FUNCTIONS

Objectives

The following is a list of objectives for this section of the workbook.

By the time the student is finished with this section of the workbook, he/she should be able to…

- ***Determine the properties of a quadratic function in standard form.***
- ***Find the x and y intercepts of a quadratic function.***
- ***Find the range and domain of a quadratic function.***
- ***Find the vertex of a quadratic function in standard form.***
- ***Graph a quadratic function.***
- ***Determine the properties of an absolute value function in standard form.***
- ***Find the x and y intercepts of an absolute value function.***
- ***Find the range and domain of an absolute value function.***
- ***Find the vertex of an absolute value function.***
- ***Graph an absolute value function.***
- ***Determine the properties of a radical function in standard form.***
- ***Find the x and y intercepts of a radical function.***
- ***Find the range and domain of a radical function.***
- ***Find the point of origin of a radical function.***
- ***Graph a radical function.***
- ***Determine the properties of an exponential function in standard form.***
- ***Find the x and y intercepts of an exponential function.***
- ***Find the range and domain of an exponential function.***
- ***Find the key point of an exponential function.***
- ***Graph an exponential function.***
- ***Determine the properties of a logarithmic function in standard form.***
- ***Find the x and y intercepts of a logarithmic function.***
- ***Find the range and domain of a logarithmic function.***
- ***Find the key point of a logarithmic function.***
- ***Graph a logarithmic function.***
- ***Determine the properties of a cubic function in standard form.***
- ***Find the x and y intercepts of a cubic function.***
- ***Find the range and domain of a cubic function.***
- ***Find the vertex of a cubic function.***
- ***Graph a cubic function.***
- ***Shift the graph of a function without actually knowing the equation, i.e. graphing*** $f_{(x+2)}$.
- ***Graph piece-wise functions.***

Math Standards Addressed

The following state standards are addressed in this section of the workbook.

Algebra II

1.0 Students solve equations and inequalities involving absolute value.

8.0 Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system.

9.0 Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as *a, b,* and *c* vary in the equation $y = a(x-b)^2 + c$.

10.0 Students graph quadratic functions and determine the maxima, minima, and zeros of the function.

11.0 Students prove simple laws of logarithms.

> 11.1 Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
>
> 11.2 Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step.

12.0 Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.

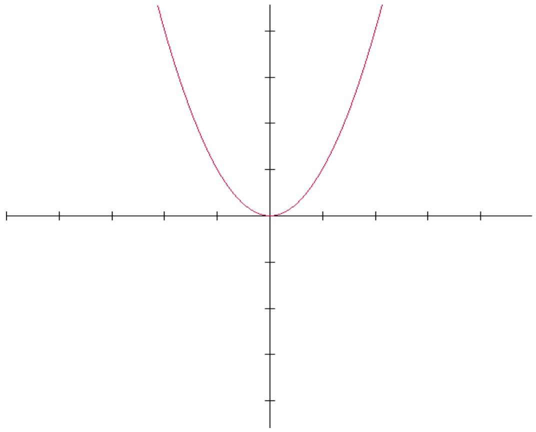
15.0 Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.

Quadratic Functions

The translation of a function is simply the shifting of a function. In this section, for the most part, we will be graphing various functions by means of shifting the parent function. We will go over the parent function for a variety of algebraic functions in this section. It is much easier to see the effects different constants have on a particular function if we use the parent function. We will begin with quadratics. Observe the following regarding a quadratic function in standard form.

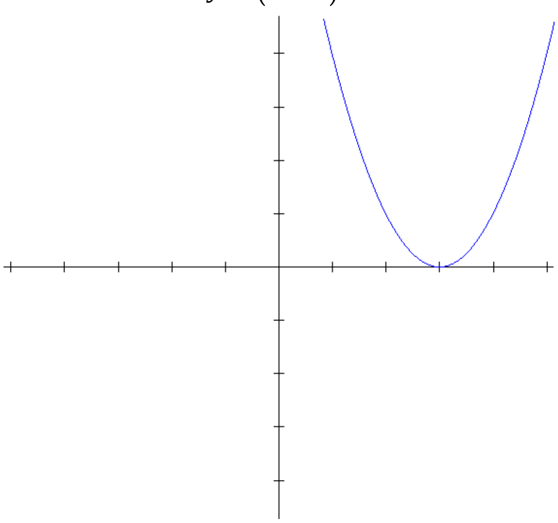
$$y = a(x-h)^2 + k$$

$$y = x^2$$



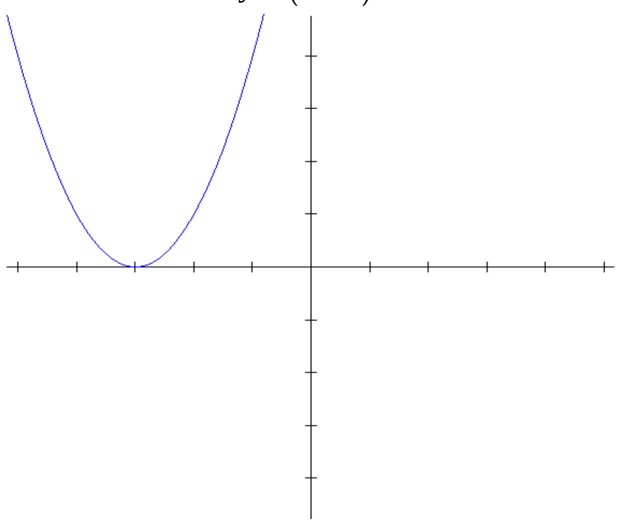
Notice that in the equation above, the h and k values are zero, while the value of a is one. This gives you the parent function for all quadratics. Everything else is merely a manipulation of the parent function.

$$y = (x-3)^2$$



The graph of the function shifts right 3.

$$y = (x+3)^2$$

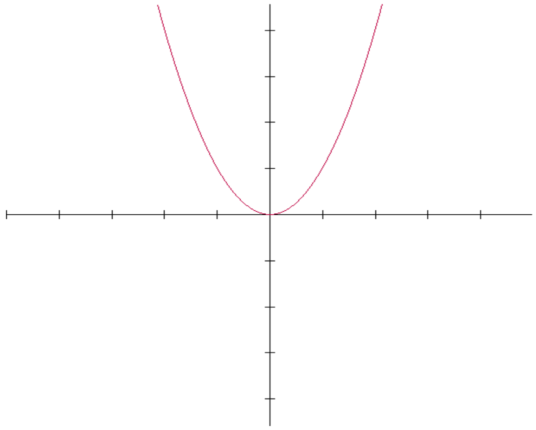


The graph of the function shifts left 3.

The number inside the parenthesis makes the graph shift to the left or right. Remember P.L.N.R., Positive Left Negative Right, tells about the horizontal shift needed to graph the function.

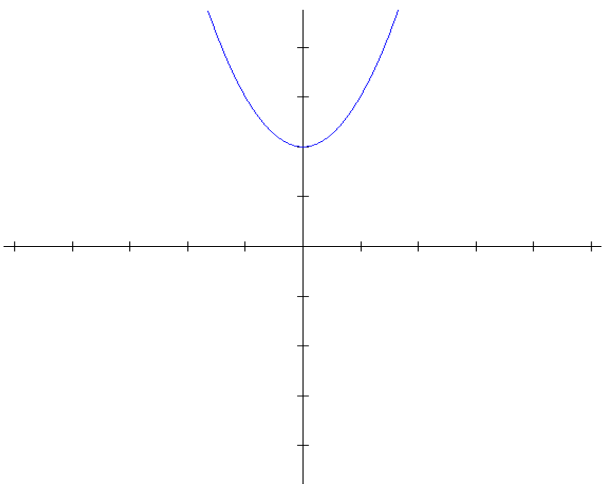
If the function above is $f_{(x)}$, the functions below would be $f_{(x-3)}$ and $f_{(x+3)}$ respectively. This is important to know, because in the future, you will be required to graph functions based solely on the picture provided. No equation will be given. You must rely solely on your knowledge of translating graphs.

$y = x^2$



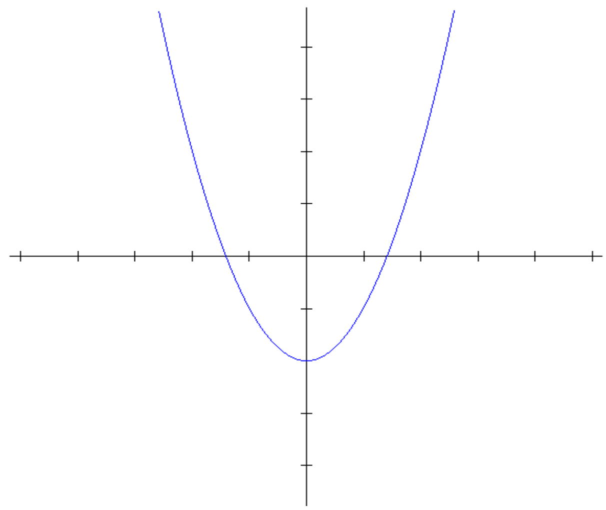
Once again, the parent function is illustrated above, and translations of it below.

$y = x^2 + 2$



The graph of the function shifts up 2.

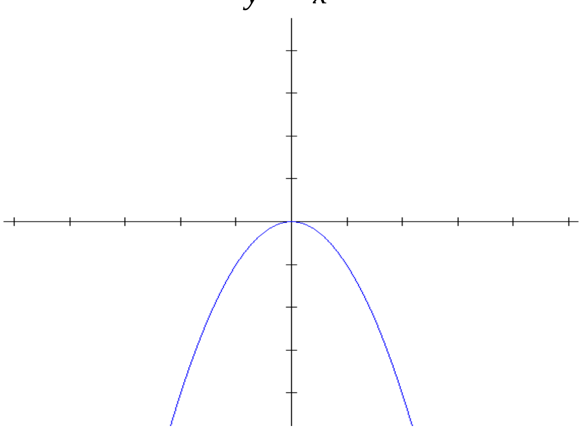
$y = x^2 - 2$



The graph of the function shifts down 2.

In these examples, the k ***value is what is changing. The value of*** k ***dictates a vertical shift of the function. In this case, consider the parent function as being*** $f_{(x)}$. ***Given no information regarding the specific equation of the function, the equations for these two translations of*** $f_{(x)}$ ***are*** $f_{(x)} + 2$, ***and*** $f_{(x)} - 2$.

$y = -x^2$



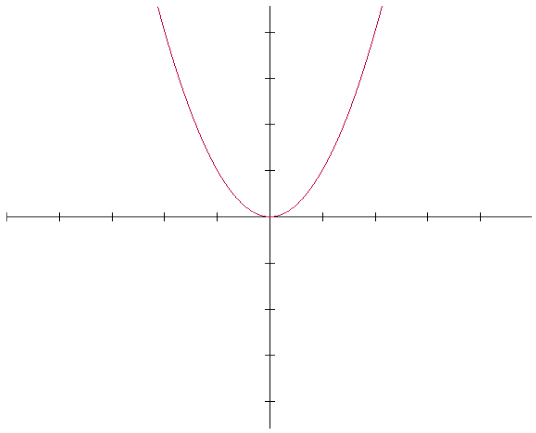
Now, on the left we have the opposite of the parent function. In this particular example, the value of a, ***in the standard form is -1. A negative reflects the graph of the function about the horizontal axis. Once again, if the parent function given is referred to as*** $f_{(x)}$, ***this function is*** $-f_{(x)}$.

We have seen how to graph a function by shifting the parent function. You may have noticed that we graphed $-f_{(x)}$, but not $f_{(-x)}$. The reason we did not see $f_{(-x)}$, is because this is the graph of an even function. That means that if a –x were plugged in to the function, it would make no difference. The outcome would be the same. However, if we are dealing with a different type of function, one that was not even, $f_{(-x)}$ would cause the graph of the function to reflect about a vertical axis. In other words, if $-f_{(x)}$ makes a graph flip upside down, $f_{(-x)}$ would make the graph flip from right to left, or left to right, whatever may be the case.

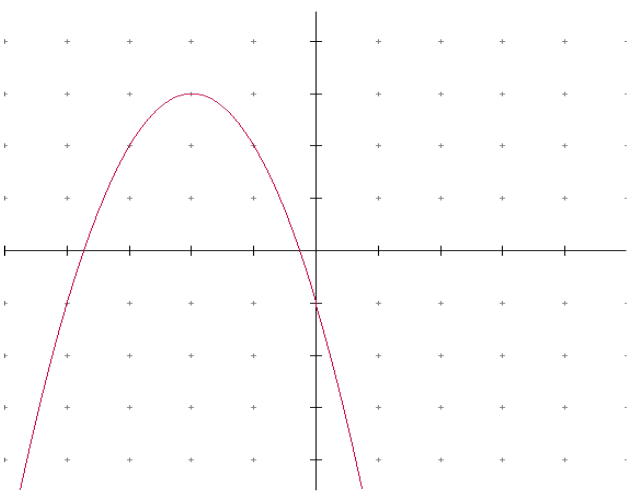
Lets see how differing values of a, h and k will cause various shifts of the function.

$$y = a(x-h)^2 + k$$

Once again, take note of the parent function $y = x^2$



$$y = -(x+2)^2 + 3$$

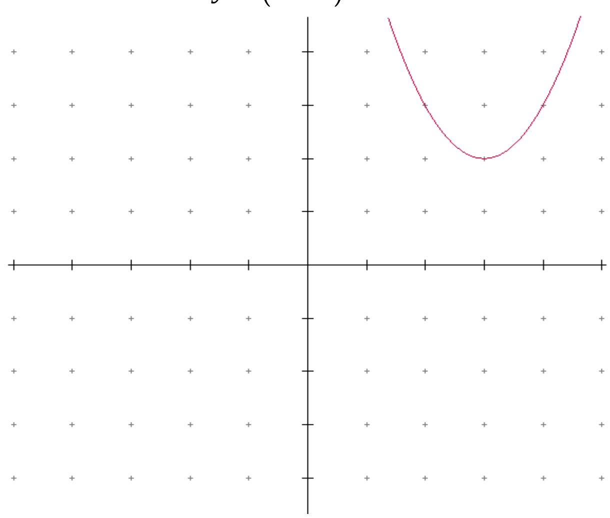


This graph opens down, and shifts left 2, up 3.

If this graph is a translation of the function $f_{(x)}$

It would by written as $-f_{(x+2)} + 3$.

$$y = (x-3)^2 + 2$$



This graph opens up and shifts right 3, and up 2

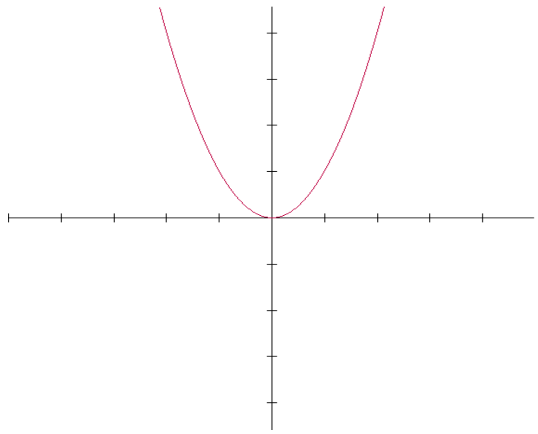
If this graph is a translation of the function $f_{(x)}$

It would by written as $f_{(x-3)} + 2$.

We will be graphing functions using only $f_{(x)}$ in the "translations of functions" section.

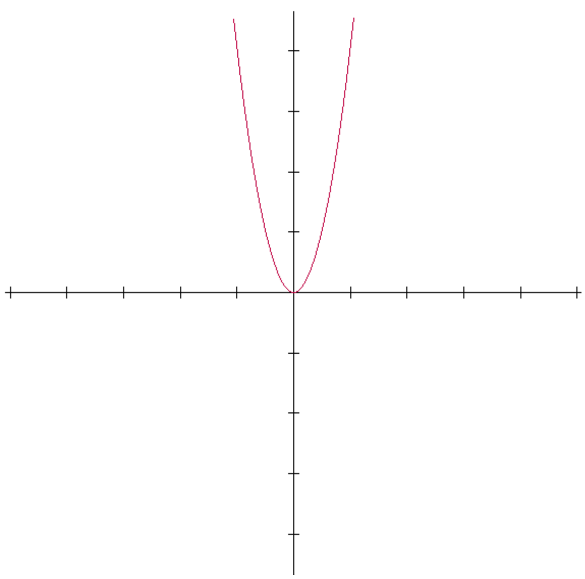
$$y = a(x-h)^2 + k$$

$y = x^2$



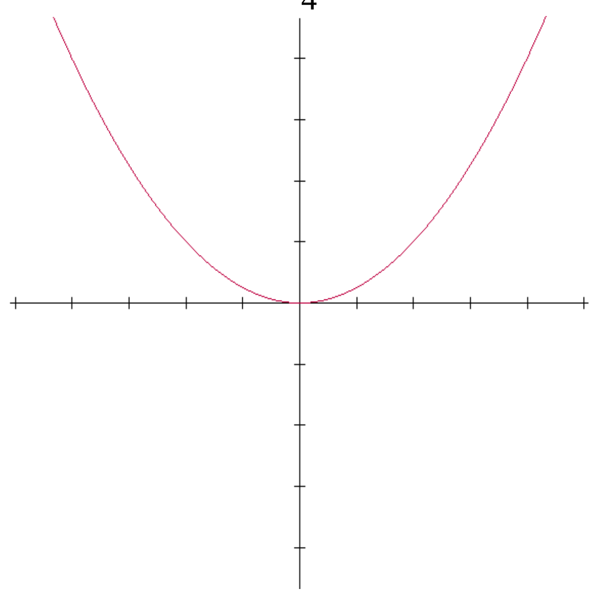
Here we will see how the value of a for the quadratic function in standard form affects the graph of the function. To illustrate this, we will look at the graph of a parabola that has its vertex on the origin.

$y = 4x^2$



This graph seems very narrow, but what is actually happening, is the value of the function is increasing very rapidly. The y values are increasing at 4 times their normal rate. The rapid increase causes the graph to appear narrow.

$y = \frac{1}{4}x^2$



This graph is wider than the parent function. In this case, the y values of the function are increasing at ¼ their normal rate, causing a more gradual increase.

As you can see, if the value of the leading coefficient is a whole number, the y values of the graph will increase rapidly causing a narrow and steeper curve. In contrast, if the leading coefficient is a fraction, the y values of the function will increase mildly, causing a more gradual curve.

Describe the movement of each of the following quadratic functions. Describe how each opens and if there is any horizontal or vertical movement. Be sure to state how many spaces it moves, for example: ***This graph opens down, and shifts left 2, up 3.***

A) $y=-3(x-4)^2+2$

B) $y=2(x+3)^2-8$

C) $y=\frac{1}{2}(x-3)^2$

D) $y=\left(x+\frac{1}{2}\right)^2-\frac{2}{3}$

E) $y=-(x+5)^2+6$

F) $y=7(x-3)^2+1$

G) $y=-\frac{1}{5}(x-7)^2+4$

H) $y=3(x+6)^2+8$

I) $y=-4(x-3)^2-2$

J) $y=x^2-3$

K) $y=-\frac{1}{5}(x+14)^2$

L) $y=-2x^2+8$

As you describe the graphs of the quadratic functions above, you wrote that it shifts to the left or right, and up or down. What is actually shifting?

Write the equation for a quadratic function in $y = a(x-h)^2 + k$ form that opens down, shifts left 3 and up 7.

Write the equation for a quadratic function in $y = a(x-h)^2 + k$ form that opens up, shifts right 4 and down 2.

Write the equation for a quadratic function in $y = a(x-h)^2 + k$ form that opens up, and only shifts down 4.

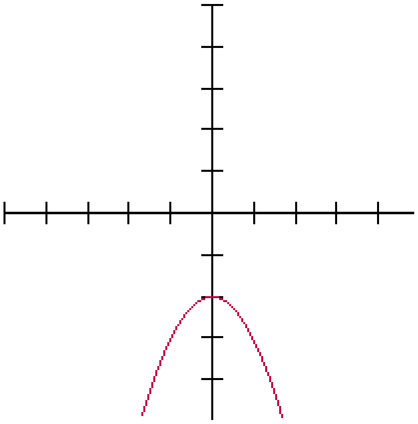
Write the equation for a quadratic function in $y = a(x-h)^2 + k$ form that opens down and shifts to the left 8 spaces.

Write the equation for a quadratic function in $y = a(x-h)^2 + k$ form that opens down and shifts up 7.

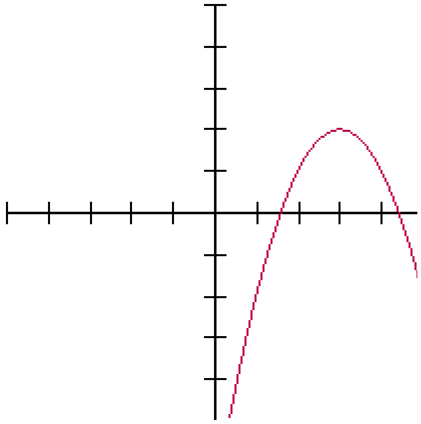
Is a quadratic function a one-to-one function? Why or why not? What does this tell you about the inverse of a quadratic function?

Match the appropriate graph with its equation below. Explain why each of your solutions is true.

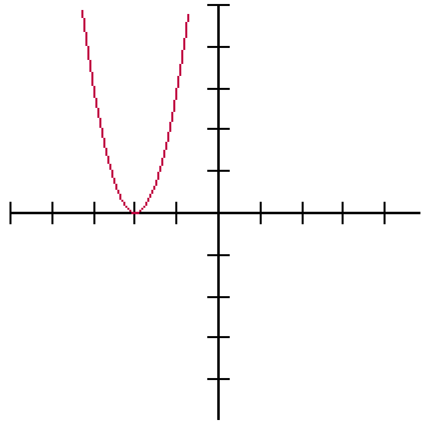
A



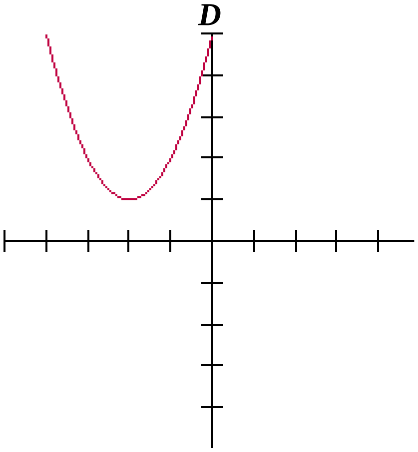
B



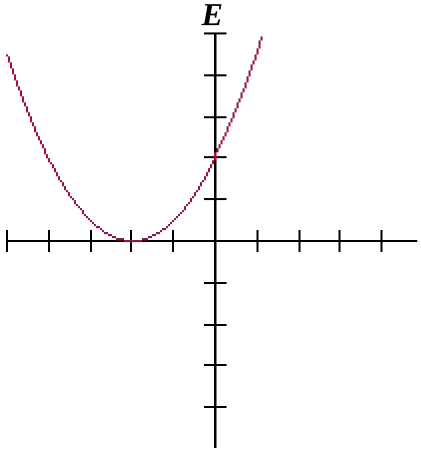
C



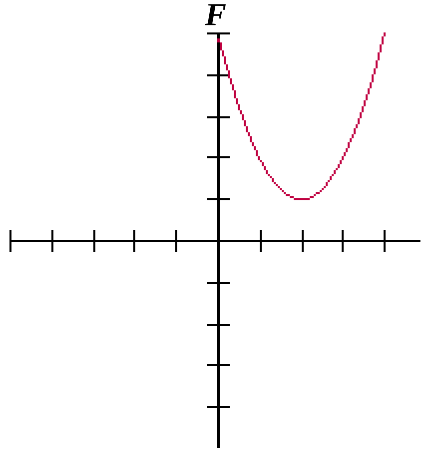
D



E



F



1) $f_{(x)} = \frac{1}{2}(x+2)^2$

2) $f_{(x)} = -x^2 - 2$

3) $f_{(x)} = (x+2)^2 + 1$

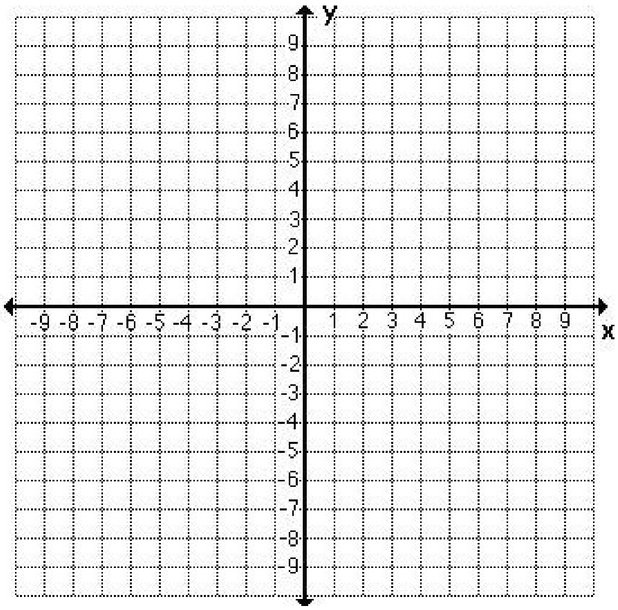
4) $f_{(x)} = (x-2)^2 + 1$

5) $f_{(x)} = 3(x+2)^2$

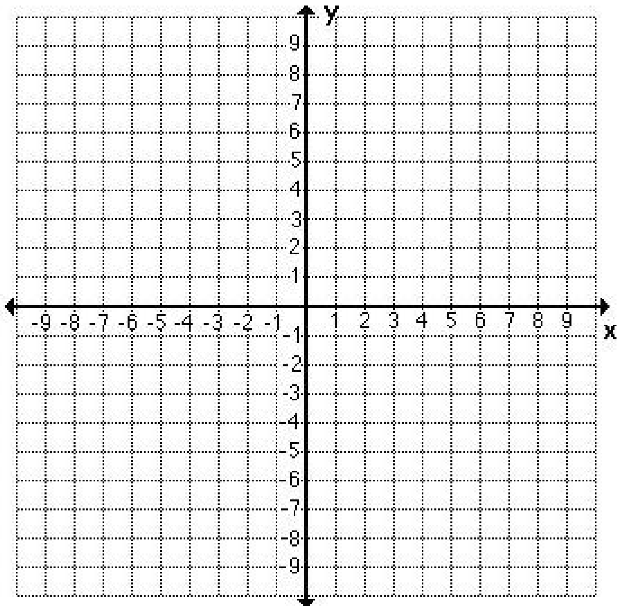
6) $f_{(x)} = -(x-3)^2 + 2$

Graph each of the following functions. You may need to use an axis of symmetry to graph some of these. **Label the vertex, y-intercept, and all x-intercepts.**

