following in exponential form $\log_9 27 = \frac{3}{2}$ $9^{3/2} = 27$

2) Write each of the following in logarithmic form $16^{1/4} = 2$ $\log_{16} 2 = \frac{1}{4}$

Evaluate each of the following logarithms without the use of a calculator.

3) $\log_4 \frac{1}{2} =$ $-\frac{1}{2}$
$4^x = \frac{1}{2}$, $2^{2x} = 2^{-1}$, $2x = -1$

4) $\log_8 4 =$ $2/3$
$8^x = 4$, $2^{3x} = 2^2$, $3x = 2$

5) $\log_3 81 =$ 4
$3^x = 81$, $3^x = 3^4$

6) $\log_4 0 =$ No solution
Cannot take log of 0 or $-\#$

Write each of the following as the sum or difference of logarithms.

7) $\log \sqrt[4]{(x+1)^3(x-2)^2}$ $= \log (x+1)^{3/4}(x-2)^{2/4}$
$\log (x+1)^{3/4} + \log (x-2)^{2/4}$
$\boxed{\frac{3}{4}\log(x+1) + \frac{2}{4}\log(x-2)}$

8) $\log_5 \frac{6x^2}{11y^5z}$ $= \log_5 6x^2 - \log_5 11y^5z$
$\log_5 6 + \log_5 x^2 - (\log_5 11 + \log_5 y^5 + \log_5 z)$
$\boxed{\log_5 6 + 2\log_5 x - \log_5 11 - 5\log_5 y - \log_5 z}$

9) $\log_2 \frac{\sqrt[5]{3(x+2)^3}}{x-1}$
$\log_2 \frac{3^{1/5}(x+2)^{3/5}}{x-1}$
$\boxed{\frac{1}{5}\log_2 3 + \frac{3}{5}\log_2(x+2) - \log_2(x-1)}$

10) $\log_3 \frac{\sqrt{5x^5y^3}}{\sqrt[3]{z^2}}$ $= \log_3 \frac{5^{1/2}x^{5/2}y^{3/2}}{z^{2/3}}$
$\boxed{\frac{1}{2}\log_3 5 + \frac{5}{2}\log_3 x + \frac{3}{2}\log_3 y - \frac{2}{3}\log_3 z}$

Rewrite each of the following logarithmic expressions using a single logarithm.

11) $\frac{1}{3}\log 6+\frac{1}{3}\log x+\frac{2}{3}\log y$

$\frac{1}{3}(\log 6 + \log x + 2\log y)$

$\frac{1}{3}\log 6xy^2$

$\boxed{\log \sqrt[3]{6xy^2}}$

12) $\ln(x+3)-\ln(2x+5)+2\ln(x-1)$

$\ln(x+3) - \ln(2x+5) + \ln(x-1)^2$

$\boxed{\ln \frac{(x+3)(x-1)^2}{2x+5}}$

13) $3\log_4 x-5\log_4 y+2\log_4 z$

$\log_4 x^3 - \log_4 y^5 + \log_4 z^2$

$\boxed{\log_4 \frac{x^3z^2}{y^5}}$

14) $\log_3(x+2)+\log_3(x-2)-\log_3(x+4)$

$\boxed{\log_3 \frac{(x+2)(x-2)}{x+4} \quad \text{or} \quad \log \frac{x^2-4}{x+4}}$

Solve each of the following logarithmic equations. **(write any answer that requires a decimal value in calculator ready form.)** ***Always check for extraneous roots!!!***

