5. Suppose \(-x^2+1 \leq g(x) \leq x^2+1\) for all \(x\) in an open interval containing 0. Find \(\lim_{x \to 0} g(x)\).

\[
\lim_{x \to 0} \left( -x^2+1 \right) \leq \lim_{x \to 0} g(x) \leq \lim_{x \to 0} \left( x^2+1 \right)
\]

both \(-x^2+1\) and \(x^2+1\) are continuous at \(x=0\) so

\[
\lim_{x \to 0} (-x^2+1) = (-0^2+1) = 1 \quad \text{and} \quad \lim_{x \to 0} (x^2+1) = (0^2+1) = 1
\]

so

\[
\lim_{x \to 0} (x^2+1) \leq \lim_{x \to 0} g(x) \leq \lim_{x \to 0} (x^2+1)
\]

\[
0 \leq \lim_{x \to 0} g(x) \leq 1
\]

by the Squeeze theorem, \(\lim_{x \to 0} g(x) = 1\)