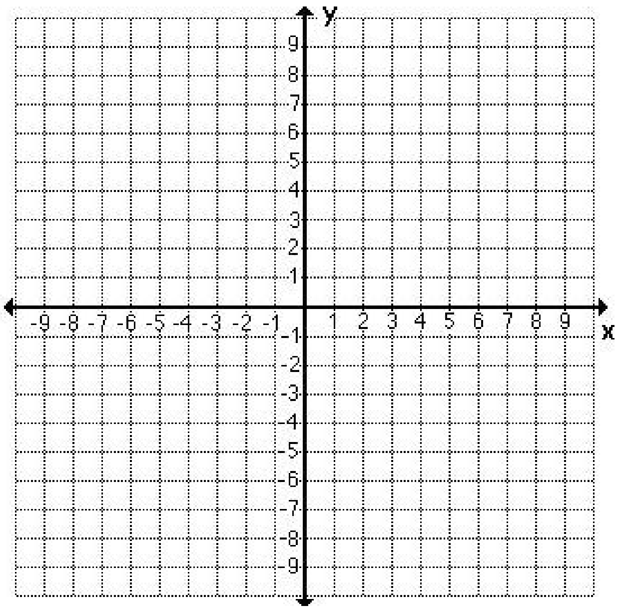
A) $f_{(x)} = (x-3)^2 + 1$



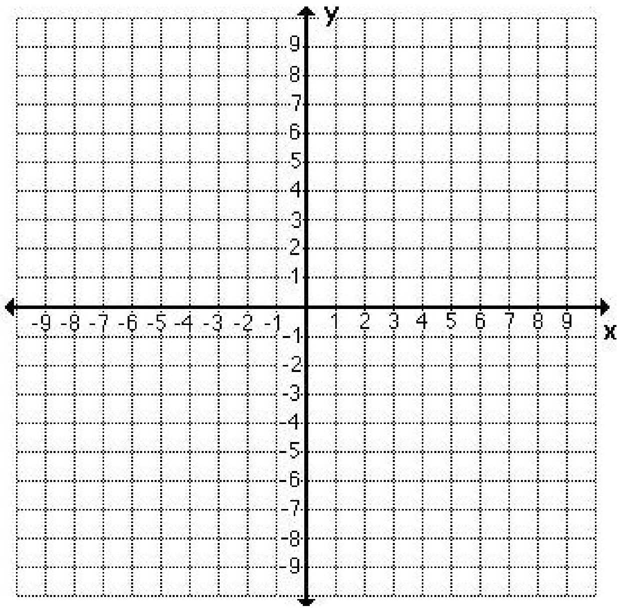
B) $f_{(x)} = -(x+4)^2 + 9$



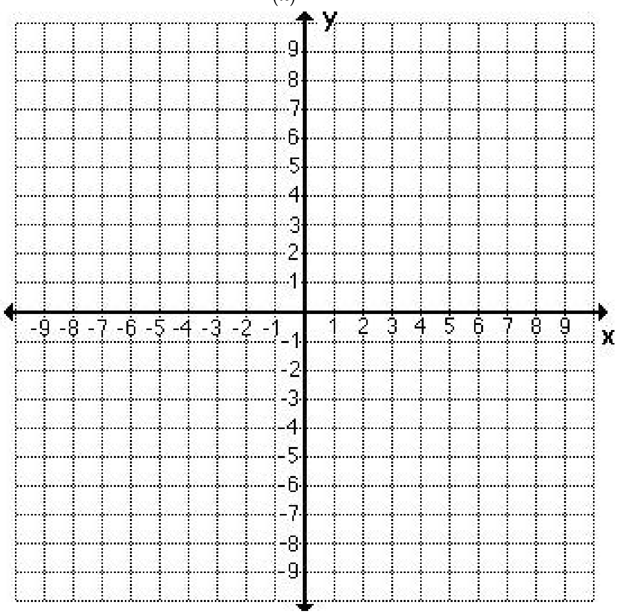
C) $f_{(x)} = \frac{1}{2}(x-5)^2$



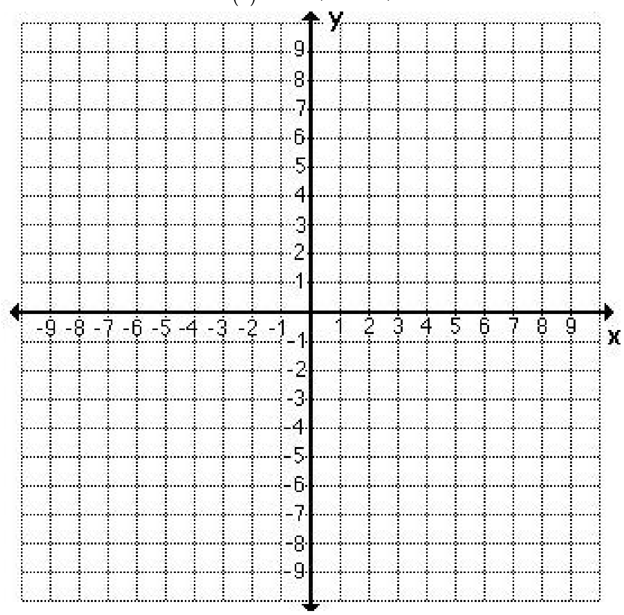
D) $f_{(x)} = -2(x+3)^2 + 5$



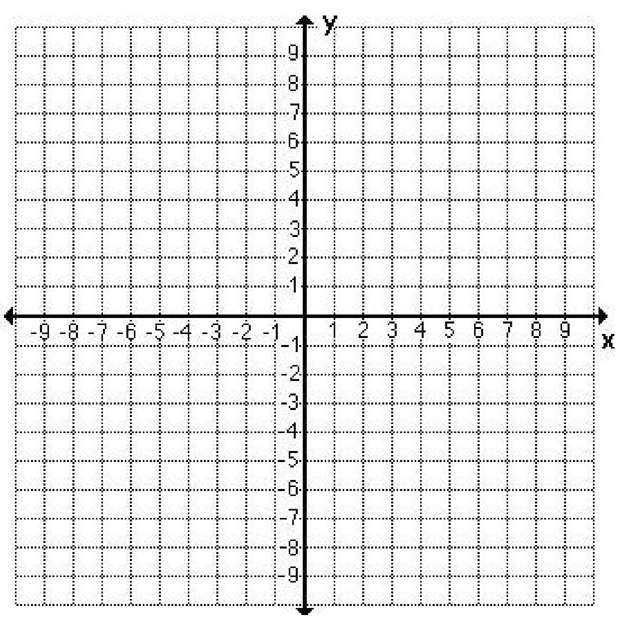
E) $f_{(x)} = x^2 + 3$



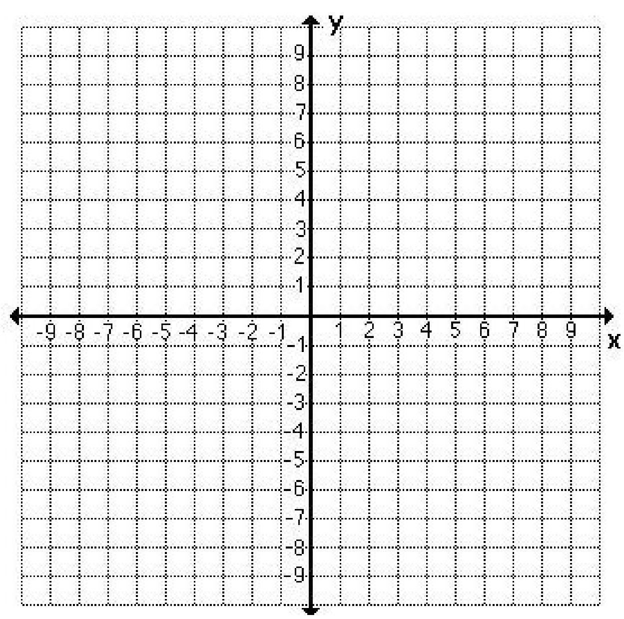
F) $f_{(x)} = -(x+3)^2 + 4$



G) $f_{(x)} = -2(x-4)^2$



H) $f_{(x)} = 2(x-1)^2 - 8$



The quadratic function given by the equation $f_{(x)} = 3(x-2)^2 + 6$ has an axis of symmetry of_____________.

The quadratic function given by the equation $f_{(x)} = -3(x+6)^2 - 4$ has an axis of symmetry of_____________.

The quadratic function given by the equation $f_{(x)} = a(x-h)^2 + k$ has an axis of symmetry of_____________.

Considering your answers to the previous questions, we can conclude that the axis of symmetry for any quadratic function is given by the x value of the ______________.

Why does the axis of symmetry look as though we are saying x equals a number ($x = \#$)?

Why is it sometimes necessary to graph a quadratic function using the axis of symmetry?

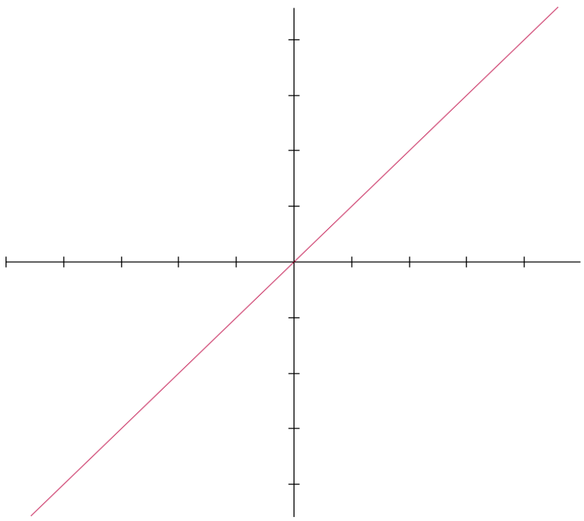
<u>Absolute Value Functions</u>

In order to graph an absolute value function, you will be using many of the same methods you did for quadratics. The standard form of an absolute value function is nearly identical to that of a quadratic function.

$$f_{(x)} = a|x-h|+k$$

The standard graph by which we translate absolute value functions comes from the equation of the diagonal line $y = x$.

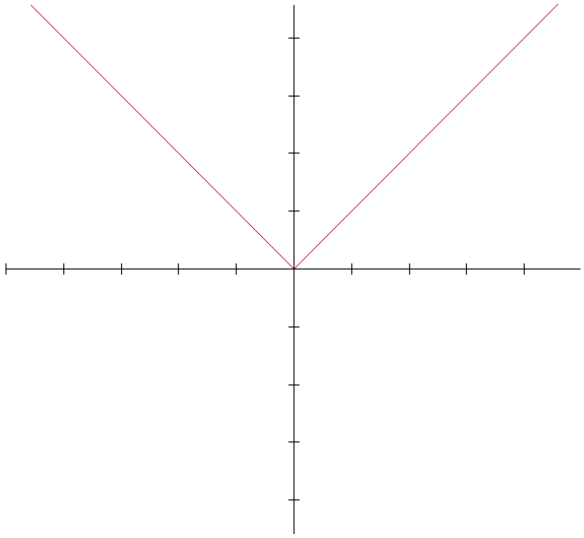
$y = x$



This is the graph of the function $y = x$***. In this case, the x and y values of coordinates are identical. For example, (-3,-3). You can see the x and y values are the same.***

Now, lets take a look at what happens when I want the absolute value of x.

$y = |x|$

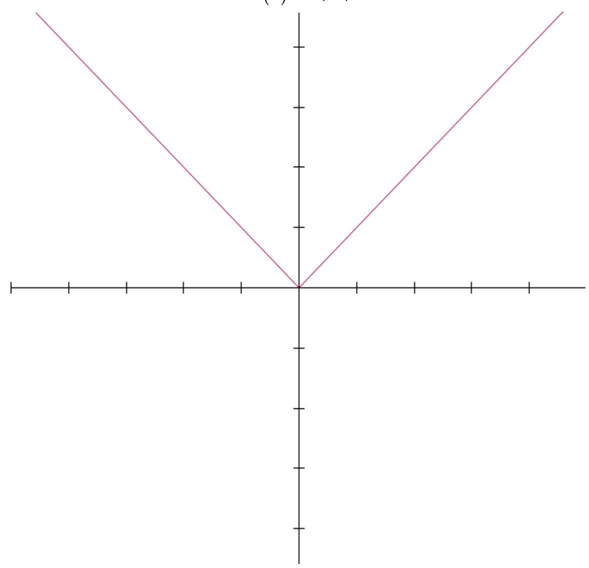


If the graph of $y = x$ ***above is*** $f_{(x)}$***, the function to the left is*** $|f_{(x)}|$***. We know that the absolute value of a number cannot be negative. If we take the absolute value of*** $f_{(x)}$***, it would cause the left portion of the graph above to reflect above the x axis. Now as you can see, all y values of the function are positive. This is where the graph of the absolute value of x comes from.***

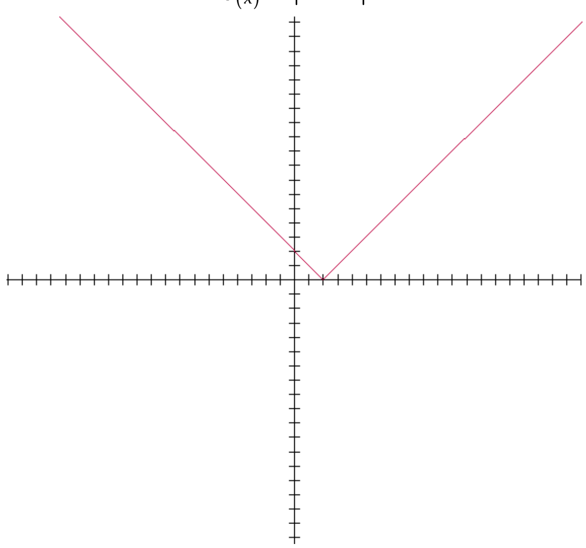
As we look at the following absolute value functions, you will notice how similar they are to quadratic functions. The vertex of an absolute value function is also given by (h,k). Horizontal and vertical shifts are identical, as well as the effect the value of a ***has on the graph. The rules for finding the range and domain of an absolute value function are also the same as a quadratic. Sometimes, an axis of symmetry must be used to graph your function. Intercepts are found by substituting zero for either x or y, and solving for the remaining variable.***

$$f_{(x)} = a|x-h|+k$$

$$f_{(x)} = |x|$$

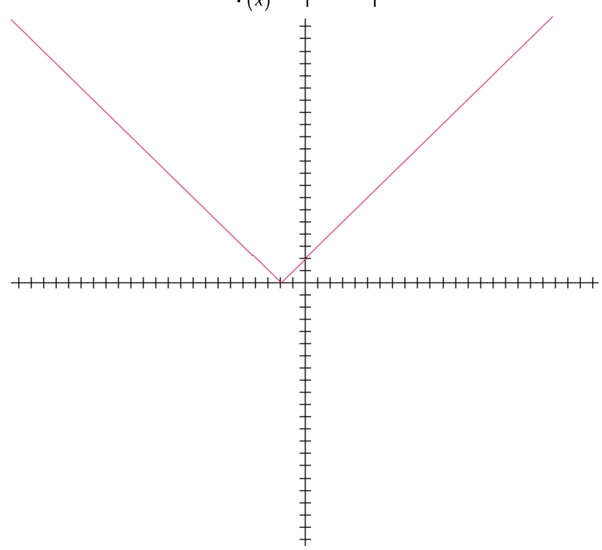


$$f_{(x)} = |x-2|$$



The graph of this function shifts to the right 2.

$$f_{(x)} = |x+2|$$

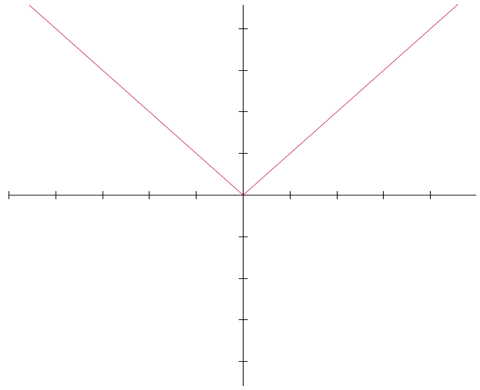


The graph of this function shifts to the left 2.

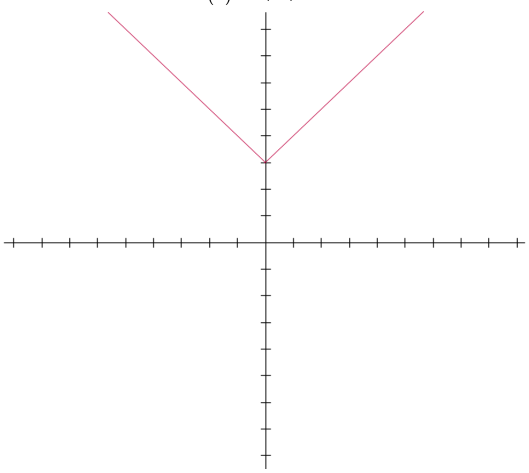
Once again, notice that the value of h ***determines the horizontal shift of the function. If the function is defined as*** $f_{(x)}$***, the graph on the left is*** $f_{(x-2)}$***, while the graph on the right is*** $f_{(x+2)}$***.***

$$f_{(x)} = a|x-h|+k$$

$$f_{(x)} = |x|$$

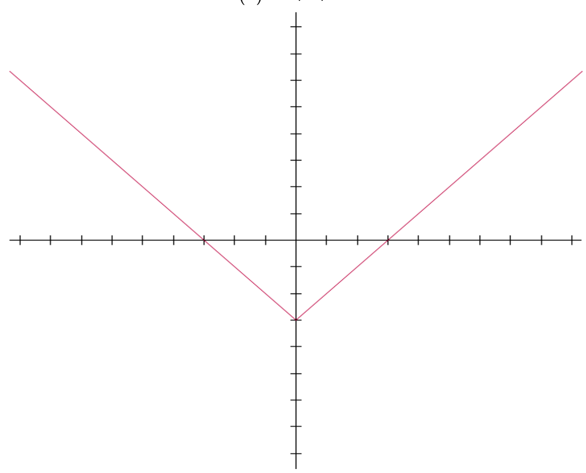


$$f_{(x)} = |x|+3$$



The graph of this function shifts up 3.

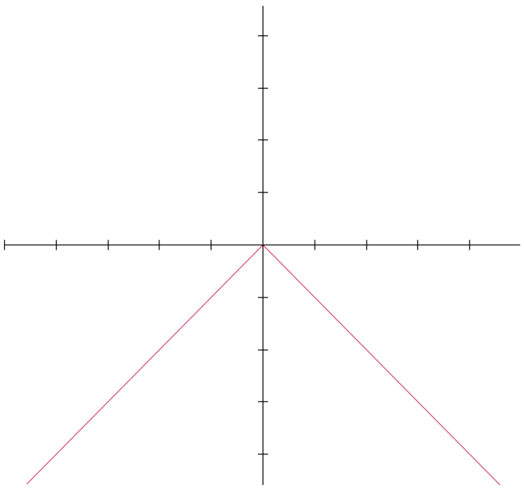
$$f_{(x)} = |x|-3$$



The graph of this function shifts down 3.

The value of k***, for an absolute value function in standard form determines the vertical shift of the function. As before, if the function is simply defined as*** $f_{(x)}$***, we are looking at*** $f_{(x)}+3$ ***and*** $f_{(x)}-3$ ***respectively.***

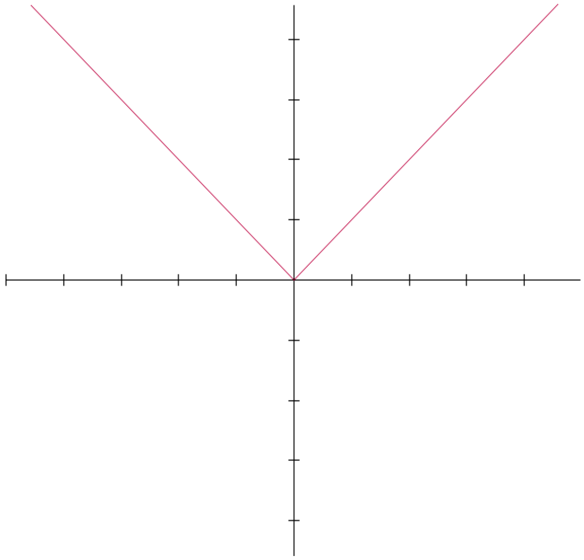
$$f_{(x)} = -|x|$$



On the left we have the opposite of the parent function. In this example, the value of a***, in the standard form is -1. A negative reflects the graph of the function about the horizontal axis. This is read as the opposite of the absolute value of x. If the parent function given is referred to as*** $f_{(x)}$***, this function is*** $-f_{(x)}$***.***

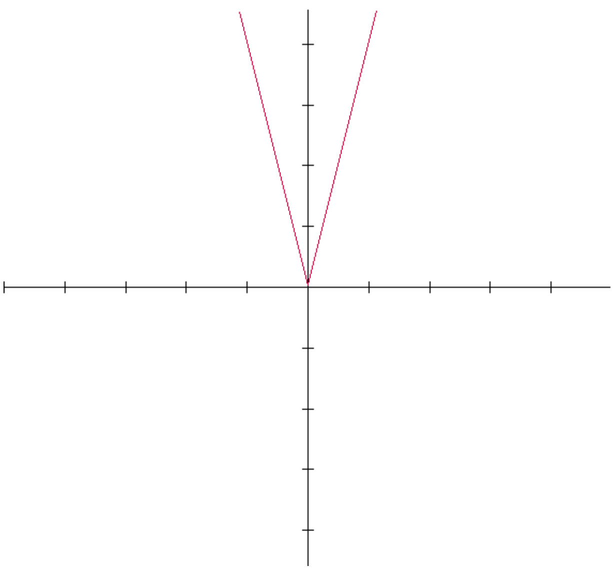
$$f_{(x)} = a|x-h|+k$$

$$f_{(x)} = |x|$$



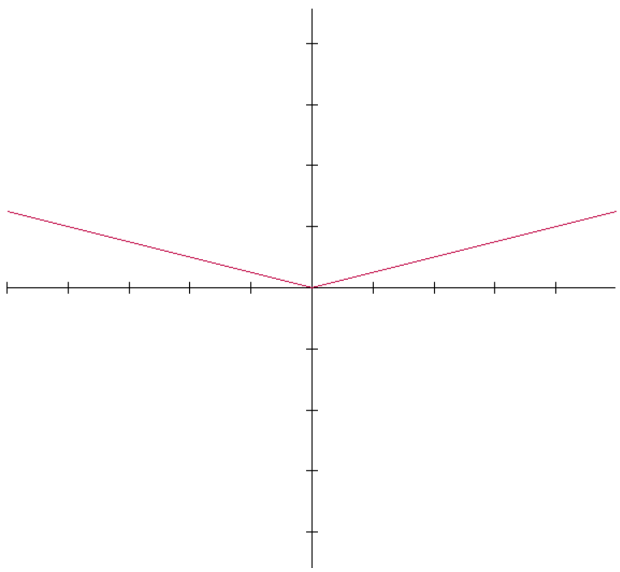
Here we will see how the value of a in an absolute value function in standard form affects the graph of the function. To illustrate this, we will look at the following graphs that have their vertices on the origin.

$$f_{(x)} = 4|x|$$



This graph seems very narrow, but what is actually happening, is the value of the function is increasing very rapidly. The y values are increasing at 4 times their normal rate. The rapid increase causes the graph to appear narrow.

$$f_{(x)} = \frac{1}{4}|x|$$



This graph is wider than the parent function. In this case, the y values of the function are increasing at ¼ their normal rate, causing a more gradual increase.