15) $\log_3(x+5)+\log_3(x+3)=\log_3 35$

$\log_3(x^2+8x+15)=\log_3 35$

$x^2+8x+15=35$

$x^2+8x-20=0$

$(x+10)(x-2)=0$

$\cancel{x=-10}$ $\quad x=2$

Cannot take log of a -#

$\log_3 7+\log_3 5$ ✓

$\log_3 x$

$\boxed{x=2}$

16) $2\log_3 x-\log_3(x-2)=2$

$\log_3 \frac{x^2}{x-2}=2$

$(x-2)\ 3^2=\frac{x^2}{x-2}\ (x-2)$

$9x-18=x^2$

$x^2-9x+18=0$

$(x-3)(x-6)=0$

$x=3 \quad x=6$

$\log_3 9-\log_3 1=2$ ✓

$\log_3 36-\log_3 4=2$ ✓

$\boxed{x=3 \quad x=6}$

17) $\log_2(x+3)+\log_2(x-3)=4$

$\log_2(x^2-9)=4$

$2^4=x^2-9$

$16=x^2-9$

$x^2-25=0$

$\cancel{x=-5}$ $\quad x=5$

$\log_2 8+\log_2 2=4$ ✓

$3+1=4$

$x=5$

18) $\ln x+\ln(x-2)=1$

$\ln(x^2-2x)=1$

$e=x^2-2x$

$x^2-2x-e=0$

$x=\frac{2\pm\sqrt{4-4(1)(-e)}}{2(1)}$

$4+4e$

$4(1+e)$

$\boxed{x=\frac{2+2\sqrt{1+e}}{2}}$

Solve each of the following exponential equations. Write your answer in calculator ready form.

19) $12^{3x+1}=7^2$

$\log 12^{3x+1}=\log 7^2$

$(3x+1)\log 12=2\log 7$

$3x\log 12+\log 12=2\log 7$

$3x\log 12=2\log 7-\log 12$

$\boxed{x=\frac{2\log 7-\log 12}{3\log 12}}$

20) $12^{3x-2}=8^{5x+1}$

$(3x-2)\log 12=(5x+1)\log 8$

$3x\log 12-2\log 12=5x\log 8+\log 8$

$3x\log 12-5x\log 8=\log 8+2\log 12$

$x(3\log 12-5\log 8)=\log 8+2\log 12$

$\boxed{x=\frac{\log 8+2\log 12}{3\log 12-5\log 8}}$

21) $8^{6x-5}=4^{2x+1}$

Both powers of 2

$(2^3)^{6x-5}=(2^2)^{2x+1}$

$2^{18x-15}=2^{4x+2}$

one-to-one property

$18x-15=4x+2$

$-4x+15 \quad -4x+15$

$14x=17$

$x=\frac{17}{14}$

22) $\left(1+\frac{0.10}{12}\right)^{12t}=2$

$\log\left(1+\frac{.10}{12}\right)^{12t}=\log 2$

$12t\log\left(1+\frac{.10}{12}\right)=\log 2$

$\boxed{t=\frac{\log 2}{12\log\left(1+\frac{.10}{12}\right)}}$

23) $e^{2x}=7$

$\ln e^{2x}=\ln 7$

$2x\ln e=\ln 7$

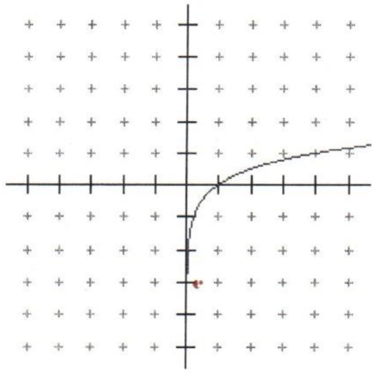
$\ln e=1$

$\frac{2x}{2}=\frac{\ln 7}{2}$

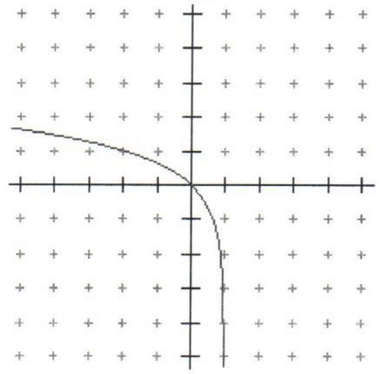
$x=\frac{\ln 7}{2}$ or $\frac{1}{2}\ln 7$

Match the appropriate graph with its equation below. Explain why each of your solutions is true.