If the value of the leading coefficient is a whole number, the y values of the graph will increase rapidly causing a narrow graph and more extreme slope. If the leading coefficient is a fraction, the y values of the function will increase mildly, yielding a more gradual slope.

Describe the movement of each of the following absolute value functions. Describe how the graph of the function opens and if there is any horizontal or vertical movement. Be sure to tell identify how many spaces it moves, for example: ***This graph opens up, and shifts left 6, up 3.***

A) $f_{(x)} = 3|x-4|+1$ **B)** $f_{(x)} = -|x+1|+6$ **C)** $f_{(x)} = |x|+4$

D) $f_{(x)} = |x+5|-2$ **E)** $f_{(x)} = |x-3|$ **F)** $f_{(x)} = -3|x+1|+3$

G) $f_{(x)} = \frac{1}{2}|x-3|+2$ **H)** $f_{(x)} = -|x+6|$ **I)** $f_{(x)} = \frac{2}{3}|x|-4$

State the range and domain for each of the following.

A) $f_{(x)} = 3|x-4|+1$ **B)** $f_{(x)} = -|x+1|+6$ **C)** $f_{(x)} = |x|+4$

D) $f_{(x)} = |x+5|-2$ **E)** $f_{(x)} = |x-3|$ **F)** $f_{(x)} = -3|x+1|+3$

G) $f_{(x)} = \frac{1}{2}|x-3|+2$ **H)** $f_{(x)} = -|x+6|$ **I)** $f_{(x)} = \frac{2}{3}|x|-4$

Find the vertex of each of the following absolute value functions.

A) $f_{(x)} = -|x+3|$

B) $f_{(x)} = 3|x-2|-4$

C) $f_{(x)} = \frac{1}{2}|x|-2$

D) $f_{(x)} = |x-2|+3$

E) $f_{(x)} = 2|x+6|+9$

F) $f_{(x)} = -|x-4|+7$

Solve each of the following absolute value equations. ***This is what you will need to do to find the x intercepts of absolute value functions. Remember, first isolate the absolute value, then set up two separate equations to find your solutions.***

$$|x+3|-6=0$$
$$|x+3| \quad =6$$

Set equation equal to 6 and solve by subtracting 3 to both sides.	$x+3=6$ $x=3$	***and***	$x+3=-6$ $x=-9$	***Set equation equal to -6 and solve by subtracting 3 to both sides.***

So the two solutions are 3 and -9. These would be the x intercepts of the graph of the function $y=|x+3|-6$ ***.***
Watch for abnormalities. Iif the absolute value equals a negative number, you cannot create two problems. If this happens, there will be no solutions to the problem, which in terms of the graph of the function, tells you that there are no x intercepts.

A) $|x-4|-2=0$

B) $2|x+4|-12=0$

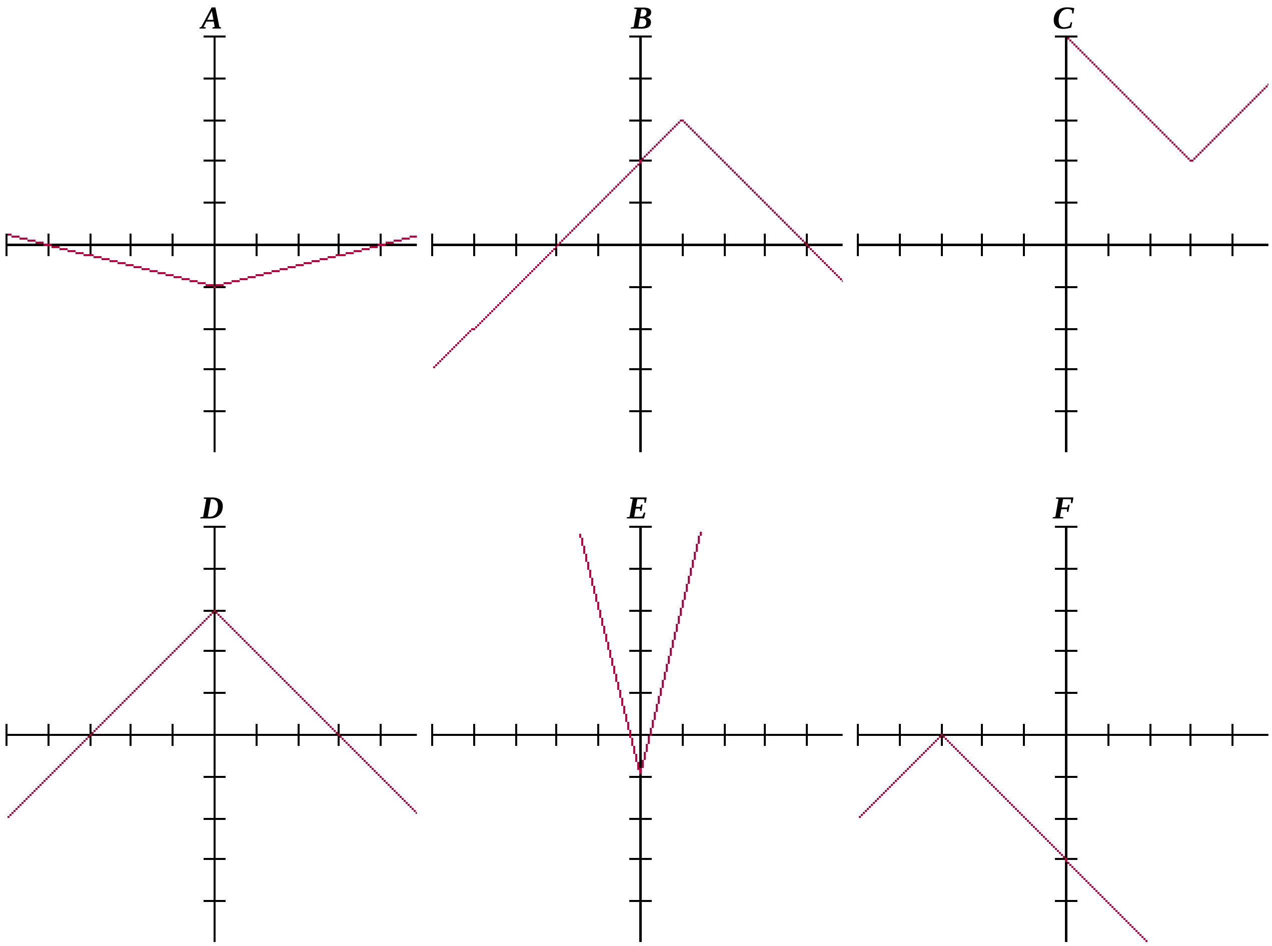
C) $-\frac{1}{2}|x|+1=0$

D) $-2|x-3|=0$

E) $|x-6|-5=0$

F) $-3|x-1|+2=0$

Match the appropriate graph with its equation below. Explain why each of your solutions is true.



1) $f_{(x)} = 4|x| - 1$

2) $f_{(x)} = \frac{1}{4}|x| - 1$

3) $f_{(x)} = -|x| + 3$

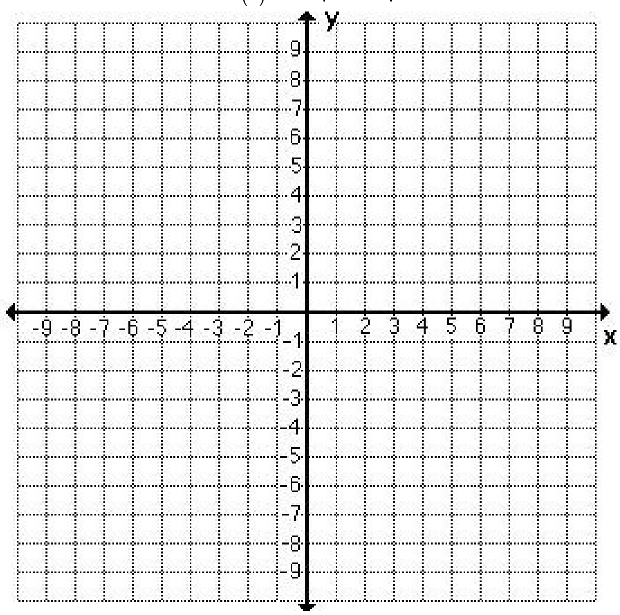
4) $f_{(x)} = |x - 3| + 2$

5) $f_{(x)} = -|x - 1| + 3$

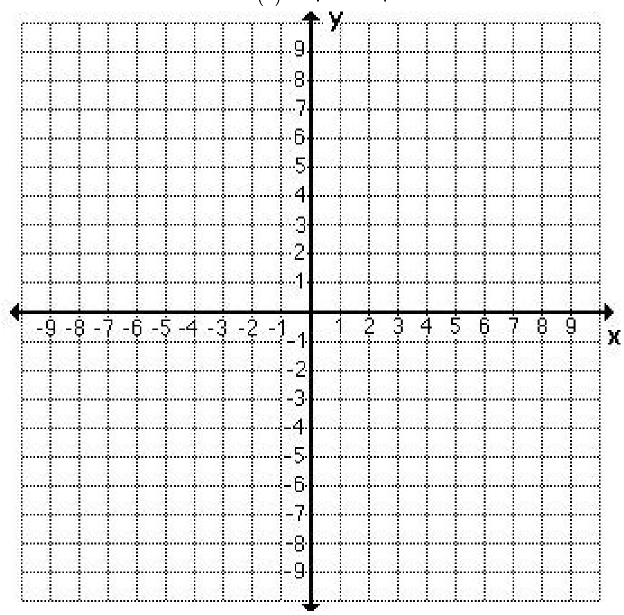
6) $f_{(x)} = -|x + 3|$

Graph each of the following functions. You may need to use an axis of symmetry to graph some of these. **Label the vertex, y-intercept, and all x-intercepts. Remember, to find the x intercepts of an absolute value function you will need to set the function equal to zero and solve an absolute value equation.**

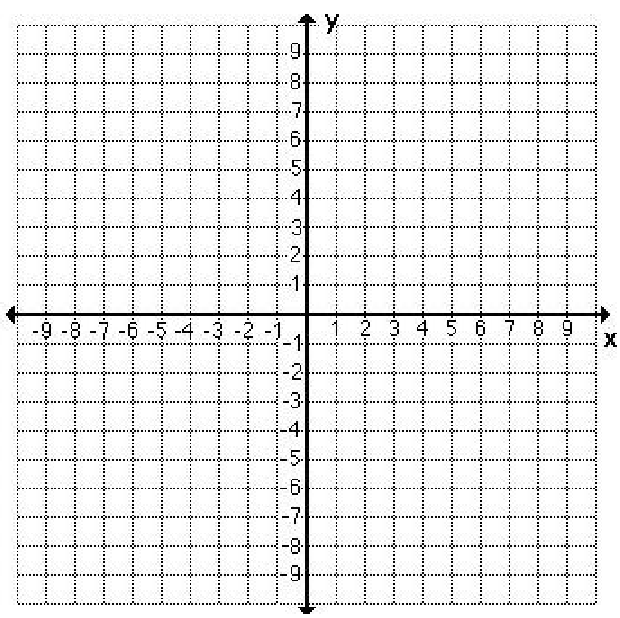
A) $f_{(x)} = -|x-2|+3$



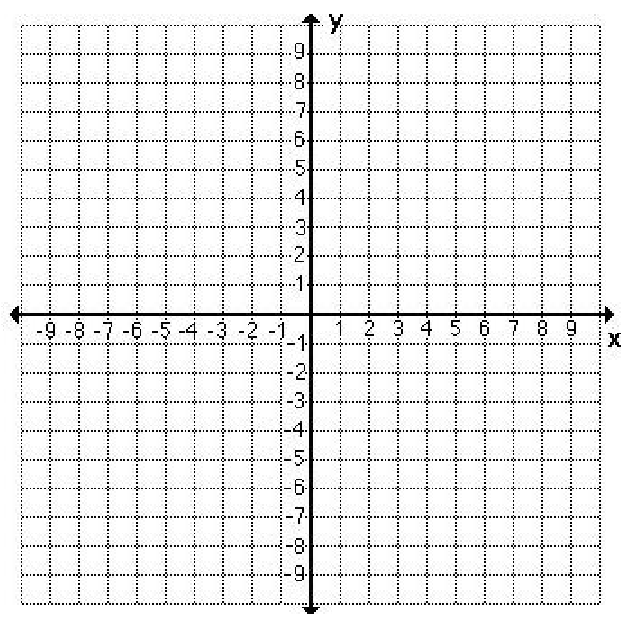
B) $f_{(x)} = |x+4|-4$



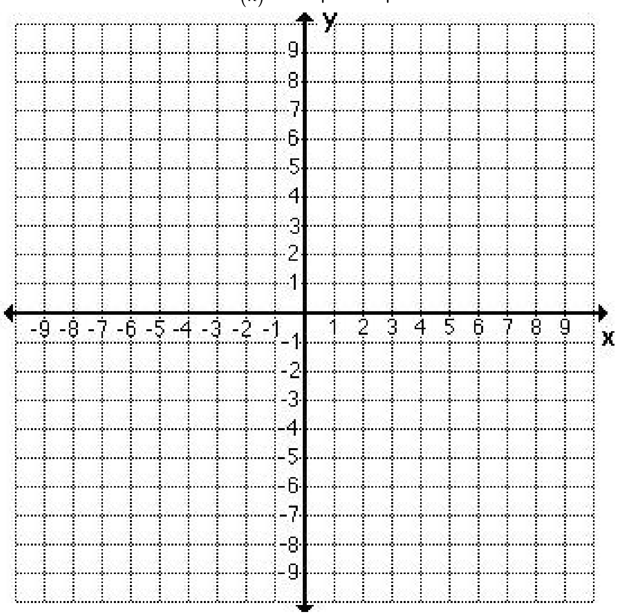
C) $f_{(x)} = -|x-5|$



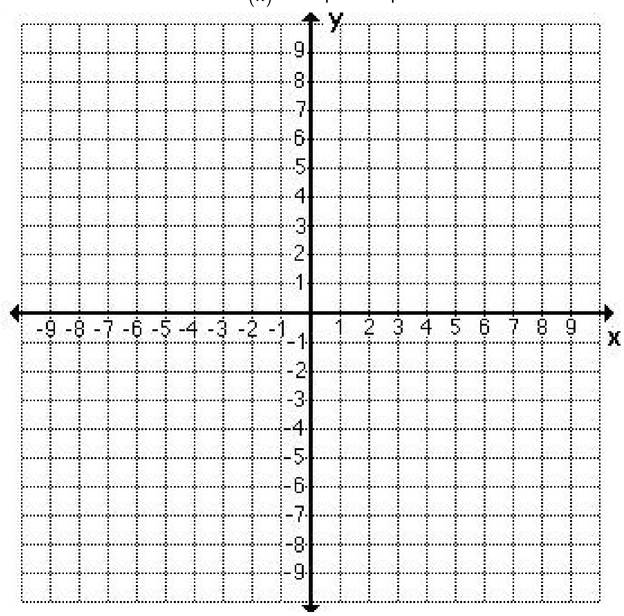
D) $f_{(x)} = \frac{1}{2}|x|-3$



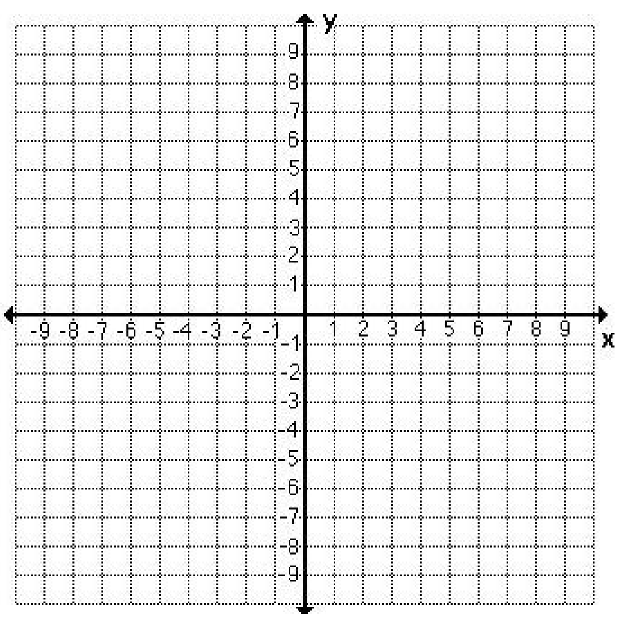
E) $f_{(x)} = -|x-3| - 2$



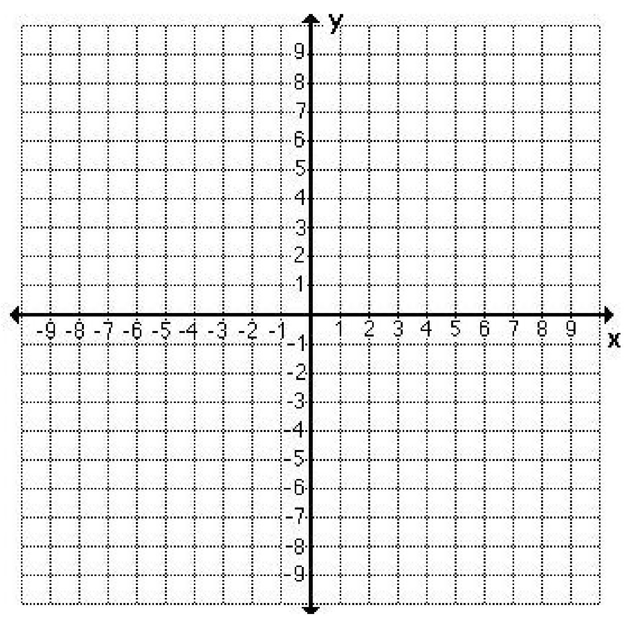
F) $f_{(x)} = 3|x+5| - 6$



G) $f_{(x)} = |x-3| + 2$



H) $f_{(x)} = -2|x| + 4$



Translations of Functions

We will now look at graphing a function without actually knowing the equation. Based on the graph of a function, it will be possible to shift, or translate the graph in any manner indicated.

For example, if given the picture of a graph and told "This is the graph of the function $f_{(x)}$." Proceed to first identify the coordinate of any vertex seen. These will serve as a guide for the graph of the function's translation.

To graph the function of $f_{(x+6)}$, the function will need to shift to the left 6 spaces. To accomplish this, subtract 6 from all x values in the original function. The results will be the coordinates for the new graph. Likewise, to graph $f_{(x-4)}$, this function will need to shift to the right 4 spaces, so add 4 to all x values.

In order to graph $f_{(x)}+5$, the function will shift up 5 spaces, requiring that 5 be added to all y values. If asked to graph $f_{(x)}-3$, the will function shift down 3 spaces, meaning subtract 3 from all y values.

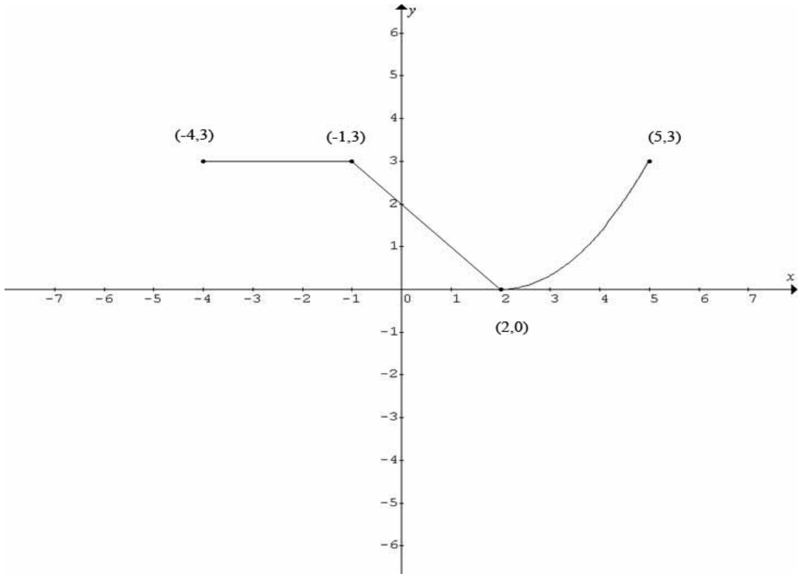
If the number is in the parenthesis, the function is shifting using P.L.N.R.. If the number is after the $f_{(x)}$, simply shift as indicated, + says shift up, - says shift down.

Any number in front of the $f_{(x)}$ will affect the scale of the function. This means it will affect the rate at which the function grows. When graphing, for example, $-f_{(x)}$, change the sign of all y values on the graph of the function. This will cause the graph of the function to flip upside down. A number other than -1 can also be used. Lets say we need to graph $3f_{(x)}$, this means the actual curve will increase 3 times as fast. It will therefore, be necessary to multiply all y values by 3. This will result in the coordinates for the new function. If the 3 were grouped with the x such as $f_{(3x)}$, the horizontal change is the inverse of what it appears to be. So instead of multiplying x values by 3, divide by 3.

When graphing $f_{(-x)}$, take the opposite of the x values of the function. This will cause the graph of the function to flip along a vertical axis.

Combinations of these rules will be encountered throughout your study of functions, for example, to shift right 3 and up 6. Just stick with the rules and the graph will be translated to its new location. If faced with a problem such as $2f_{(x)}+3$, <u>follow the order of operations</u>. Multiply all y values by 2 first, then add 3 to each. Referring to the previous two topics, quadratic functions and absolute value functions, you will find references to these rules and examples throughout.

The following is the graph of the function $f_{(x)}$. Use this to graph each function for letters A-D.



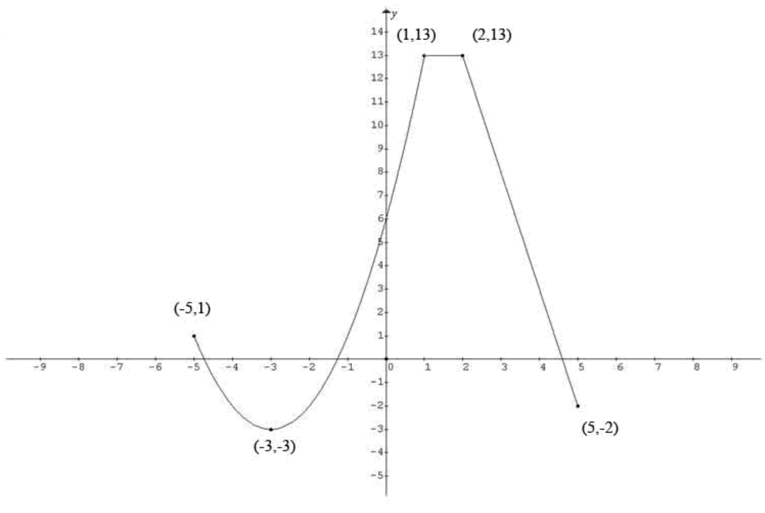
A) $f_{(x+2)}$

B) $f_{(x)} - 3$

C) $-f_{(x)} + 1$

D) $f_{(-x)}$

The following is the graph of the function $f_{(x)}$. Use this to graph each function for letters E-H.



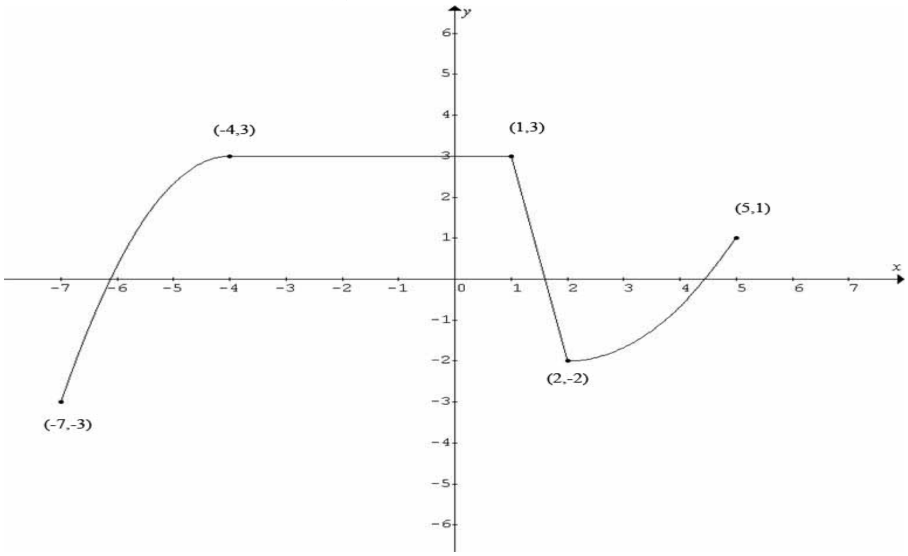
E) $f_{(x-1)}$

F) $\left|f_{(x)}\right|$

G) $f_{(x)} - 4$

H) $f_{(x+2)} - 3$

The following is the graph of the function $f_{(x)}$. Use this to graph each function for letters I-L.



I) $f_{(-x)}$

J) $f_{(x-2)} - 1$

K) $-f_{(x)} + 1$

L) $\frac{1}{3} f_{(x)}$

Radical Functions

For radical functions we will use the equation $f_{(x)} = a\sqrt{x-h}+k$ to denote the standard form of the equation. Be aware, that the variable x may have a coefficient from time to time. Follow the standard procedure to find the x and y intercepts of any radical function. Set the x or y equal to zero, depending on which one you wish to find, and solve for the remaining variable. Finding the domain of a radical function is a little tricky. To find the domain of any radical function with an even index, set the radicand greater that or equal to zero ($\geq$) and solve. If the radicand is a polynomial, you will need to solve the polynomial inequality by finding critical points, and testing intervals. To find the range of the radical function, find y value of the point of origin, and use the constant a to determine the range of the function.

Given the radical function $f_{(x)} = -\sqrt{x+4}-3$, the following can be determined.

First find the domain of the function. This will give you the x value needed for the point of origin.

$$f_{(x)} = -\sqrt{x+4}-3$$

Finding the domain.

$$x+4 \geq 0$$

$$x \geq -4$$

You can see the domain of the function is $[-4,\infty)$.
The -4 is the x value the point of origin.

Finding the range.

Since the constant a is -1, the function will go downwards. Meaning that the range is $(-\infty,-3]$.

Finding the "point of origin" of a radical function.

To find the point of origin of a radical function use the rules discussed in previous sections. The point of origin for the parent function $y = \sqrt{x}$ is (0, 0). This particular graph will shift left 4 and down 3, so the point of origin is $(-4,-3)$. Be careful when using these rules. Make sure to find the domain of the function before you attempt to find the point of origin. Consider a function such as $y = \sqrt{3-x}$. Since there is a positive 3 inside the radicand, you would normally shift to the left 3. However, If you were to find the domain of this function by setting the radicand ≥ 0, You will find the domain is actually $x \leq 3$. This says the graph is shifting to the right 3 spaces.

Finding the x-intercept.

Substitute 0 for y and solve for x.

$$0 = -\sqrt{x+4}-3$$

$$\sqrt{x+4} = -3$$

This is not possible. That means there is no x intercept for this function.

Finding the y-intercept.

Substitute 0 for x and solve for y.

$$y = -\sqrt{(0)+4}-3$$

$$y = -\sqrt{4}-3$$

$$y = -2-3$$

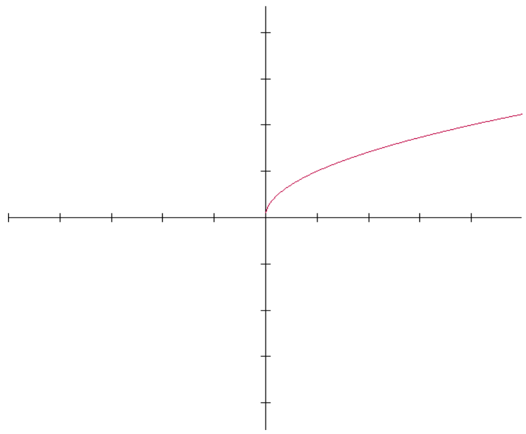
$$y = -5$$

The y intercept of this function is (0, -5).

We will now look at the parent function, and some translations of it.

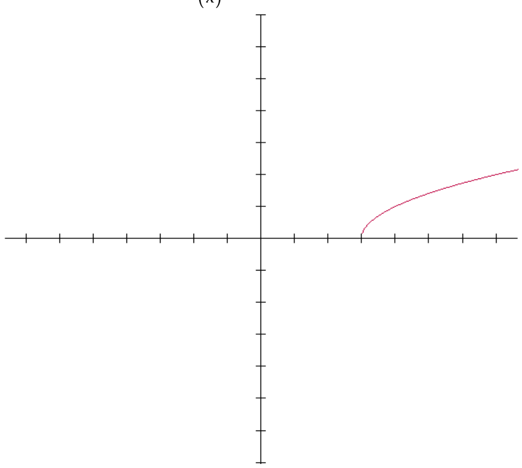
$$f_{(x)} = a\sqrt{x-h}+k$$

$$f_{(x)} = \sqrt{x}$$



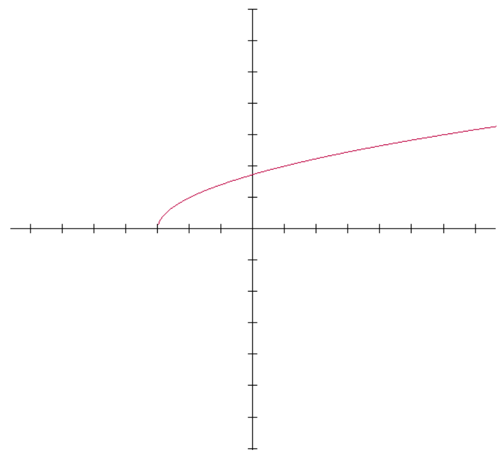
The parent function has the point of origin at (0, 0)

$$f_{(x)} = \sqrt{x-3}$$



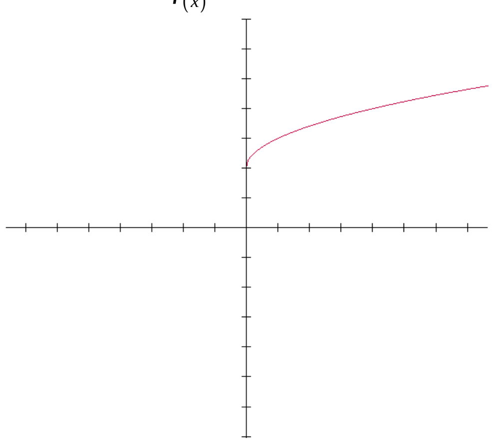
The graph of this function shifts right 3.

$$f_{(x)} = \sqrt{x+3}$$



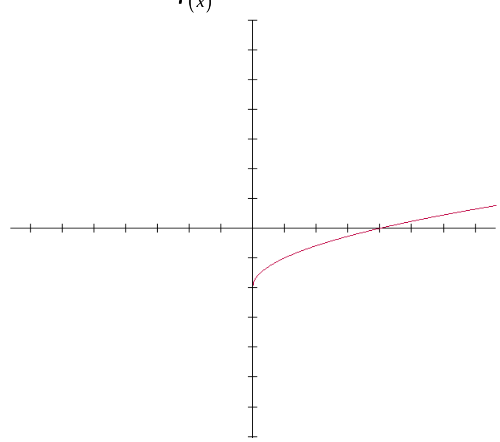
The graph of this function shifts left 3.

$$f_{(x)} = \sqrt{x}+2$$



Here the graph shifts up 2.

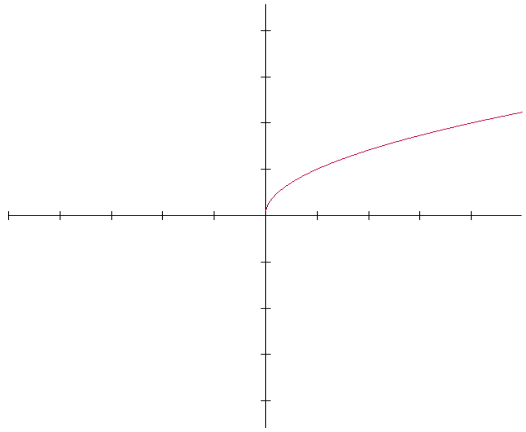
$$f_{(x)} = \sqrt{x}-2$$



The graph of this function shifts down 2.

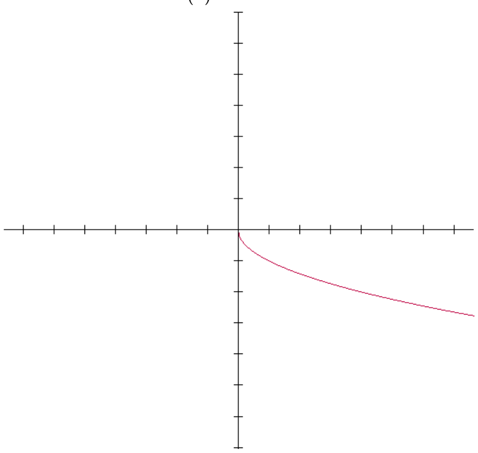
$$f_{(x)} = a\sqrt{x-h} + k$$

$$f_{(x)} = \sqrt{x}$$



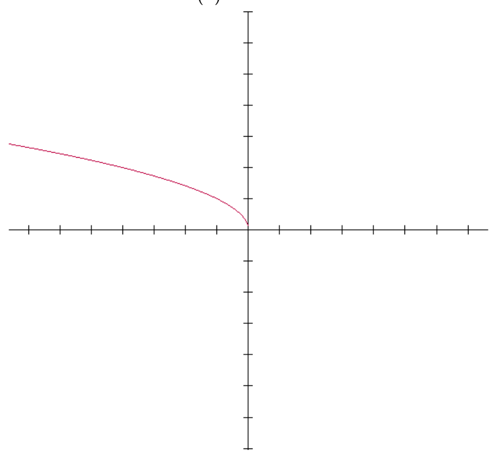
The parent function has the point of origin at (0, 0)

$$f_{(x)} = -\sqrt{x}$$



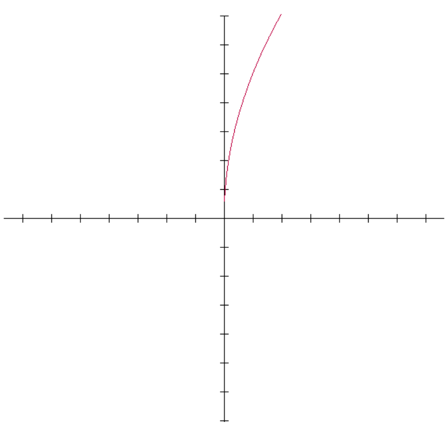
The graph of this function flips upside down.

$$f_{(x)} = \sqrt{-x}$$



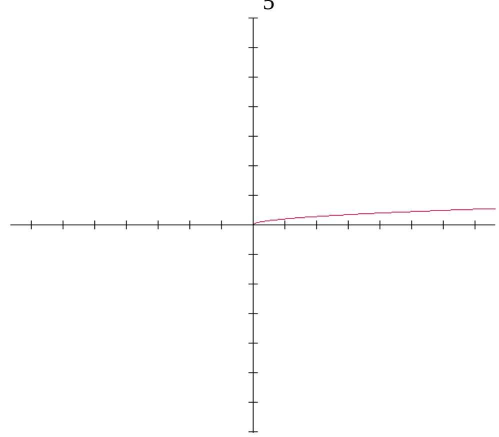
The graph of this function flips from right to left as the –x affects the domain of the function.

$$f_{(x)} = 5\sqrt{x}$$



Scale increased by a factor of 5.

$$f_{(x)} = \frac{1}{5}\sqrt{x}$$



This is 1/5 the normal scale.

Find the domain of each of the following radical functions in interval notation.

A) $f_{(x)} = \sqrt{x+4} - 2$

B) $f_{(x)} = 2\sqrt{4-x} + 1$

C) $f_{(x)} = \sqrt{2x+3} + 1$

D) $f_{(x)} = \sqrt{x^2 - 4}$

E) $f_{(x)} = \sqrt{x^2}$

F) $f_{(x)} = \frac{1}{2}\sqrt{6-x} - 3$

G) $f_{(x)} = -\sqrt{x+5} - 8$

H) $f_{(x)} = \sqrt{2-x} + 1$

I) $f_{(x)} = 2\sqrt{x+7} - 5$

The range of a radical function in $f_{(x)} = a\sqrt{x-h} + k$ ***form can be found using the value of the "*** a ***" term, and the y value of the point of origin.***

If $a > 0$***, the range of the function is*** $[k, \infty)$***.***
If $a < 0$***, the range of the function is*** $(-\infty, k]$***.***

Find the range for each of the following.

A) $f_{(x)} = \sqrt{x+5} - 3$

B) $f_{(x)} = -\sqrt{x-3} + 2$

C) $f_{(x)} = 2\sqrt{x-4} + 3$

D) $f_{(x)} = -3\sqrt{5-x} + 6$

E) $f_{(x)} = \sqrt{4-x} - 3$

F) $f_{(x)} = \sqrt{x-7} + 5$

Find the point of origin for each of the following radical functions.

A) $f_{(x)} = \sqrt{x+4} - 2$

B) $f_{(x)} = 2\sqrt{4-x} + 1$

C) $f_{(x)} = \sqrt{x} - 4$

D) $f_{(x)} = -\sqrt{x-3}$

E) $f_{(x)} = \sqrt{x^2}$

F) $f_{(x)} = \frac{1}{2}\sqrt{6-x} - 3$

G) $f_{(x)} = -\sqrt{x+5} - 8$

H) $f_{(x)} = \sqrt{2-x} + 1$

I) $f_{(x)} = 2\sqrt{x+7} - 5$

Why is the graph of the function $f_{(x)} = \sqrt{-x}$ **moving towards the left rather than the right?**

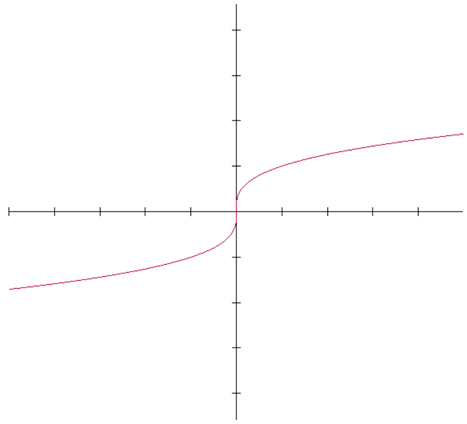
Explain why the graph of the function $f_{(x)} = \sqrt{x^2}$ **is identical to that of** $f_{(x)} = |x|$**.**

To find the domain of a radical function that has an even index, why do you need to set the radicand ≥ 0?

We will now look at the cube root function.

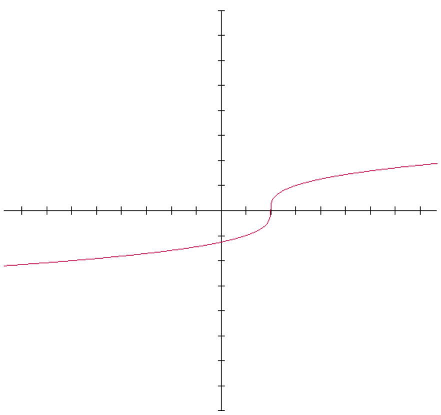
$$f_{(x)} = a\sqrt[3]{x-h}+k$$

$$f_{(x)} = \sqrt[3]{x}$$



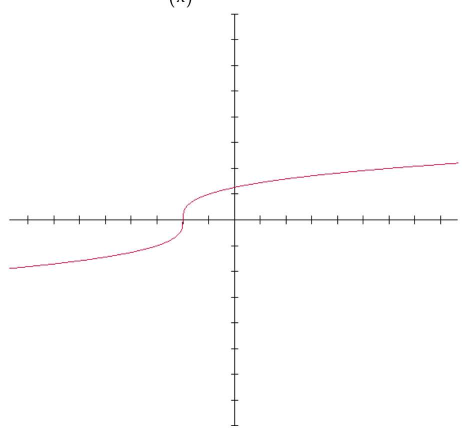
The parent function has the point of origin at (0, 0)

$$f_{(x)} = \sqrt[3]{x-2}$$



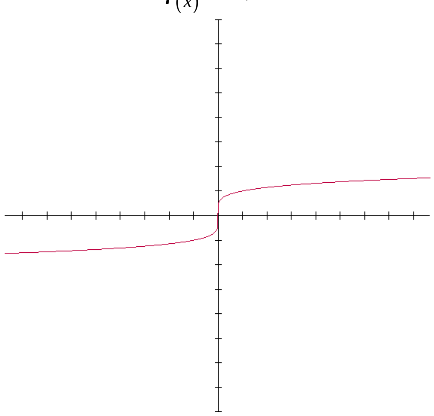
The graph of this function shifts right 2.

$$f_{(x)} = \sqrt[3]{x+2}$$



The graph of this function shifts left 2.

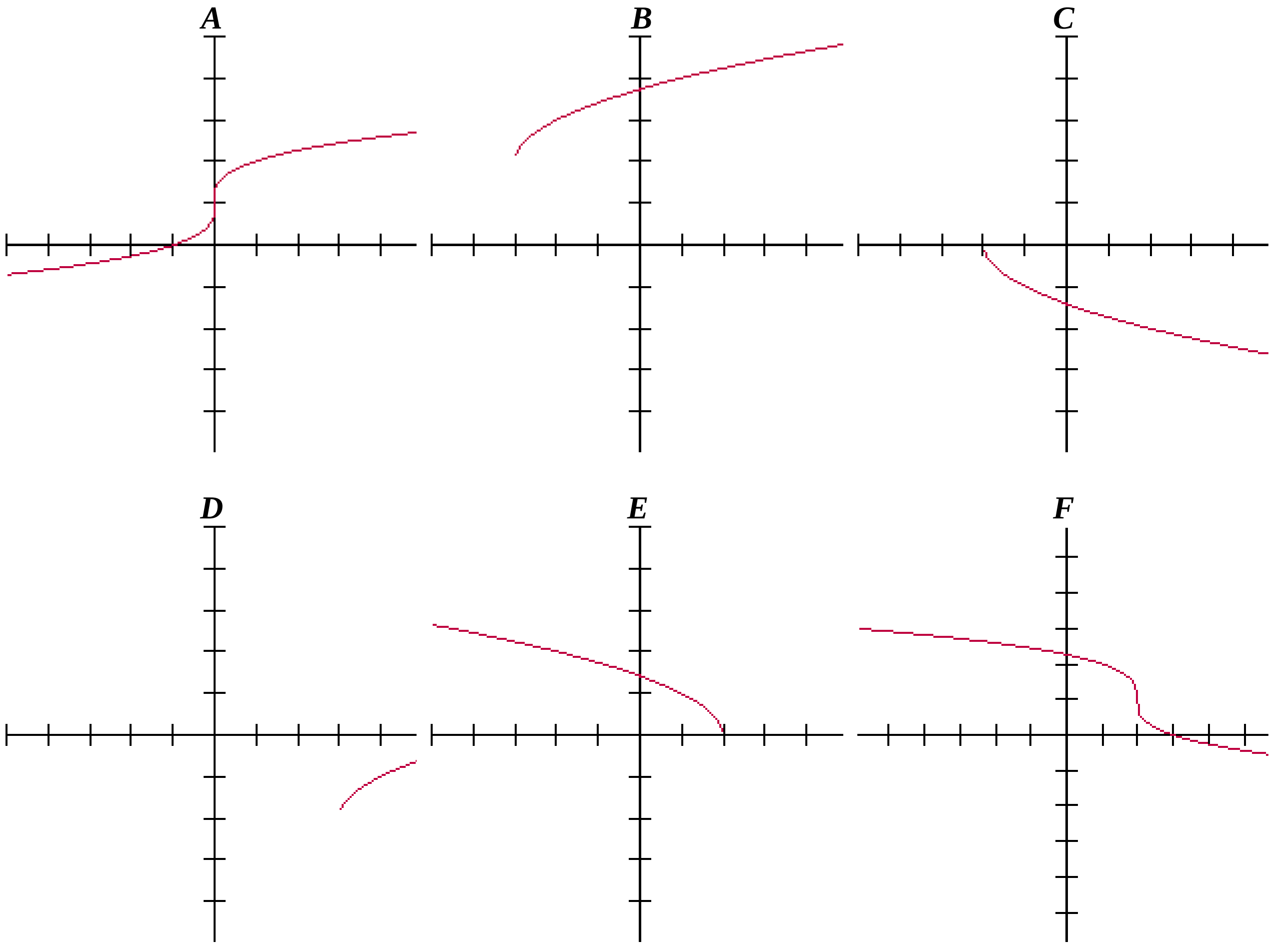
$$f_{(x)} = \sqrt[5]{x}$$



As you can see on the left, the curve is just about the same for a 5th root, verses a cubed root. This will be the same case for any radical function where the index is odd. This also means that any radical function where the index is even will look like a normal square root function. The curves of these functions are a little "flatter" than a regular square root or cubed root.

Vertical translations of the function are identical to that of a regular square root function. As you can see, the domain and range of any radical function with an odd index is all real numbers.

Match the appropriate graph with its equation below. Explain why each of your solutions is true.



1) $f_{(x)} = \sqrt{x+3}+2$

2) $f_{(x)} = \sqrt{x-3}-2$

3) $f_{(x)} = \sqrt[3]{x}+1$

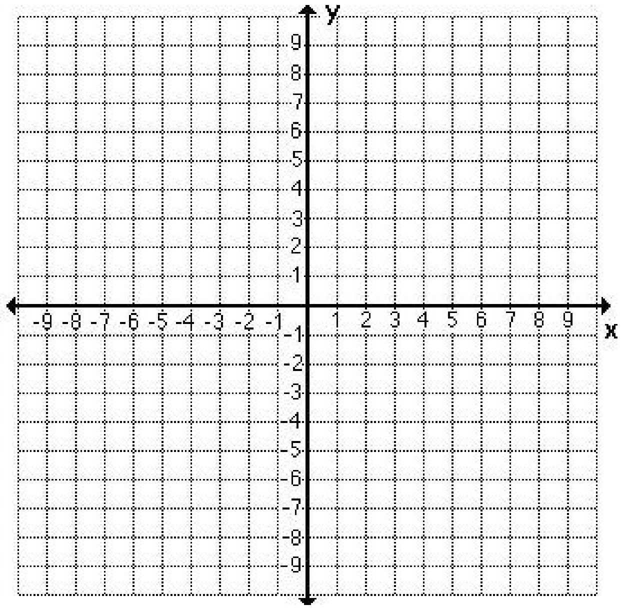
4) $f_{(x)} = -\sqrt[3]{x-2}+1$

5) $f_{(x)} = -\sqrt{x+2}$

6) $f_{(x)} = \sqrt{2-x}$

Graph each of the following radical functions. ***Find all required information.***

A) $f_{(x)} = \sqrt{x-3}+2$



Point of Origin:

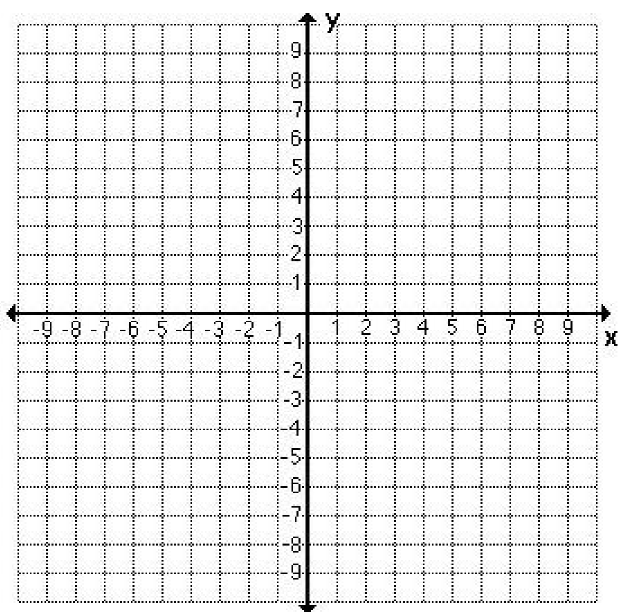
Y-intercept:

X-intercepts:

Range:

Domain:

B) $f_{(x)} = -\sqrt{x-3}+1$



Point of Origin:

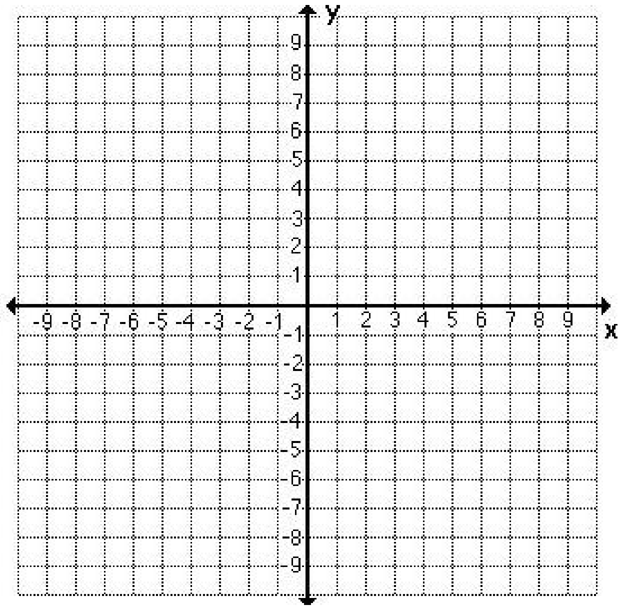
Y-intercept:

X-intercepts:

Range:

Domain:

C) $f_{(x)} = \sqrt{3-x}+1$



Point of Origin:

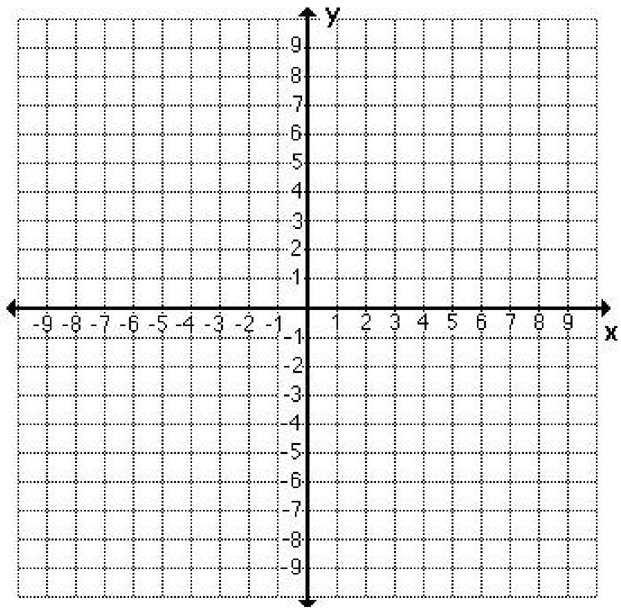
Y-intercept:

X-intercepts:

Range:

Domain:

D) $f_{(x)} = 2\sqrt{x}-4$



Point of Origin:

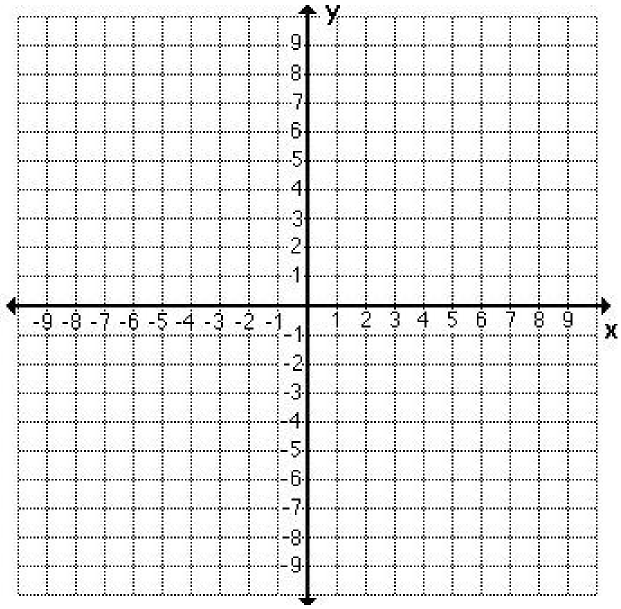
Y-intercept:

X-intercepts:

Range:

Domain:

E) $f_{(x)} = -\sqrt{-x}$



Point of Origin:

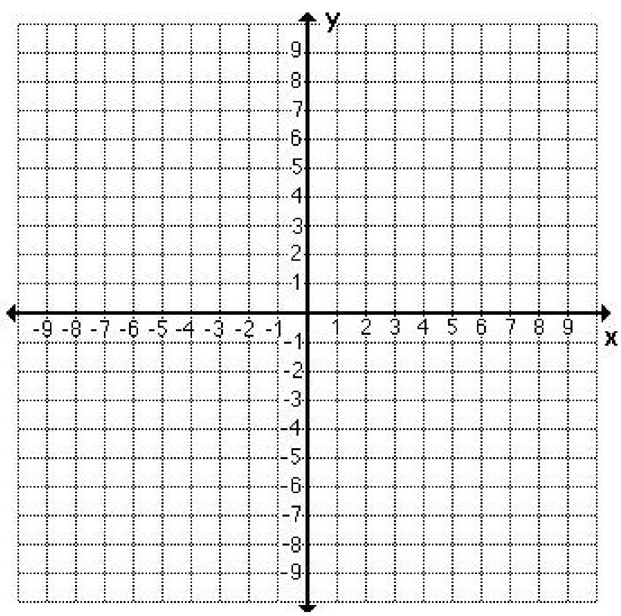
Y-intercept:

X-intercepts:

Range:

Domain:

F) $f_{(x)} = \sqrt[3]{x+2} + 3$



Point of Origin:

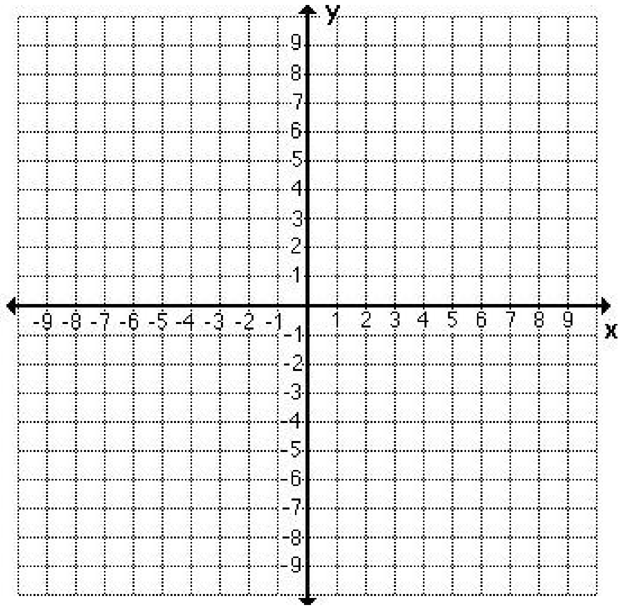
Y-intercept:

X-intercepts:

Range:

Domain:

G) $f_{(x)} = -\sqrt[3]{x-3} - 2$



Point of Origin:

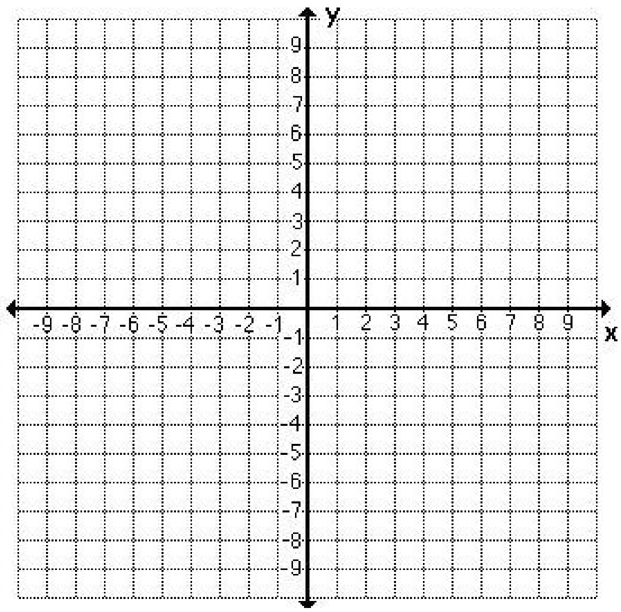
Y-intercept:

X-intercepts:

Range:

Domain:

H) $f_{(x)} = \sqrt[3]{x-6}$



Point of Origin:

Y-intercept:

X-intercepts:

Range:

Domain:

Why are the graphs of $y = \sqrt[3]{x}$ **and** $y = -\sqrt[3]{-x}$ **identical?**

Exponential Functions

This information was covered in a previous section of the workbook, but it won't hurt to go over it again.

Standard exponential function

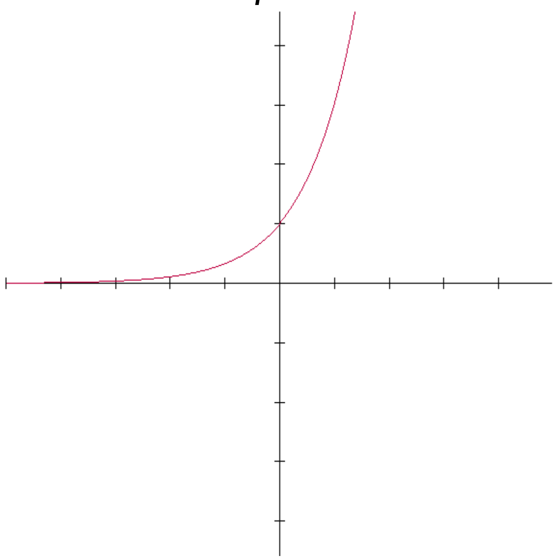
$$f_{(x)} = ca^{x-h} + k$$

The c term is a constant that can make to graph reflect about a horizontal axis or change to scale of the graph of the function proportionately. If c is a positive value, then you will have a standard looking growth or decay curve. If c is negative, the growth or decay curve will flip upside down. We will get into the effects different values of h and k have on this function shortly. What we will concentrate on here is identifying an exponential function as being growth or decay, and finding the range, domain and key point of the function.

Exponential Growth

$$f_{(x)} = ca^{x-h} + k$$

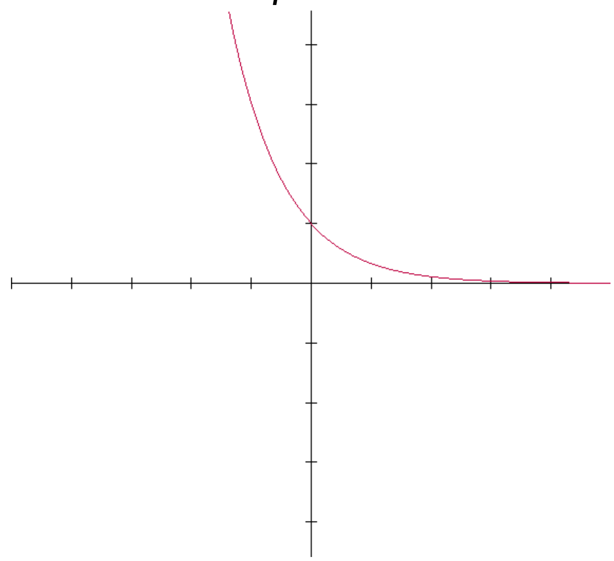
If a>1



Exponential Decay

$$f_{(x)} = ca^{x-h} + k$$

If 0<a<1



Obviously if $a = 1$, we are raising 1 to various powers, and we wind up getting a horizontal line because no matter what you do, raising one to any power still yields a result of one. Pay special attention to the exponential decay function. The statement $0 < a < 1$ is saying that the value of a is a fraction whose value is between zero and one. Do not make the mistake of just looking for a fraction to determine whether or not the function is decay. Make sure the value of the fraction is between zero and one.

The values for variables $h, \; k, \; and \; c$ act to make the graph shift left/right, up/down, change the scale or will reflect the function about a horizontal axis.

Notice the key point for each of these functions is the point $(0,1)$. This information is vital. This key point will shift depending on the values of h, k and c. To find the x value of the key point, evaluate $x-h=0$. In other words, find the value of x that would create a problem such as 3 to the zero power. This number is the x value of the key point. To find the y value, substitute the x value back in. Refer to the following example.

$$f_{(x)}=2^{x-3}+5$$

$$to\quad find\quad the\quad key\quad point\quad evaluate\quad x-3=0$$

$$x-3=0$$

$$x=3\quad this\quad is\quad the\quad x\quad value\quad of\quad the\quad key\quad point$$

$$now\quad substitute\quad 3\quad back\quad into\quad the\quad problem\quad for\quad x$$

$$f_{(3)}=2^{3-3}+5$$

$$f_{(3)}=2^{0}+5$$

$$f_{(3)}=1+5$$

$$f_{(3)}=6$$

$$so\quad the\quad key\quad point\quad is\quad (3,6)$$

$$f_{(x)}=ca^{x-h}+k$$

As we work to translate these functions, use $(0,1)$, as the default key point to any exponential growth or decay curve that is above the horizontal asymptote where the value of "c" is 1. In other words, if the value of "c" is positive one, use the point $(0,1)$ to assist you in shifting the function. If the graph of the function is below the horizontal asymptote, and the "c" value is -1, you will use $(0,-1)$ as the key point. If the value of "c" is <u>any</u> other number, you must find the key point algebraically. Consider the example above.

$$f_{(x)}=2^{x-3}+5$$

The first thing I did was notice that the graph of this function shifts right 3 and up 5. Pay special attention to the value of k. That tells you where your new horizontal asymptote is going to be, in this case, at $y=5$.

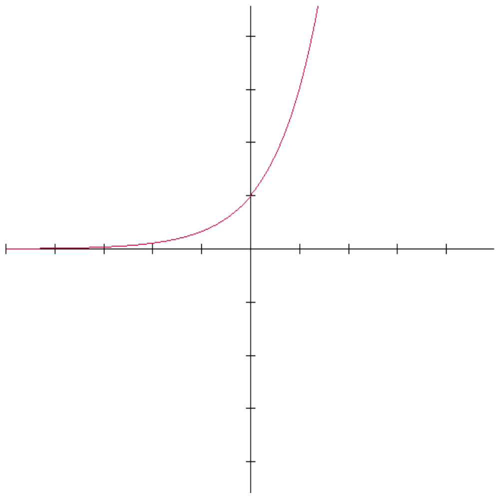
Since the value of c is positive one, begin at the key point $(0,1)$. Since the graph will shift right 3 and up 5, simply add 3 to the x value, and 5 to the y value of the key point. This produces a new key point of $(3,6)$. Observe how this information matches the work above. The key point in these functions acts as the vertex in a parabola. It gives you a point of reference with which to shift the function. Make sure the correct key point is used from the beginning, either $(0,1)$ or $(0,-1)$. <u>Remember</u>, if the "c" term is a number other that 1 or -1, the key point is actually multiplied by that number. For example, the function $f_{(x)}=3(2)^{x}$ has a key point of $(0,3)$, not $(0,1)$.

Here we will look at how to graph these functions by means of translation.

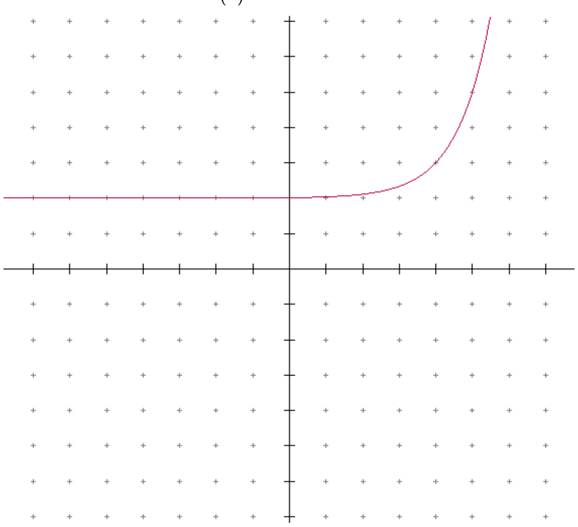
Exponential Growth

$$f_{(x)} = ca^{x-h} + k$$

$$f_{(x)} = 3^x$$

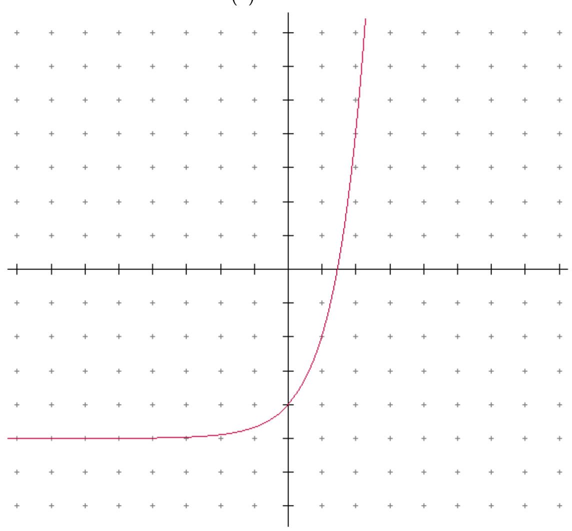


$$f_{(x)} = 3^{x-4} + 2$$



Notice that the horizontal asymptote is at y=2. Since the graph of this function is going to be above the x axis, begin with the key point (0,1). This function shifts right 4 and up 2. The dots have been left on the graph so it would be easier to see. Adding 4 to the x value of the key point, and 2 to the y value, the new key point is at (4,3). Just remember where to begin, and do not cross the horizontal asymptote.

$$f_{(x)} = 3^x - 5$$



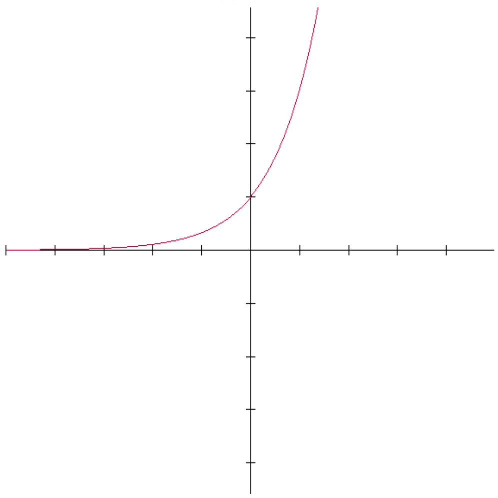
In this function, the value of k is -5. This tells you the new horizontal asymptote will be at y = -5. Since the value of the constant "c" is a positive one, begin with the key point (0,1). This function will only shift down 5 spaces. Therefore, subtract 5 from the y value of the key point which is 1. This results in: (1 – 5 = -4) therefore, the new key point is at (0,-4).

Graphing exponential functions by translation is relatively simple. The most difficult part will be finding the x and y intercepts as the x-intercept will involve the use of logarithms.

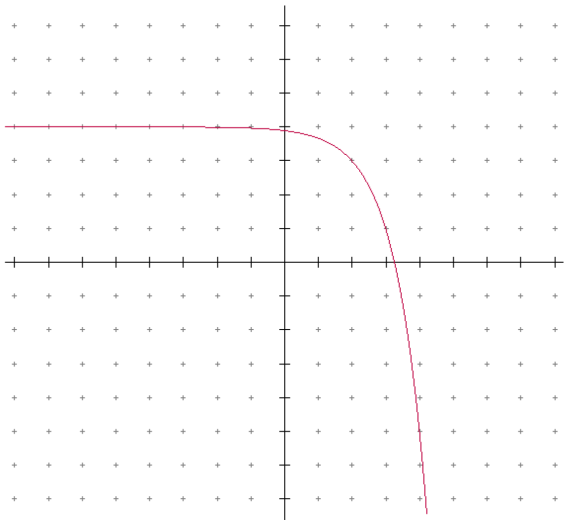
Exponential Growth

$$f_{(x)} = ca^{x-h} + k$$

$$f_{(x)} = 3^x$$

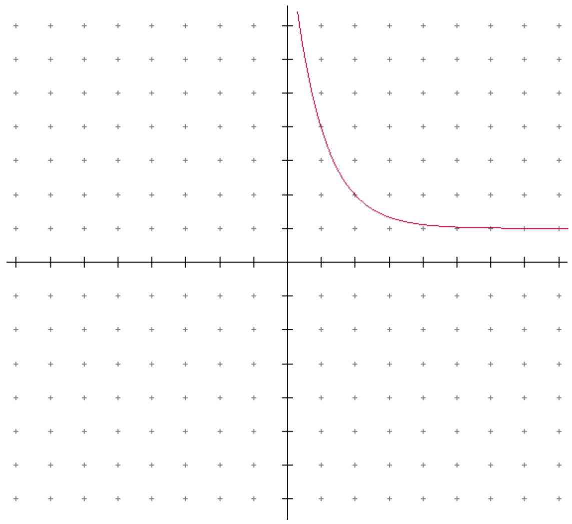


$$f_{(x)} = -3^{x-2} + 4$$



Since the value of "c" in the equation of this function is -1, we must begin with the key point of (0,-1). This is the key point, because that value of "c" caused the graph to reflect about the horizontal asymptote. The entire function will shift up 4, so the new horizontal asymptote is y = 4. The curve is going to shift right 2 and up 4. By adding 2 to the x value of the key point, and 4 to the y value, the new point can be found at (2,3). Notice the graph runs right though that point.

$$f_{(x)} = 3^{2-x} + 1$$

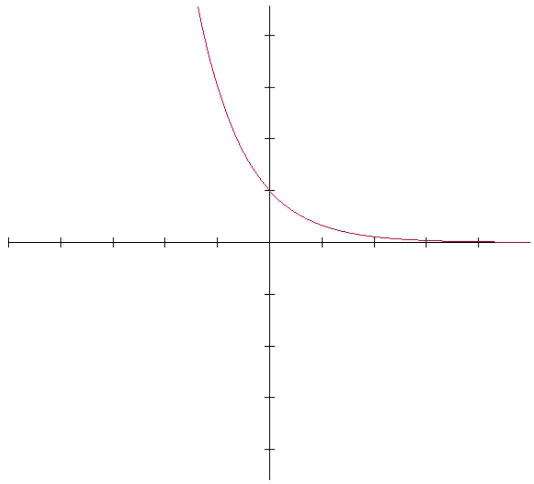


Here we have something that looks like decay. The value of "a" in this function is greater that one, so it should be growth. What really happened here, is the laws of exponents went to work. 3^{2-x} ***Is the same thing as*** $3^{-(x-2)}$***. The power of a power rule says this can be seen as*** $\left(3^{-1}\right)^{x-2}$***. This simplifies to*** $\left(\frac{1}{3}\right)^{x-2}$ ***, a decay curve. OK, so we begin with a decay curve that has a key point of (0,1). Add 2 to the x value, and 1 to the y value of the key point, and the new key point is (2,2), with a horizontal asymptote of y = 1.***

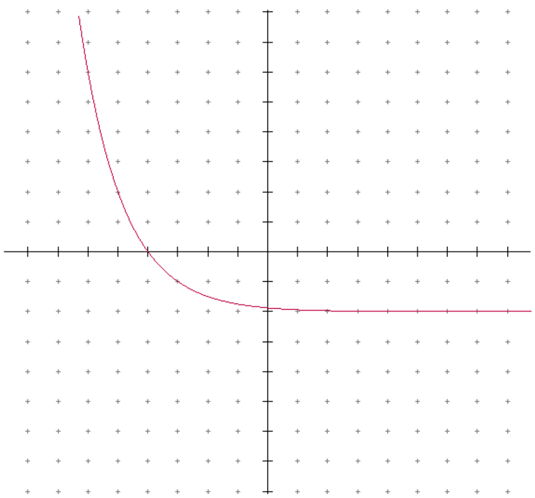
<u>*Exponential Decay*</u>

$$f_{(x)} = ca^{x-h} + k$$

$$f_{(x)} = \left(\frac{1}{2}\right)^{x}$$

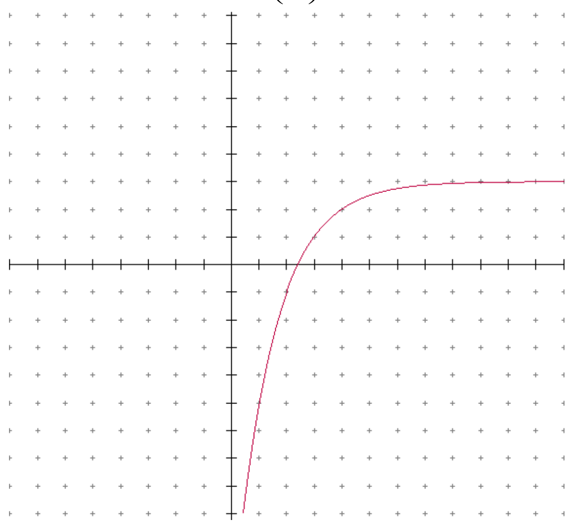


$$f_{(x)} = \left(\frac{1}{2}\right)^{x+3} - 2$$



This is an exponential decay curve. The original graph will lie above the x axis. Therefore, begin with the key point (0,1). The key point will shift to the left 3, and down 2, so subtract 3 from the x value and 2 from the y value of the key point, and the coordinates of the new point will be at (-3,-1). The graph of this function shifts down 2, so the horizontal asymptote of this function is y = -2.

$$f_{(x)} = -\left(\frac{1}{2}\right)^{x-4} + 3$$



This is an exponential decay function that is reflected and lies below the horizontal asymptote. The initial key point here is (0,-1). Since this graph will shift right 4 and up 3, add 4 to the x value of the key point and 3 to the y value. This yields a result of (4,2). Since the entire graph shifted upwards 3 spaces, the horizontal asymptote is y = 3. Once again, when graphing, do not cross the horizontal asymptote.

The translations of these functions are very similar to that of other functions we have seen. A point of reference with which to shift is all that is needed. Most important is to make sure to always use the appropriate key point to start with. <u>Draw the horizontal asymptote first</u>, that way the graph of the function does not accidentally cross it.

$$f_{(x)} = ca^{x-h} + k$$

The domain of any exponential function is $(-\infty, \infty)$. The values of c and k terms will determine the range of the function. Since the horizontal asymptote of an exponential function is given by y=k, the value of k will determine where the horizontal asymptote of the function lies, whereas the value of c will determine if the function is above or below that asymptote. Be careful not to use brackets when describing the range of an exponential function. The horizontal asymptote <u>must not be touched</u>, so only parenthesis may be used to describe the range in interval notation.