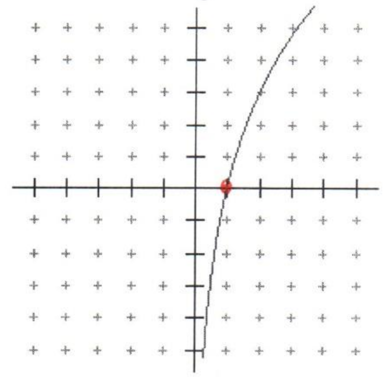
A



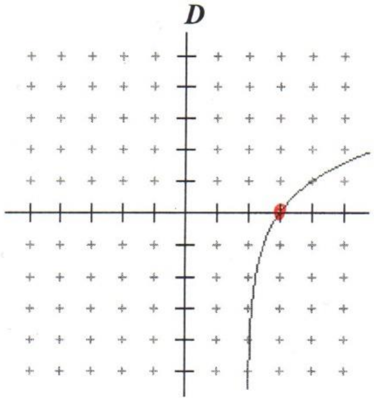
B



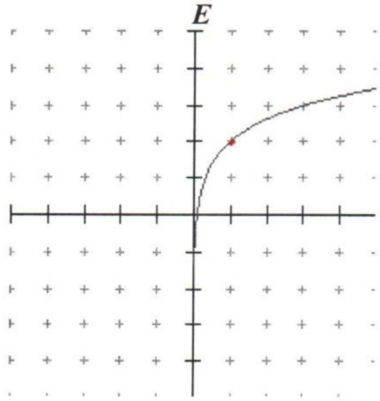
C



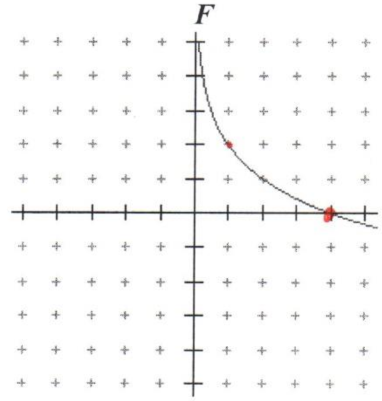
D



E



F



1) $f_{(x)} = \log_2(x-2)$

VA : X=2   (1, 0) +2 +0

Key pt. (3,0)

$\boxed{D}$

2) $f_{(x)} = \log_3(1-x)$ 1-x > 0, 1 > x

VA: X=1   Domain x < 1 goes left

$\boxed{B}$

3) $f_{(x)} = -\log_2 x + 2$

Flips upside down

Key pt (1,0) +0 +2

Key pt: (1,2)

$\boxed{F}$

4) $f_{(x)} = \log_3 x + 2$

Key pt (1,0) +0 +2

Key pt (1,2)

V.A. x=0

$\boxed{E}$

5) $f_{(x)} = \frac{1}{2}\log_2 x$

shrinks by a factor of $\frac{1}{2}$

Key pt (1,0)

V.A.: x=0

$\boxed{A}$

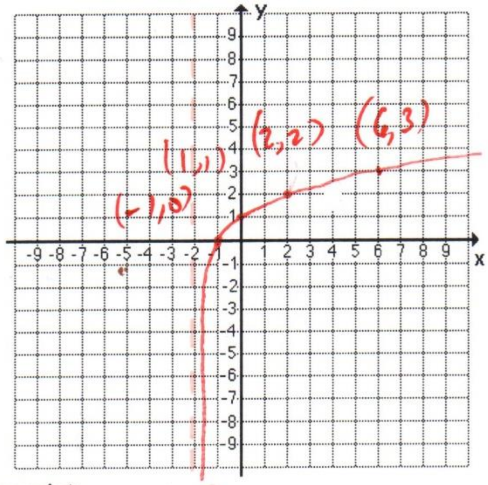
6) $f_{(x)} = 3\log_2 x$

increases 3 times faster

$\boxed{C}$

Graph the following logarithmic functions finding all indicated values.

A) $f(x) = \log_2(x+2)$



(1,0)
(-2, +0)
(-1,0)

Key point: (−1,0)

Vertical Asymptote: x = −2

Y-intercept: (0,1)

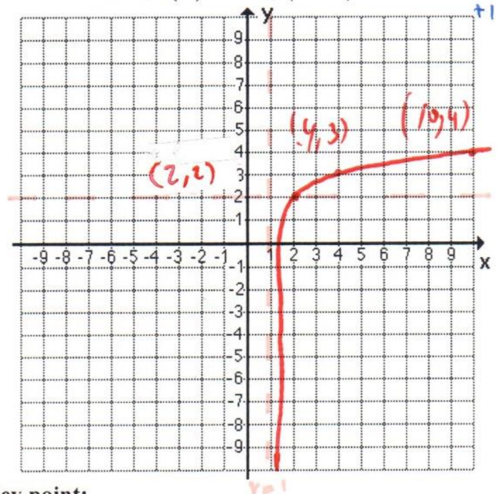
Range: (−∞,∞)