Find the range and domain of each of the following exponential functions.

A) $f_{(x)} = 2^{x+6} - 4$

B) $f_{(x)} = -\left(\frac{1}{2}\right)^{x-1} + 3$

C) $f_{(x)} = 2(3)^{x+1} - 5$

D) $f_{(x)} = 5^{-x} - 3$

E) $f_{(x)} = -2(5)^{x+2} - 3$

F) $f_{(x)} = e^{x+2} - 3$

G) $f_{(x)} = \left(\frac{5}{4}\right)^{x-8} + 2$

H) $f_{(x)} = -2^{x-3} - 7$

I) $f_{(x)} = -4^{3-x} + 2$

J) $f_{(x)} = 2\left(\frac{1}{3}\right)^{x-5} + 1$

K) $f_{(x)} = -6^{x-7} - 1$

L) $f_{(x)} = -e^{x-2} + 3$

Here is an example of finding the x and y intercept of an exponential function.

$$f_{(x)} = 3^{x+2} - 4$$

Finding the x intercept.

Begin by substituting 0 for $f_{(x)}$

$$0 = 3^{x+2} - 4$$
$$4 = 3^{x+2}$$
$$\log 4 = \log 3^{x+2}$$
$$\log 4 = (x+2)\log 3$$
$$\log 4 = x\log 3 + 2\log 3$$
$$\log 4 - 2\log 3 = x\log 3$$

Now divide both sides by log 3.

$$\frac{\log 4 - 2\log 3}{\log 3} = \frac{x\log 3}{\log 3}$$
$$x = \frac{\log 4 - 2\log 3}{\log 3}$$
$$x \approx -0.7381$$

Finding the y intercept.

Begin by substituting 0 for x.

$$f_{(x)} = 3^{0+2} - 4$$
$$f_{(x)} = 3^{2} - 4$$
$$f_{(x)} = 9 - 4$$
$$f_{(x)} = 5$$

As you can see, this function has an x intercept of approximately (-0.74,0), and a y intercept of (0,5).

Find the key point to each of the following functions.

A) $f_{(x)} = 3^{x+4} - 2$

B) $f_{(x)} = -4^{x-2} + 1$

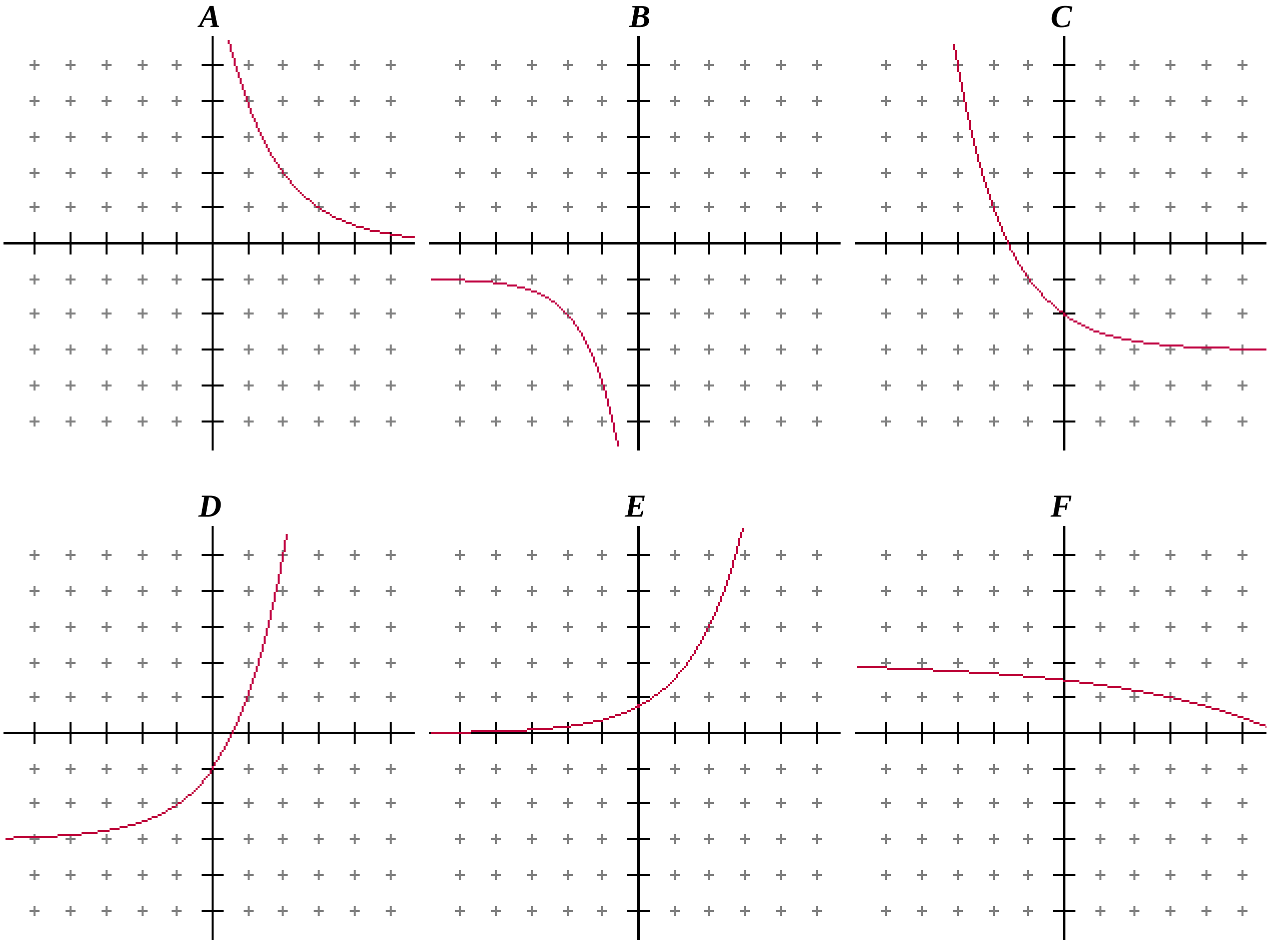
C) $f_{(x)} = 2^{4-x} + 5$

D) $f_{(x)} = 3(2)^{x+1} - 5$

E) $f_{(x)} = 2\left(\frac{1}{2}\right)^{x+4} - 3$

F) $f_{(x)} = -3^{x+2} - 4$

Match the appropriate graph with its equation below. Explain why each of your solutions is true.



1) $f_{(x)} = 2^{x+1} - 3$

2) $f_{(x)} = \left(\frac{1}{2}\right)^{x} - 3$

3) $f_{(x)} = 3(2)^{x-2}$

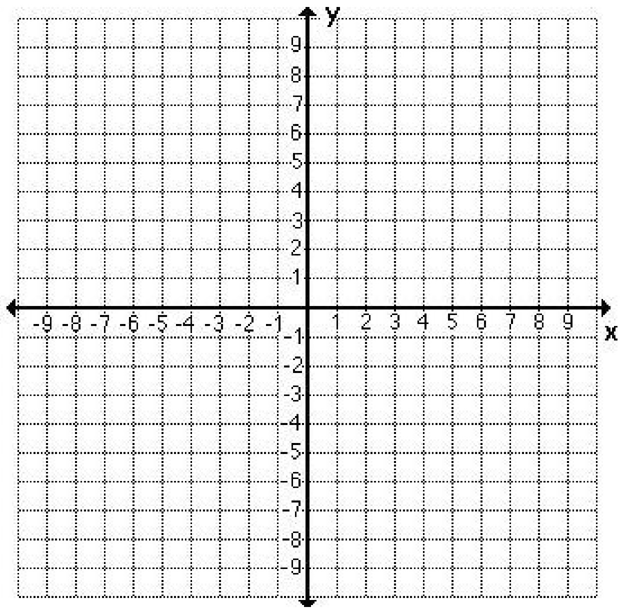
4) $f_{(x)} = -3^{x+2} - 1$

5) $f_{(x)} = -\left(\frac{5}{4}\right)^{x-3} + 2$

6) $f_{(x)} = 2^{3-x}$

Graph each of the following exponential functions. Be sure to label the key point of the function. Find the x intercept (if it exists) and y intercept of each function.

A) $f_{(x)} = 3^{x-2} - 1$



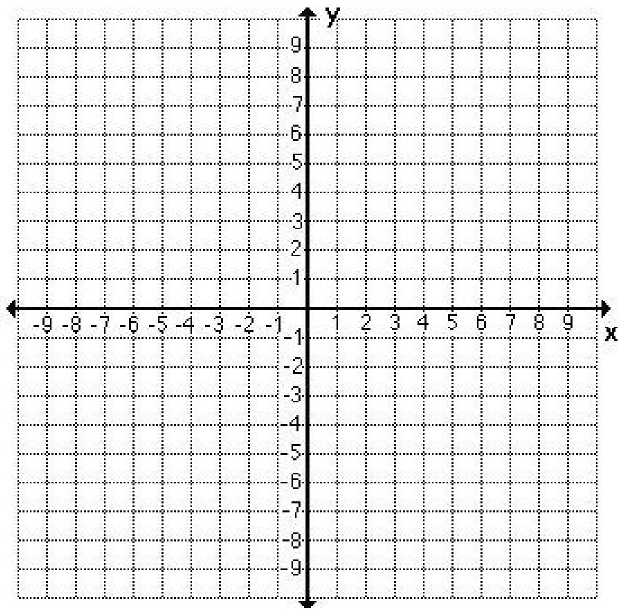
Y-intercept:

X-intercepts:

Range:

Domain:

B) $f_{(x)} = -2^{x+3} - 4$



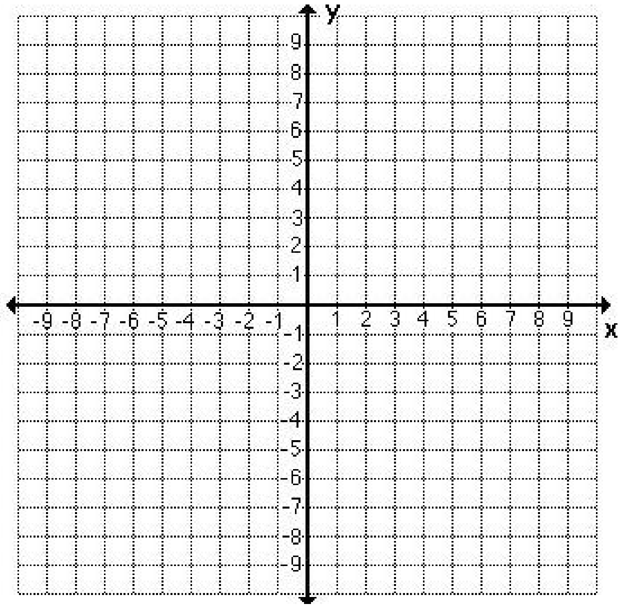
Y-intercept:

X-intercepts:

Range:

Domain:

C) $f_{(x)} = \left(\frac{1}{2}\right)^{x+2} - 3$



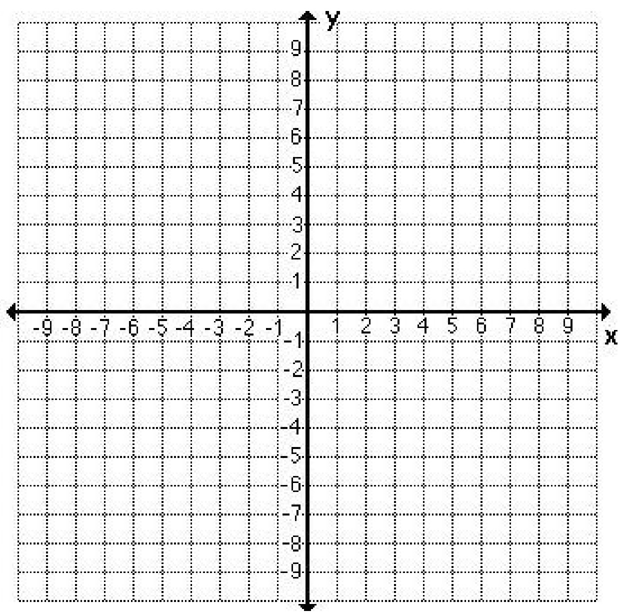
Y-intercept:

X-intercepts:

Range:

Domain:

D) $f_{(x)} = 2^{4-x}$



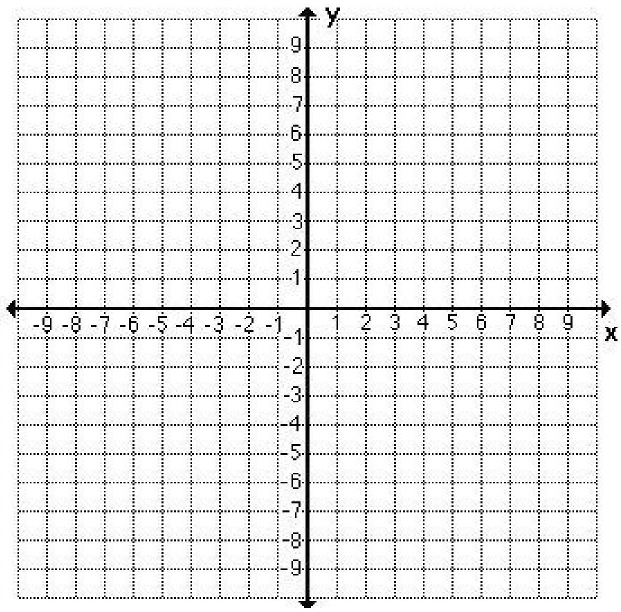
Y-intercept:

X-intercepts:

Range:

Domain:

E) $f_{(x)} = -\left(\frac{1}{3}\right)^{x-2} - 3$



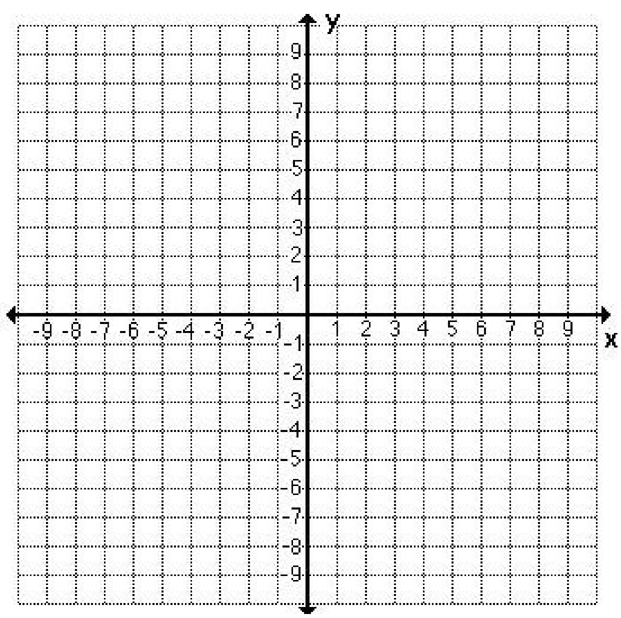
Y-intercept:

X-intercepts:

Range:

Domain:

F) $f_{(x)} = 2(3)^{x-2} + 1$



Y-intercept:

X-intercepts:

Range:

Domain:

Logarithmic Functions

$$f_{(x)} = a\log_n(bx+c)+d \qquad f_{(x)} = a\ln(bx+c)+d$$

Logarithmic functions will be graphed in the same manner as radical functions. It is first necessary to find the domain of the logarithmic function. The range of a logarithmic function is all real numbers, so only the domain needs to be found. To find the domain of a logarithmic function evaluate $bx+c>0$. Remember, this is not ≥, because you cannot take the log of zero. Once the domain is found, it will tell in which direction the function is moving. This inequality will also help find the vertical asymptote for the function.

If the x inside the log does not have a negative coefficient, the curve will be on the right side of the vertical asymptote. If the coefficient in front of x is 1, begin with the key point of (1,0). From that point on, treat the function just like an exponential function. Adding or subtracting to either the x or y values to find the new key point making the graph shift.

If the x inside the log has a negative coefficient, the curve will be on the left side of the vertical asymptote. If the coefficient in front of x is -1, begin with the key point of (-1,0) and shift from there.

***Once again, just like exponential growth and decay functions, watch the value of "a", as it affects the scale of the function. If the value of "a" is some number other that 1 or -1, find the key point algebraically before you translate the function.**

As the function shifts, it will be helpful to draw a broken line for both the horizontal and vertical asymptotes. It is OK to cross the horizontal asymptote, as you will find the key point always rests on it. The vertical asymptote, however, may never be crossed.

$$f_{(x)} = a\log_n(bx+c)+d \qquad f_{(x)} = a\ln(bx+c)+d$$

Solving for $bx+c=0$, will yield the equation for the vertical asymptote. The equation for the horizontal asymptote is $y=d$.

$$f_{(x)} = \log_3(x-4)+2$$

Finding the domain.

$$x-4>0$$
$$x>4$$

Notice the similarity in the procedures.

Finding the vertical asymptote.

$$x-4=0$$
$$x=4$$

Finding the horizontal asymptote.

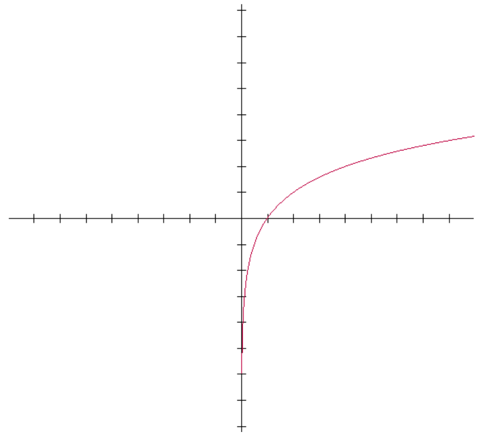
$$y=2$$

There is no real work involved with finding the horizontal asymptote. Identify the vertical shift. This is the equation of the horizontal asymptote.

***If the variable x inside the log has a coefficient other than 1 or -1, the key point will be different. The key point must then be found algebraically. To find the x value of the key point solve for $bx+c=1$. Substitute that solution back into the problem to find the y value.**

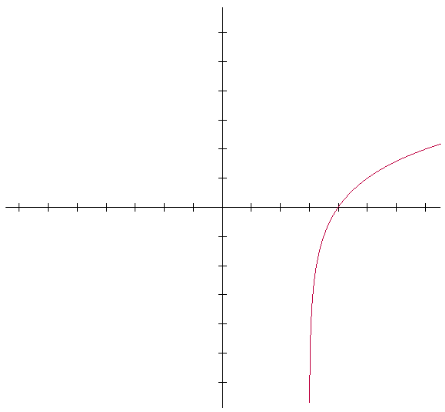
$$f_{(x)} = a\log_n(bx+c)+d \qquad f_{(x)} = a\ln(bx+c)+d$$

$$f_{(x)} = \log_2 x$$



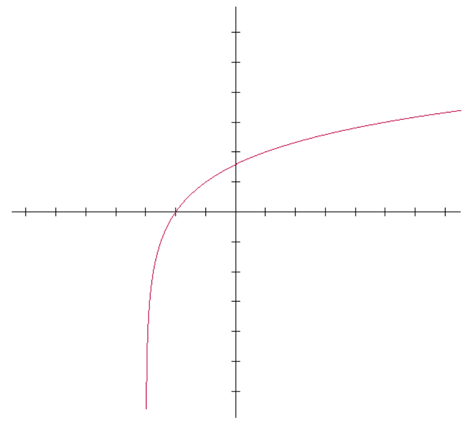
The parent function has the key point at (1, 0)

$$f_{(x)} = \log_2(x-3)$$



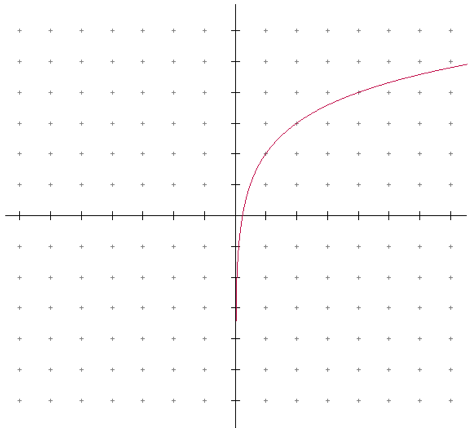
The graph of this function shifts right 3. Notice the key point moved to the right 3 places to (4,0).

$$f_{(x)} = \log_2(x+3)$$



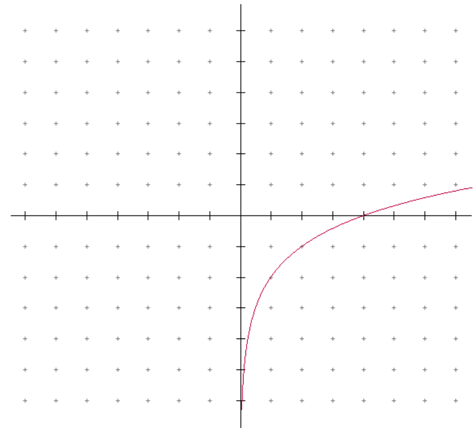
The graph of this function shifts to the left 3. The new key point is (-2,0).

$$f_{(x)} = \log_2 x+2$$



This function shifts up 2. Add 2 to the y value of the key point, and it is now at (1,2).

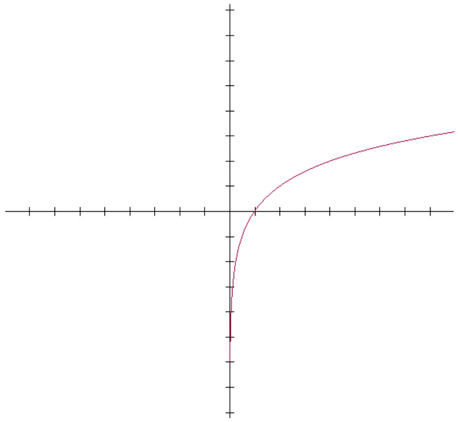
$$f_{(x)} = \log_2 x-2$$



This function shifts down 2. Subtracting 2 from the y value of the key point results in (1,-2).