X-intercepts: (−1,0)

Domain: (−2,∞)

B) $f(x) = \log_3(x-1)+2$

(1,0)
+1 +2



Key point: (2,2)

x int

$0 = \log_3(x-1)+2$

Vertical Asymptote: x = 1

~~**Y-intercept:**~~ none

$\log_3(x-1) = -2$

Range: (−∞,∞)

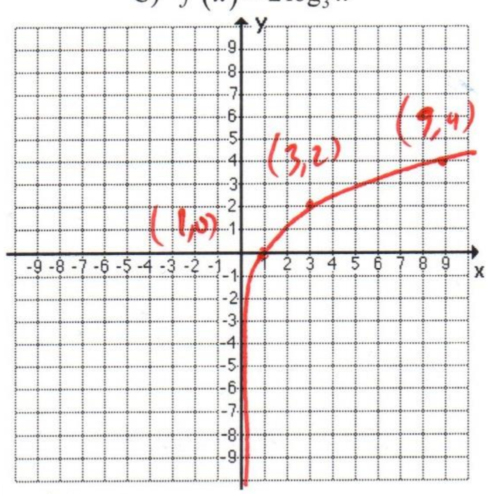
X-intercepts: $(1\frac{1}{9}, 0)$

$3^{-2} = x-1$

$x = 1 + \frac{1}{9}$

Domain: (1,∞)

C) $f(x) = 2\log_3 x$



Key point: (1,0)

Vertical Asymptote: x = 0

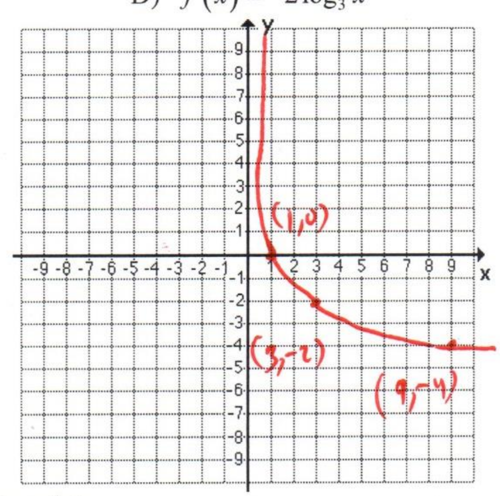
Y-intercept: none

Range: (−∞,∞)

X-intercepts: (1,0)

Domain: (0,∞)

D) $f(x) = -2\log_3 x$



Key point: (1,0)

Vertical Asymptote: x = 0

Y-intercept: none

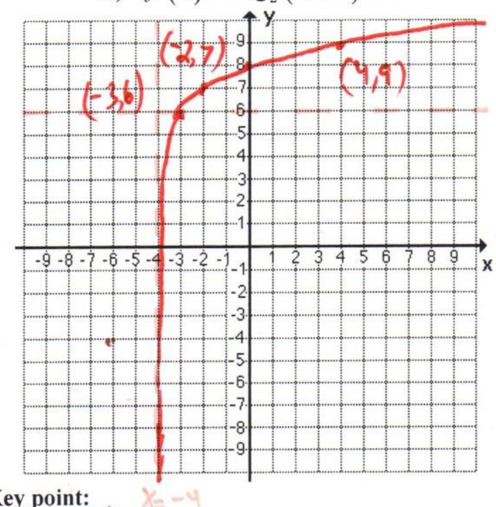
Range: (−∞,∞)

X-intercepts: (1,0)

Domain: (0,∞)

E) $f(x) = \log_2(x+4)+6$

$(1,0)$
$-4\ +6$



Key point:
$(-3, 6)$

Y-intercept:
$(0, 8)$

X-intercepts:
$\left(-3\frac{63}{64}, 0\right)$

x-int
$\log_2(x+4) = -6$
$2^{-6} = x+4$
$x = -4 + \frac{1}{64}$

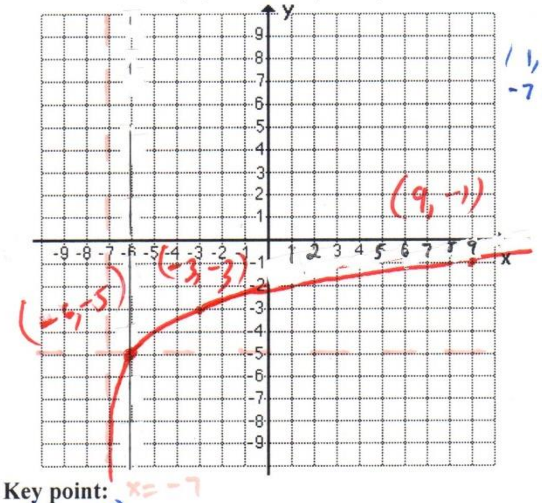
Vertical Asymptote:
$x = -4$

Range:
$(-\infty, \infty)$

Domain:
$(-4, \infty)$

F) $f(x) = 2\log_4(x+7)-5$

$(1,0)$
$-7\ -5$



Key point:
$(-6, -5)$

Y-intercept:
$(0, -2.2)$

X-intercepts:
$(25, 0)$

x int
$2\log_4(x+7) = 5$
$\log_4(x+7) = \frac{5}{2}$
$4^{5/2} = x+7$
$32 = x+7$
$25 = x$

Vertical Asymptote:
$x = -7$

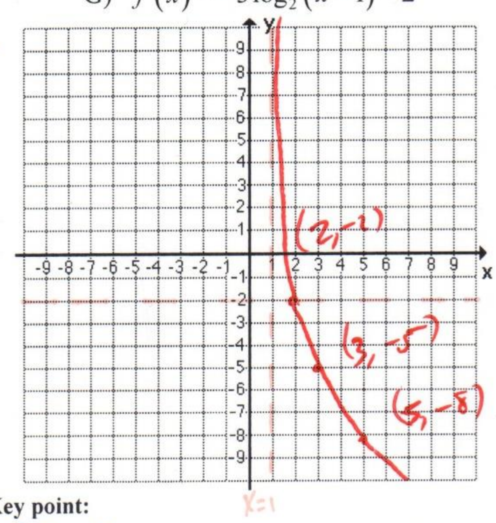
Range:
$(-\infty, \infty)$

Domain:
$(-7, \infty)$

y int.
$f(0) = 2\log_4(7) - 5$
$f(0) \approx 2.2$

G) $f(x) = -3\log_2(x-1)-2$

$(1,0)$
$+1\ -2$



Key point:
$(2, -2)$

Y-intercept:
none

X-intercepts:
$\left(\frac{2+\sqrt[3]{2}}{2}, 0\right)$

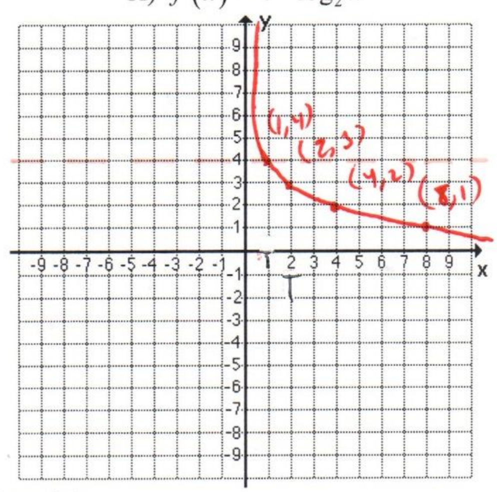
x int
$3\log_2(x-1) = -2$
$\log_2(x-1) = -\frac{2}{3}$
$2^{-2/3} = x-1$
$x = 1 + 2^{-2/3}$
$1 + \frac{1}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = 1 + \frac{\sqrt[3]{2}}{2}$