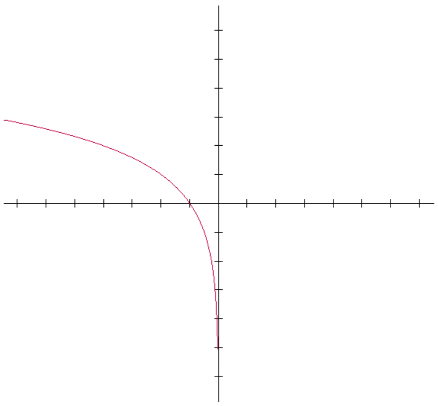
$$f_{(x)} = a\log_n(bx+c)+d \qquad f_{(x)} = a\ln(bx+c)+d$$

$$f_{(x)} = \log_2 x$$



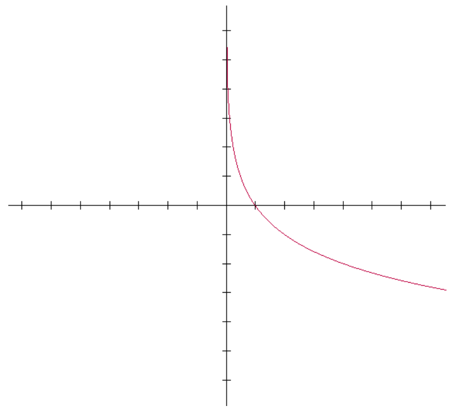
The parent function has the key point at (1, 0)

$$f_{(x)} = \log_2(-x)$$



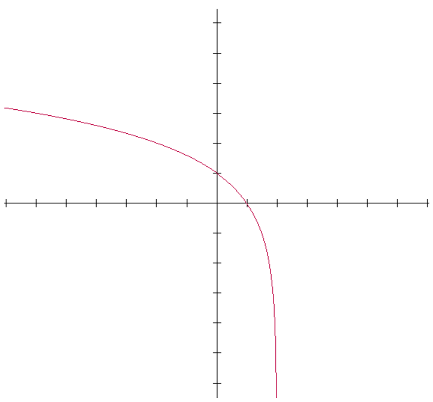
The graph of this function reflects about the vertical asymptote. Key point is now (-1,0).

$$f_{(x)} = -\log_2 x$$



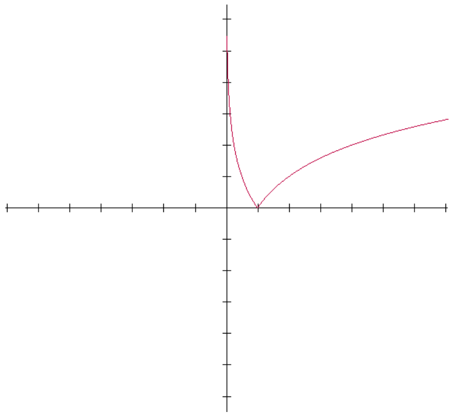
The graph of this function is reflected about the horizontal asymptote. Key point is still at (1,0).

$$f_{(x)} = \log_2(2-x)$$



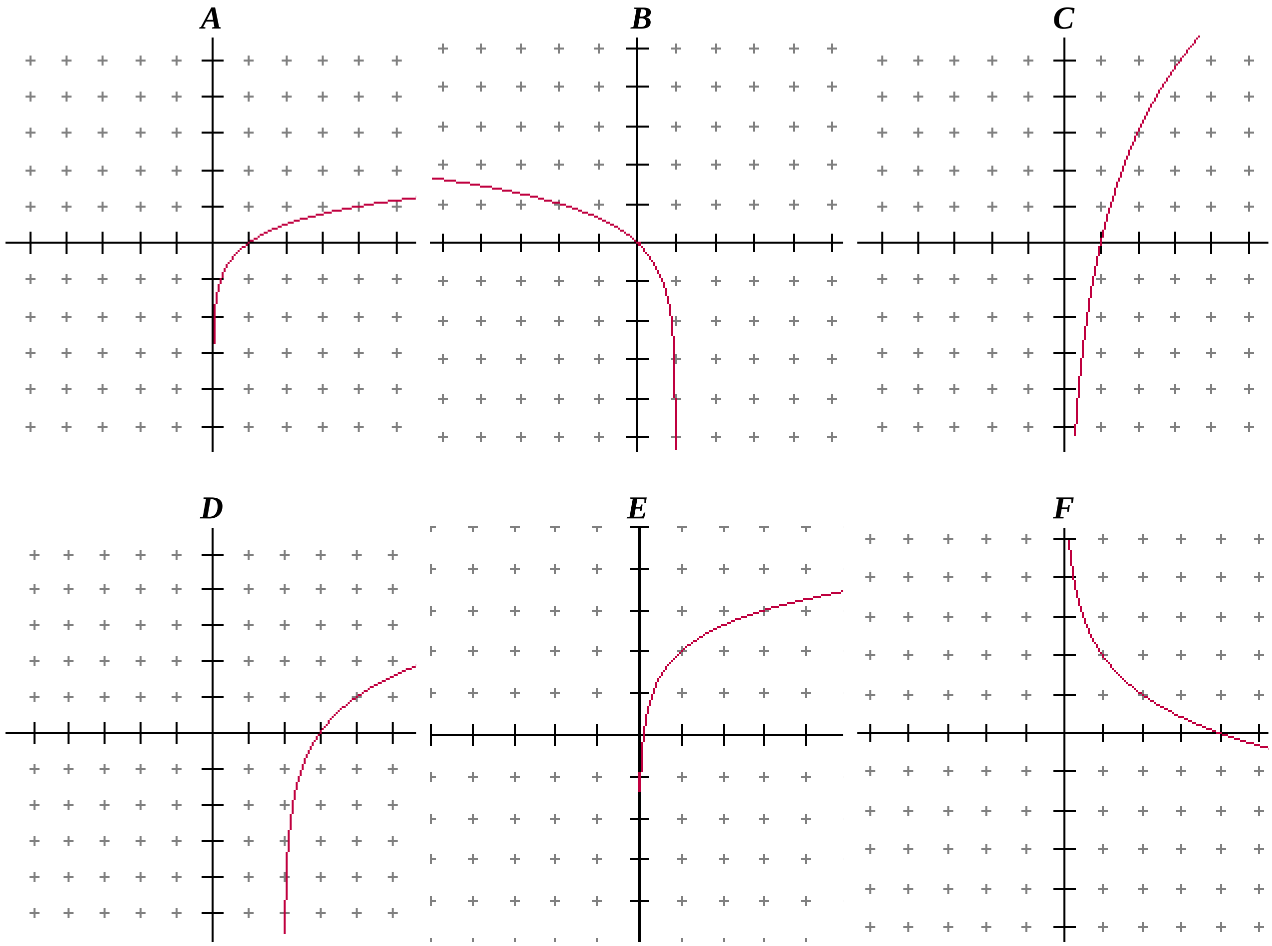
Since the coefficient of x is -1, this graph will be on the left side of the vertical asymptote. Begin with the key point (-1,0), and shift right 2 because it is positive. Add 2 to the x value of the key point. The new key point is (1,0).

$$f_{(x)} = \left|\log_2 x\right|$$



Notice the negative portion of the graph reflected above the x axis.

Match the appropriate graph with its equation below. Explain why each of your solutions is true.



1) $f_{(x)} = \log_2(x-2)$

2) $f_{(x)} = \log_3(1-x)$

3) $f_{(x)} = -\log_2 x + 2$

4) $f_{(x)} = \log_3 x + 2$

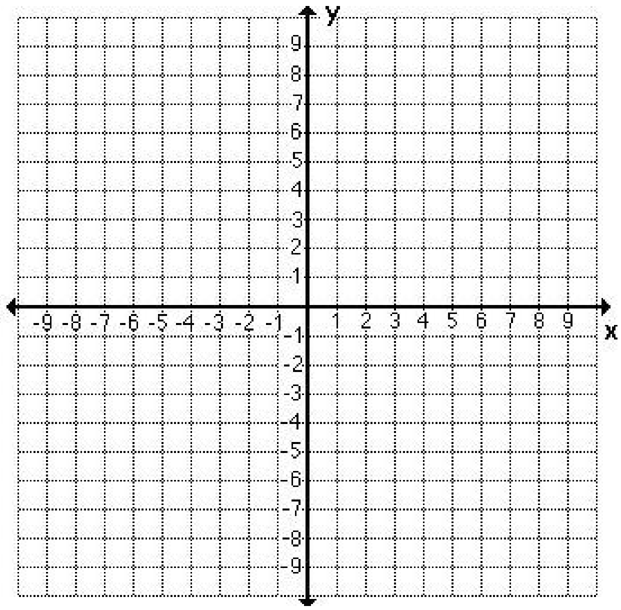
5) $f_{(x)} = \frac{1}{2}\log_2 x$

6) $f_{(x)} = 3\log_2 x$

The translation of a logarithmic function is almost identical to that of an exponential function. Just make sure to identify on which side of the vertical asymptote the graph of the function will reside. This will determine which key point to begin with. Remember to draw both asymptotes to graph the function and watch for the value of "a" which will affect key point.

Graph each of the following logarithmic functions by finding the asymptotes and labeling the key point. Be sure to find the x intercept and y intercept (if they exist).

A) $f_{(x)} = \log_3 x + 2$



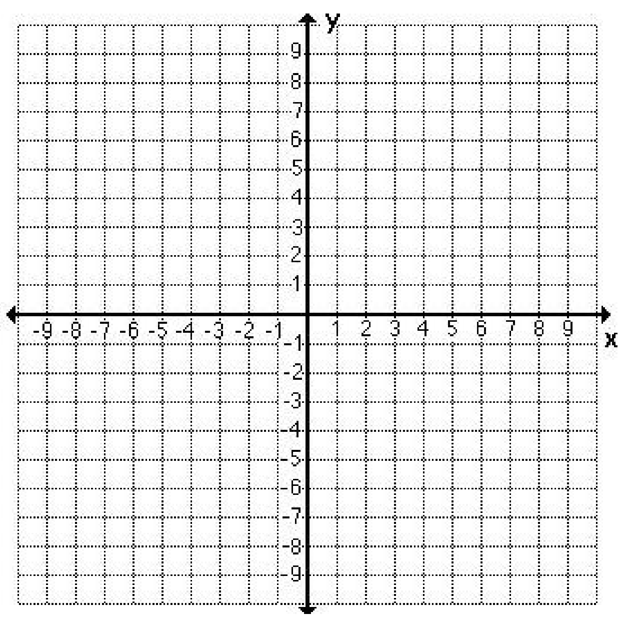
Y-intercept:

X-intercepts:

Range:

Domain:

B) $f_{(x)} = \log_2 (x + 3)$



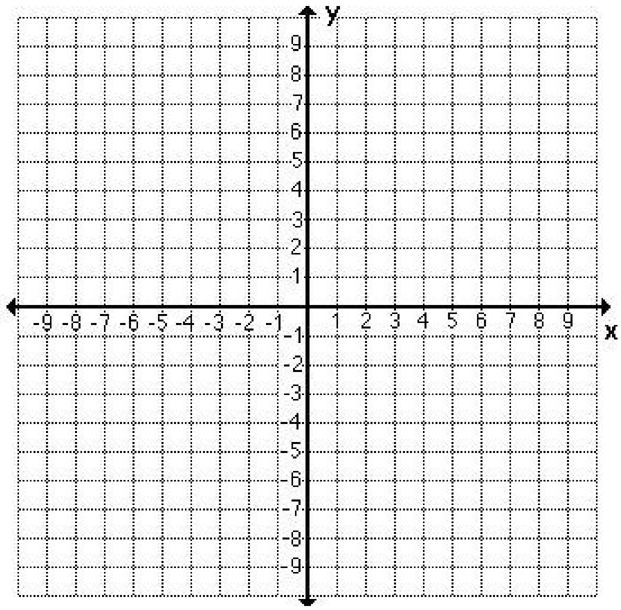
Y-intercept:

X-intercepts:

Range:

Domain:

C) $f_{(x)} = -\log_2 x + 4$



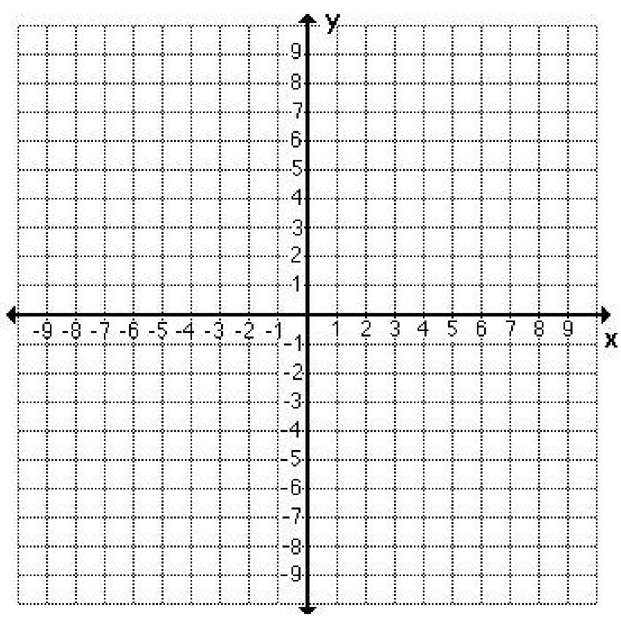
Y-intercept:

X-intercepts:

Range:

Domain:

D) $f_{(x)} = \ln(-x)$



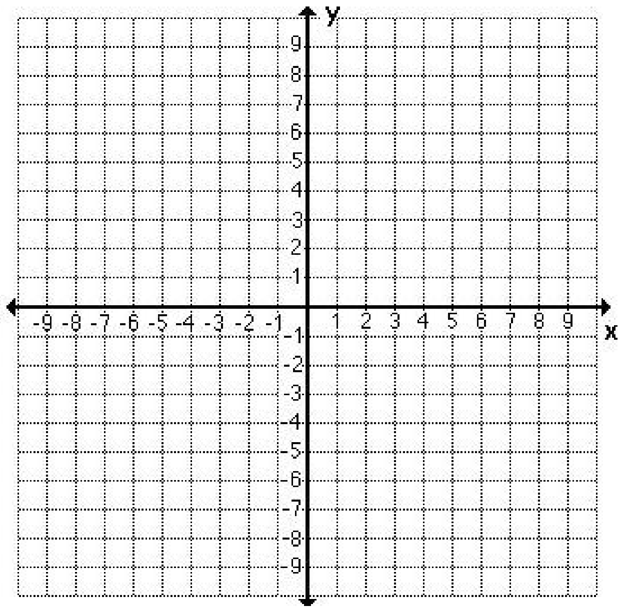
Y-intercept:

X-intercepts:

Range:

Domain:

E) $f_{(x)} = -\ln(x-2)$



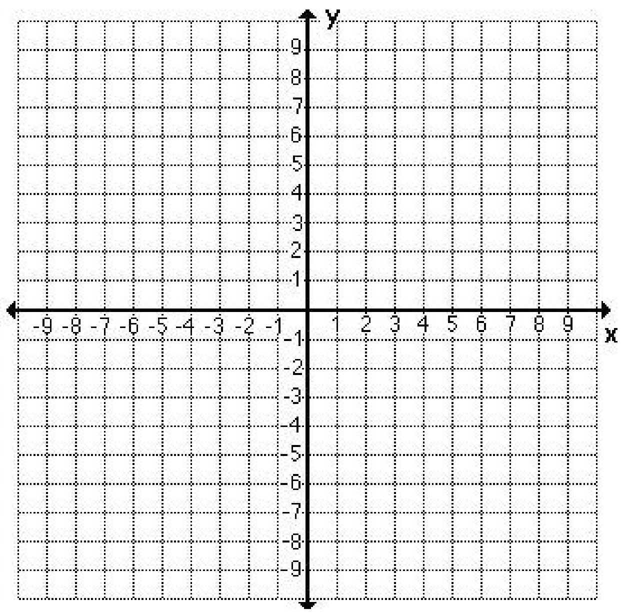
Y-intercept:

X-intercepts:

Range:

Domain:

F) $f_{(x)} = \log_2(3-x)+2$



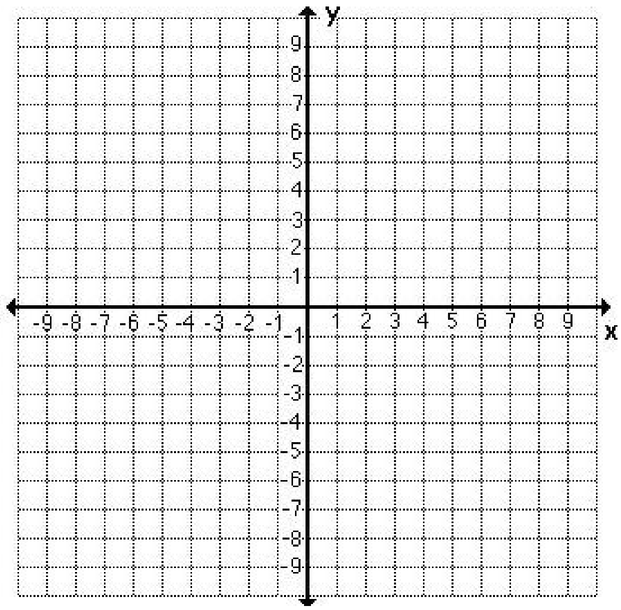
Y-intercept:

X-intercepts:

Range:

Domain:

G) $f_{(x)} = |\ln x|$



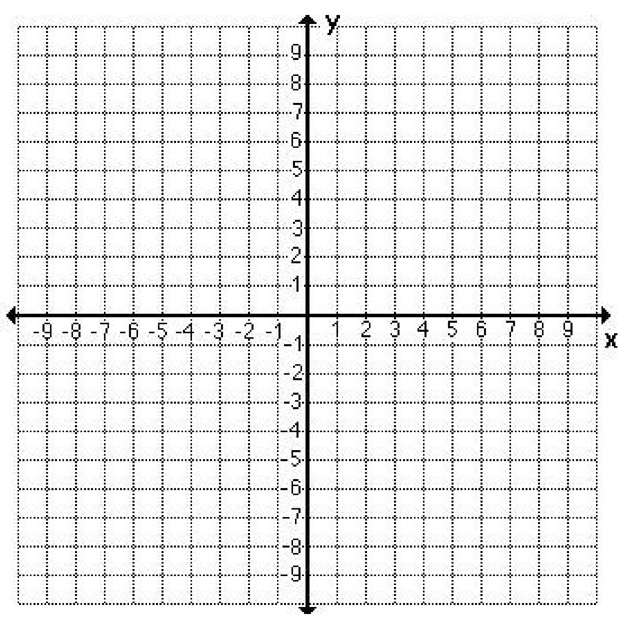
Y-intercept:

X-intercepts:

Range:

Domain:

H) $f_{(x)} = \ln|x|$



Y-intercept:

X-intercepts:

Range:

Domain:

All standard logarithmic functions (meaning a function without absolute value symbols), must have an x intercept. All standard exponential growth and decay functions must have a y intercept. Are these two statements true? Why or why not?

In order to find the domain of the logarithmic function $f_{(x)} = \log_4(x+5) - 3$, we need to evaluate $x+5>0$. Why must we use this inequality?

What is the problem with relying on a graphing calculator to graph a logarithmic function?

Cubic Functions

The cubic function is similar to the cubed root. You will notice similarities in the shape of the curve. Translations are the same as any standard function. The range and domain of any cubic function is all real numbers.

$$f_{(x)} = a(x-h)^3 + k$$

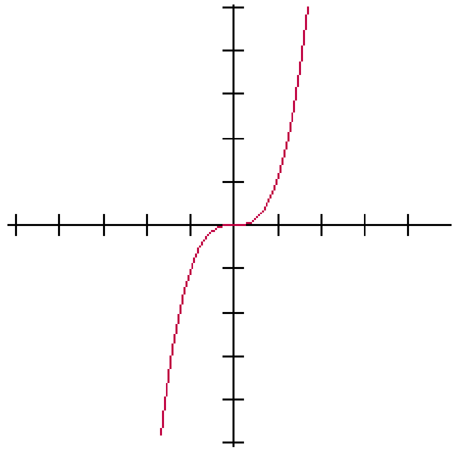
Let us look at this as the standard form of a cubic function. The center of the cubic function is given by (h,k). To find the x and y intercepts of the function, follow the standard procedures of substituting zero for one of the values, and solving for the remaining variable.

All cubic functions in this section will be given to you in this standard form. In the next section of the workbook, we will address polynomial functions that are greater than 2^{nd} degree. These functions will have no standard from with which to work. We will be graphing them by alternative means.

For now we will concentrate on the function $f_{(x)} = x^3$, and translations of this.

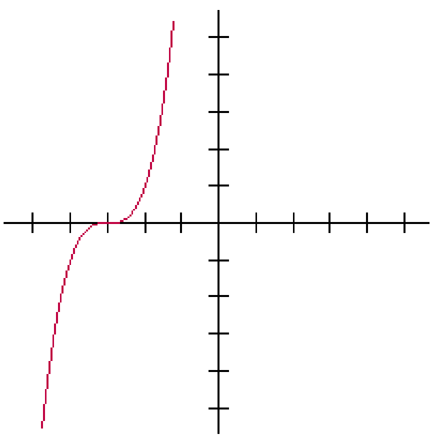
$$f_{(x)} = a(x-h)^3 + k$$

$$f_{(x)} = x^3$$



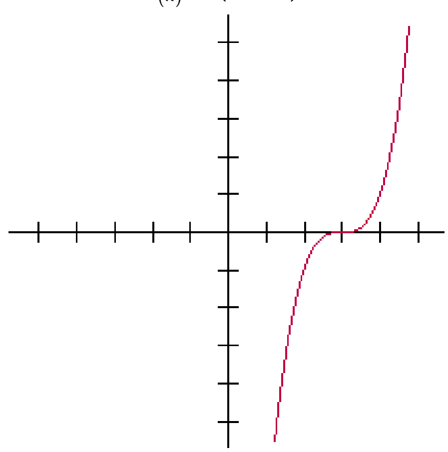
The parent function has the point of origin at (0, 0)

$$f_{(x)} = (x+3)^3$$



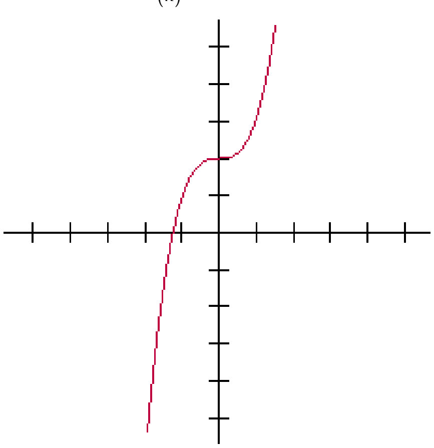
The graph of this function shifts left 3.

$$f_{(x)} = (x-3)^3$$



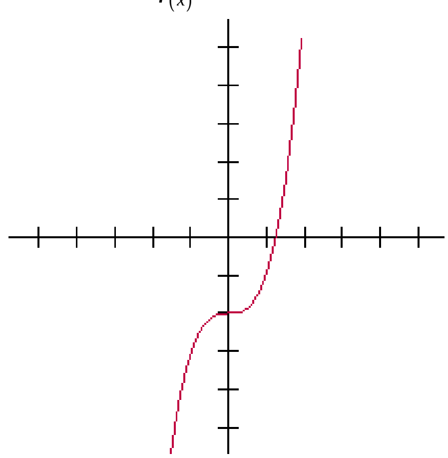
The graph of this function shifts right 3.

$$f_{(x)} = x^3 + 2$$



Graph shifts up 2.

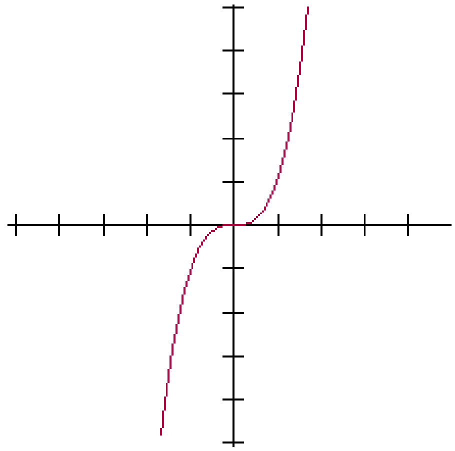
$$f_{(x)} = x^3 - 2$$



Graph shifts down 2.

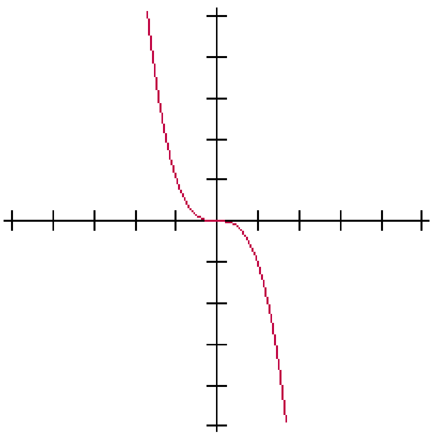
$$f_{(x)} = a(x-h)^3 + k$$

$$f_{(x)} = x^3$$



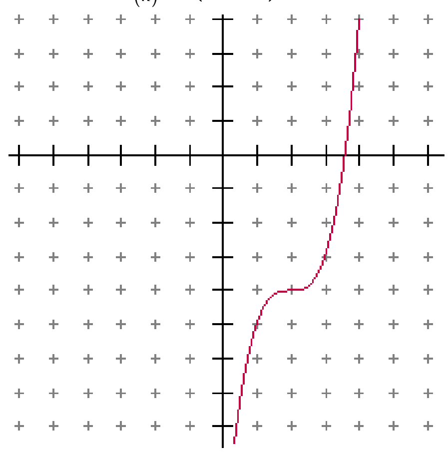
The parent function has the point of origin at (0, 0)

$$f_{(x)} = -x^3$$



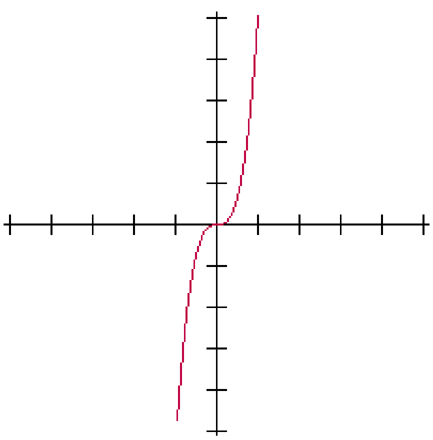
The graph of this function flips upside down.

$$f_{(x)} = (x-2)^3 - 4$$

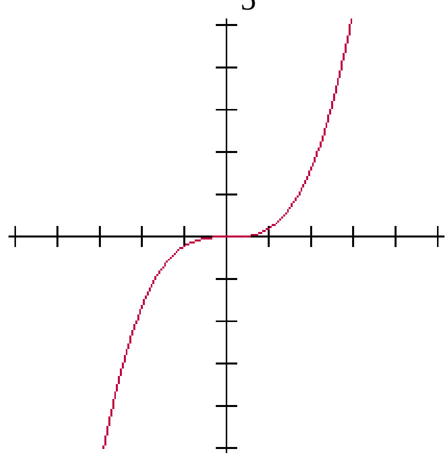


The graph of this function shifts right 2, down 4. vertex at (2,-4).

$$f_{(x)} = 5x^3$$



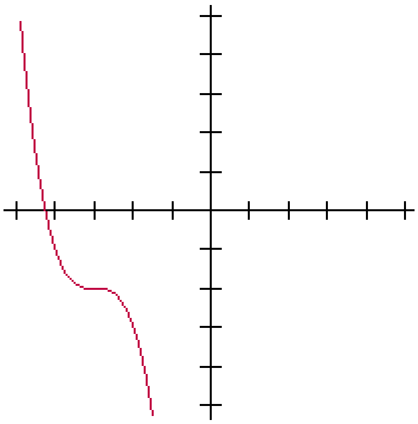
$$f_{(x)} = \frac{1}{5}x^3$$



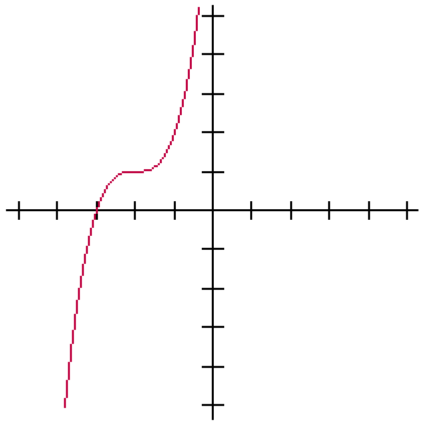
Once again, note difference the value of "a" makes in terms of the scale of the graph.