Vertical Asymptote:
$x = 1$

Range:
$(-\infty, \infty)$

Domain:
$(1, \infty)$

H) $f(x) = 4 - \log_2 x$



Key point:
$(1, 4)$

Y-intercept:
none

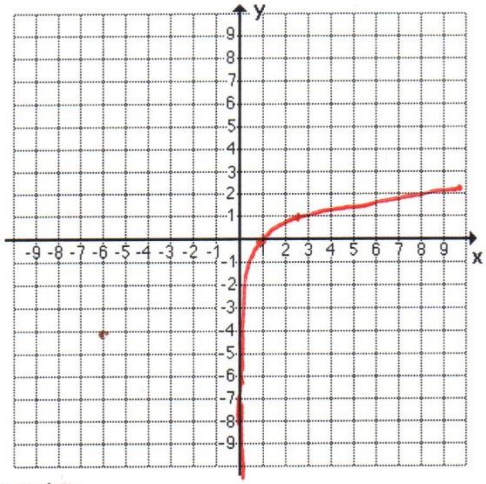
X-intercepts:
$(16, 0)$

Vertical Asymptote:
$x = 0$

Range:
$(-\infty, \infty)$

Domain:
$(0, \infty)$

I) $f(x) = \ln x$



Key point: (1, 0)

Vertical Asymptote: $x = 0$

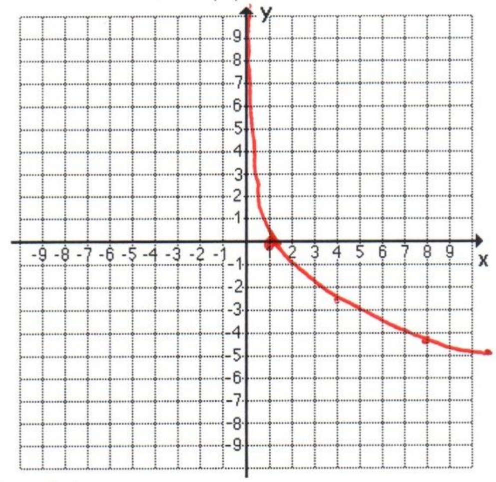
Y-intercept: none

Range: $(-\infty, \infty)$

X-intercepts: (1, 0)

Domain: $(0, \infty)$

J) $f(x) = -2\ln x$



Key point: (1, 0)

Vertical Asymptote: $x = 0$

Y-intercept: none

Range: $(-\infty, \infty)$

X-intercepts: (1, 0)

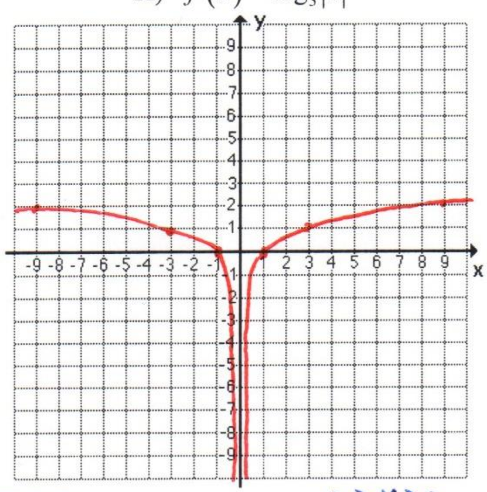
Domain: $(0, \infty)$

Absolute value

Asy – VA

reflects up

K) $f(x) = \log_3 |x|$



Key point: (-1, 0) and (1, 0)

$0 > x > 0$

Vertical Asymptote: $x = 0$

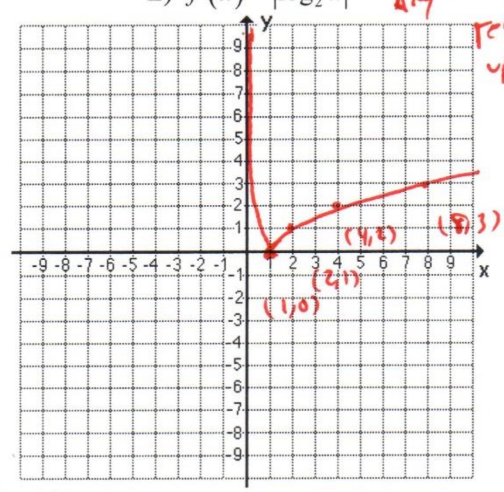
Y-intercept: none

Range: $(-\infty, \infty)$

X-intercepts: (-1, 0) and (1, 0)

Domain: $(-\infty, 0) \cup (0, \infty)$

L) $f(x) = |\log_2 x|$



Key point: (1, 0)

Vertical Asymptote: $x = 0$

Y-intercept: none

Range: $(0, \infty)$

X-intercepts: (1, 0)

Domain: $(0, \infty)$