Match the appropriate graph with its equation below. Explain why each of your solutions is true.

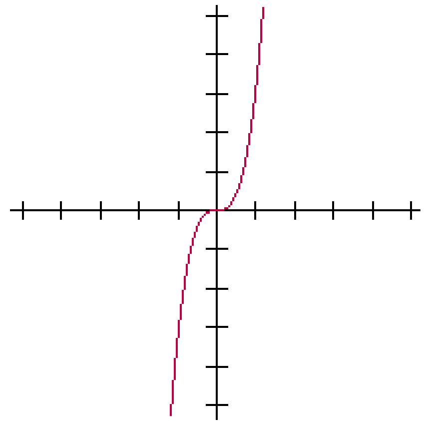
A



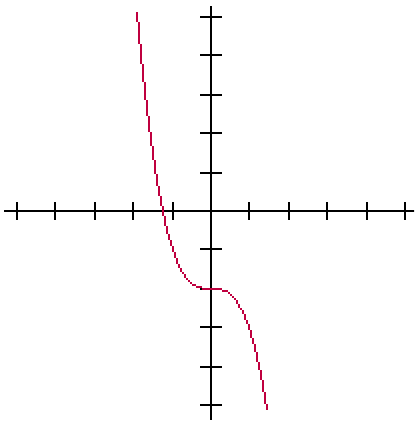
B



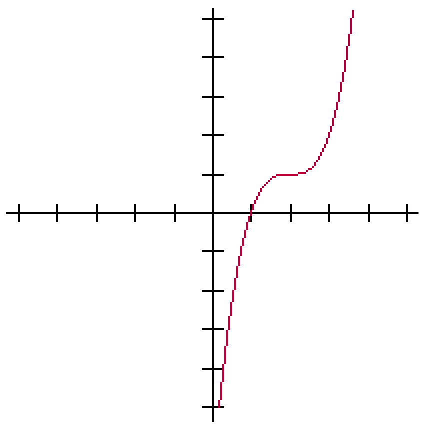
C



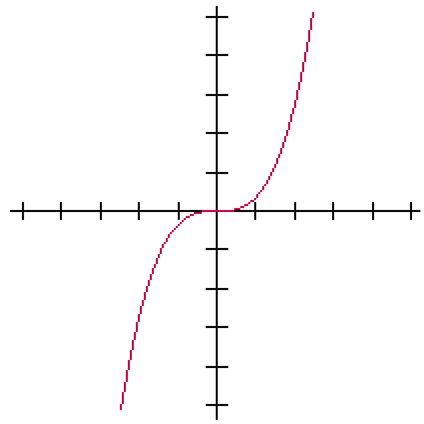
D



E



F



1) $f_{(x)} = -x^3 - 2$

2) $f_{(x)} = (x-2)^3 + 1$

3) $f_{(x)} = (x+2)^3 + 1$

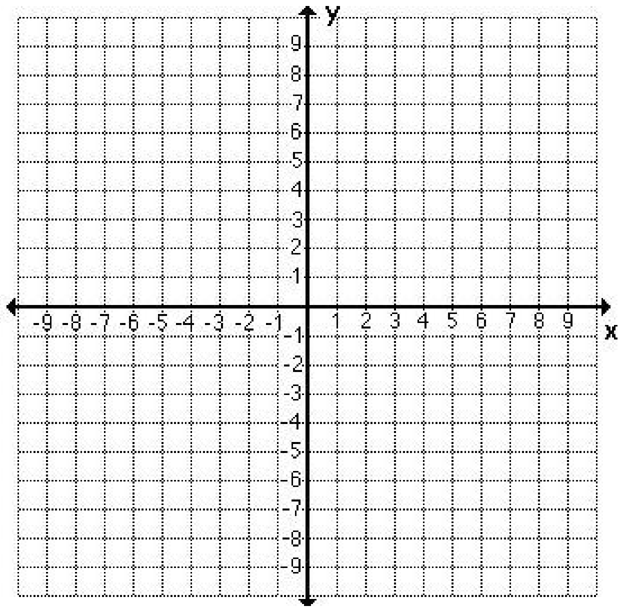
4) $f_{(x)} = \frac{1}{3}x^3$

5) $f_{(x)} = 3x^3$

6) $f_{(x)} = -(x+3)^3 - 2$

Graph each of the following cubic functions. ***Label the vertex, find all intercepts, and the range and domain of each of the following. Don't worry about graphing the intercept if it is too far off the chart.***

A) $f_{(x)} = (x-2)^3 + 3$



Vertex:

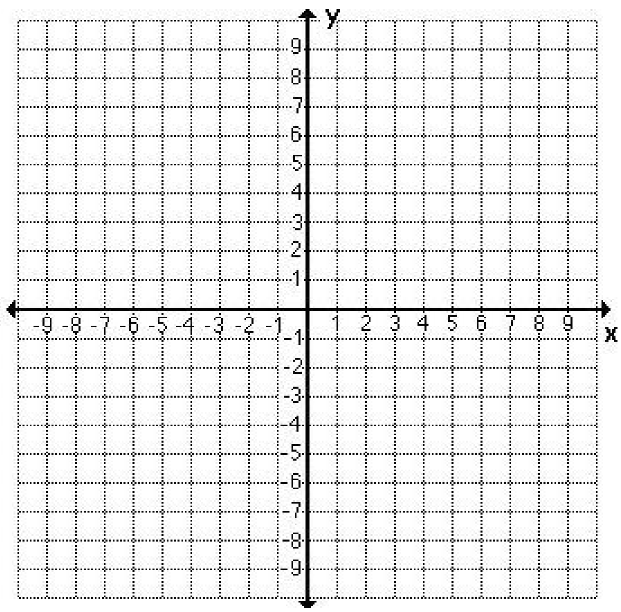
Y-intercept:

X-intercepts:

Range:

Domain:

B) $f_{(x)} = -(x+2)^3 - 1$



Vertex:

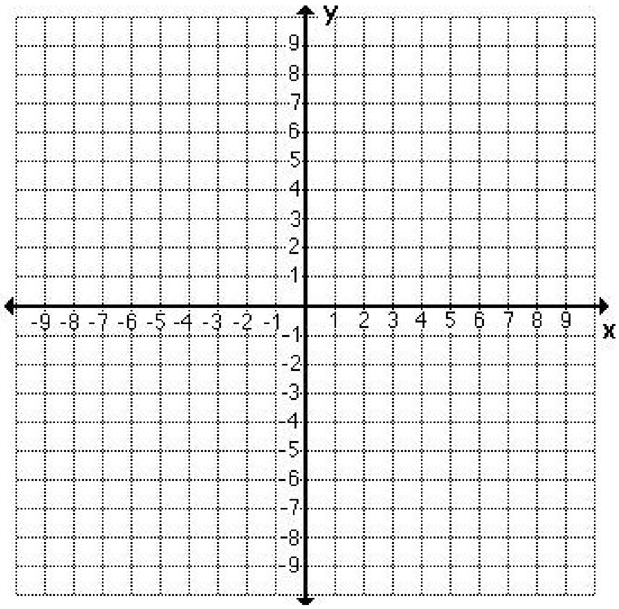
Y-intercept:

X-intercepts:

Range:

Domain:

C) $f_{(x)} = x^3 + 3$



Vertex:

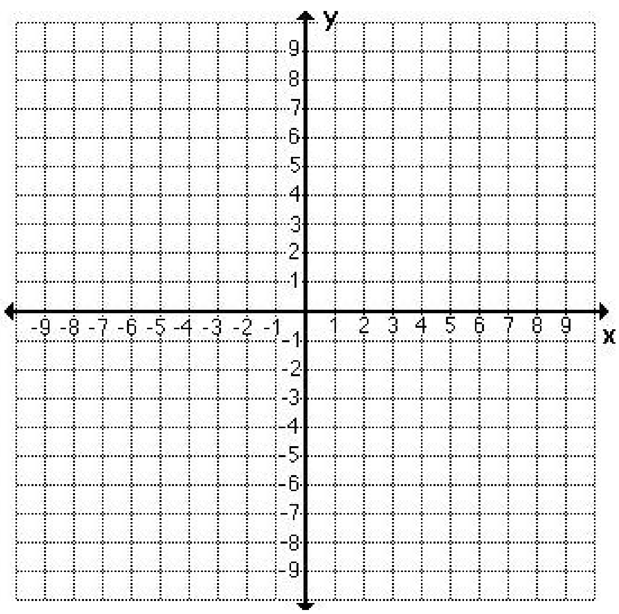
Y-intercept:

X-intercepts:

Range:

Domain:

D) $f_{(x)} = (x+3)^3 - 5$



Vertex:

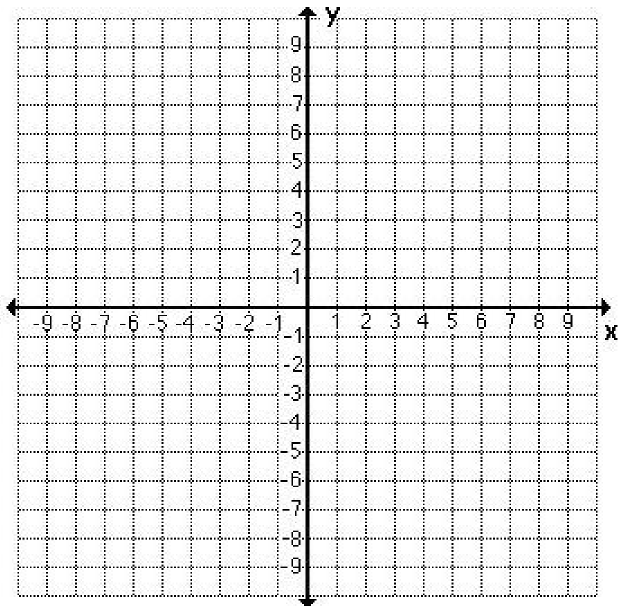
Y-intercept:

X-intercepts:

Range:

Domain:

E) $f_{(x)} = -(x-4)^3 + 2$



Vertex:

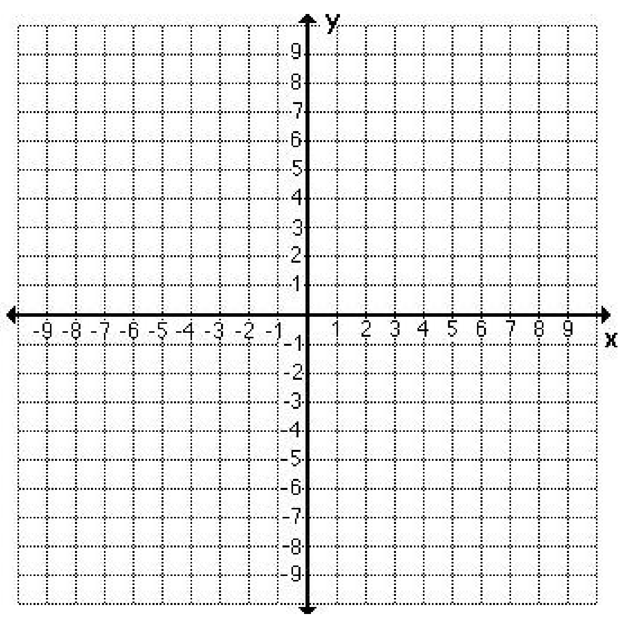
Y-intercept:

X-intercepts:

Range:

Domain:

F) $f_{(x)} = (x-3)^3 - 5$



Vertex:

Y-intercept:

X-intercepts:

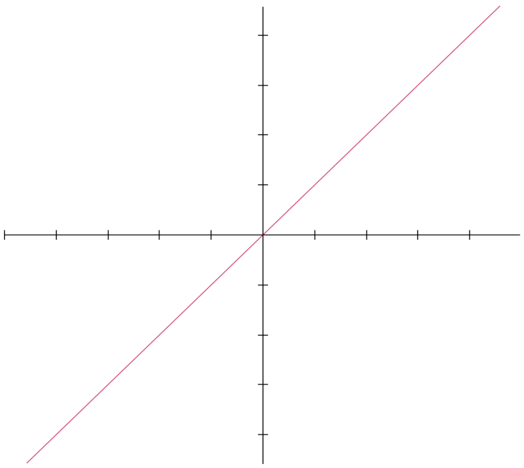
Range:

Domain:

Piece-Wise Functions

The best way to describe a piece-wise function is to look at a simple example. Consider the absolute value function.

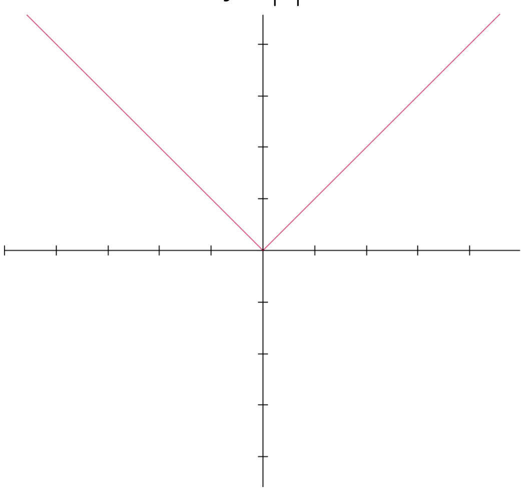
$y = x$



This is the graph of the function $y = x$. In this case, the x and y values of coordinates are identical. For example, (-3,-3). You can see the x and y values are the same.

Now, lets take a look at what happens when we want the absolute value of x.

$y = |x|$



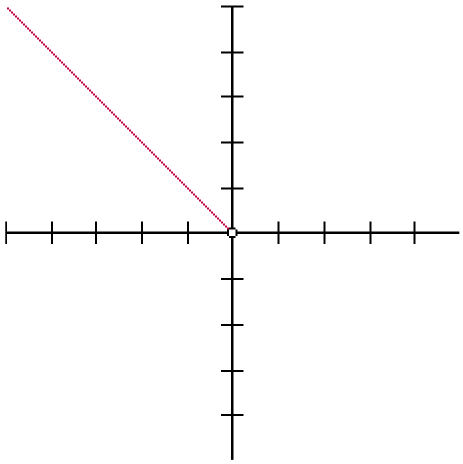
If the graph of $y = x$ above is $f_{(x)}$, the function to the left is $\left|f_{(x)}\right|$. We know that the absolute value of a number cannot be negative. If we take the absolute value of $f_{(x)}$, it would cause the left portion of the original graph to reflect above the x axis. This results in all y values of the function being positive. This is where the graph of the absolute value of x comes from.

It is possible to get this same graph if the linear function $y = x$ is graphed in the interval $[0,\infty)$, and the function $y = -x$ in the interval $(-\infty, 0)$. What happens here is a specific section of two different graphs is drawn. If these two sections are placed together on the same plane, it would result in the graph of $y = |x|$. Such an equation would appear as follows.

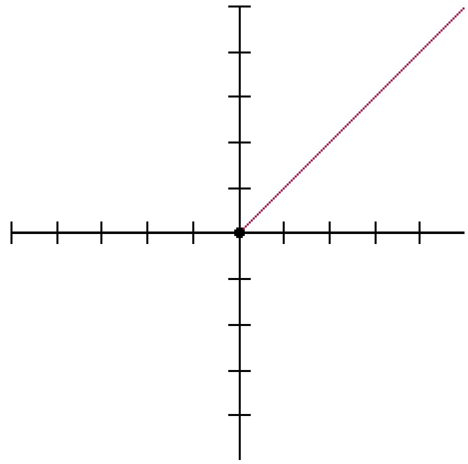
$$f_{(x)} = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$f_{(x)} = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$f_{(x)} = -x, \quad x < 0$

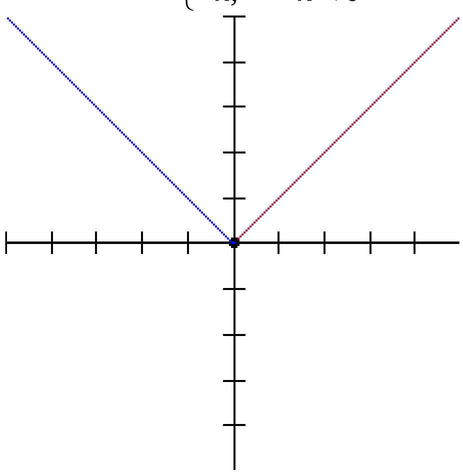


$f_{(x)} = x, \quad x \geq 0$



Notice the graph of the function on the left has a hole when x = 0. Since the function does not exist when x is zero, because of the domain of the function, an open circle must be used at that point. A pronounced dot is placed on the function to the right, showing the graph of the function in the interval $x \geq 0$***. This is how you plot a point when you have the greater than or equal to sign in the domain of the function. If both of these functions are placed on the same plane, the result would be as follows.***

$$f_{(x)} = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

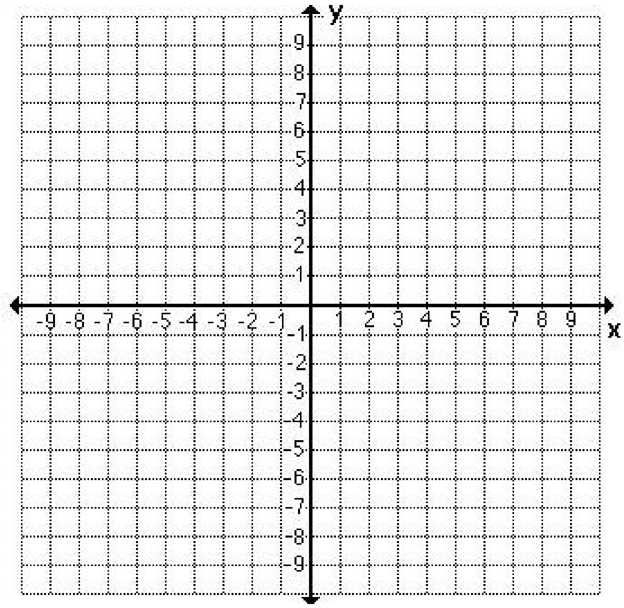


Since the open circle and solid dot are right on top of each other, the dot would "fill in" the hole on the first curve. This results in a continuous function. If the hole were still there, the function would have a removable discontinuity. If there is a complete break in the curve, from one portion of the graph to the next, it would be a discontinuous function.

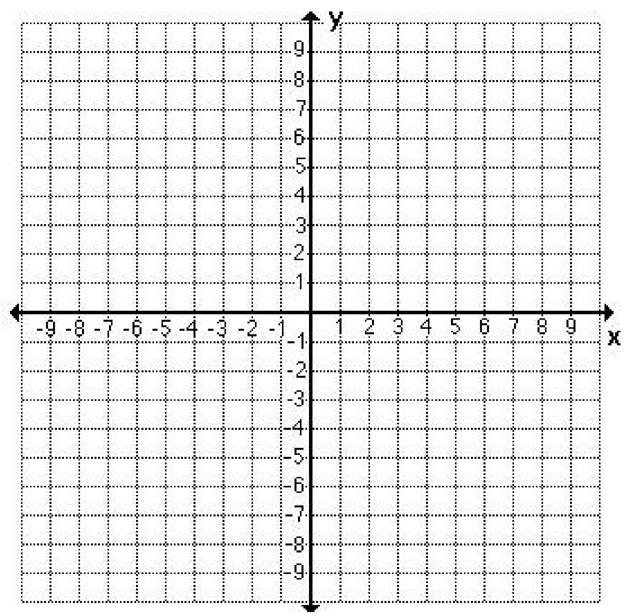
The simplest way to graph a piece-wise function is to graph the entire function and erase the portions that are not needed. This will be done for each part of the overall graph until a complete picture has been created. Very simply, a piece-wise function is just as it sounds, pieces of different functions put together to create one graph. Some examples of piecewise functions can be found in the **"Translations of Functions"** ***section of this chapter.***

Graph each of the following piece-wise functions.

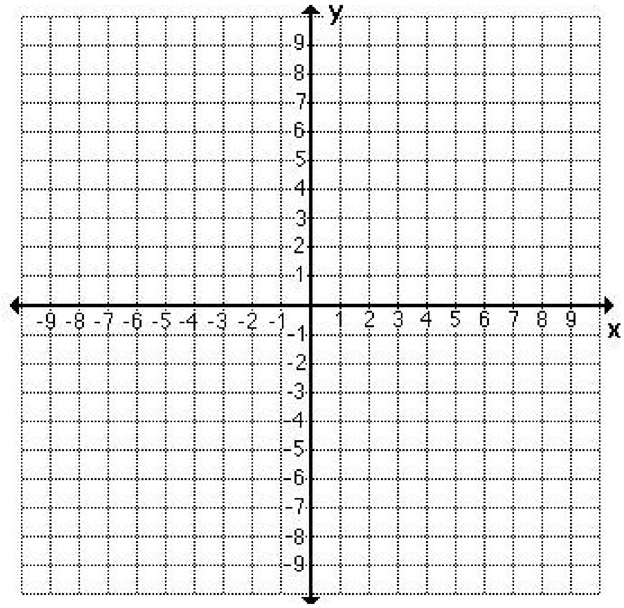
A) $f_{(x)} = \begin{cases} x+5, & x \le -3 \\ 2, & -3 < x < 1 \\ x+1, & x \ge 1 \end{cases}$



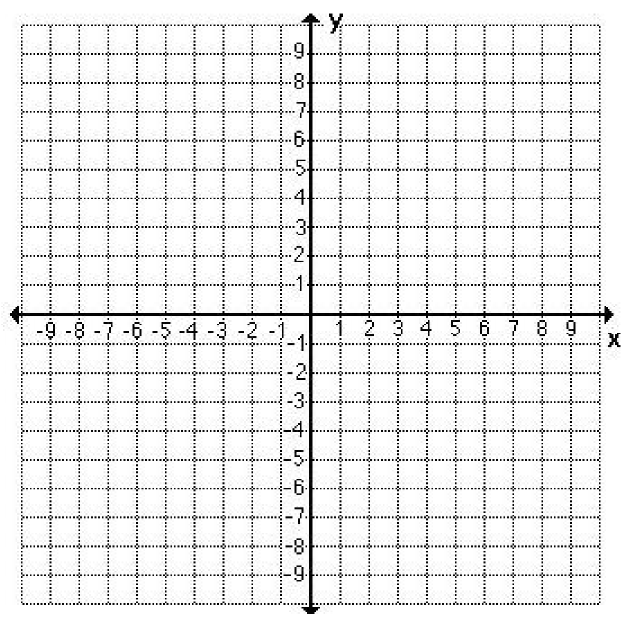
B) $f_{(x)} = \begin{cases} -(x-2)^2, & x < 2 \\ (x-2)^2, & x \ge 2 \end{cases}$



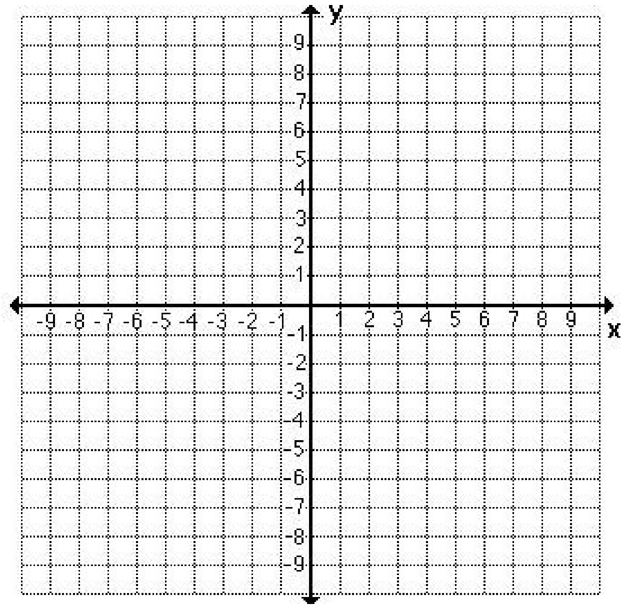
C) $f_{(x)} = \begin{cases} x+2, & x \le -4 \\ -2, & -4 < x < 3 \\ (x-3)^2 - 3, & x \ge 3 \end{cases}$



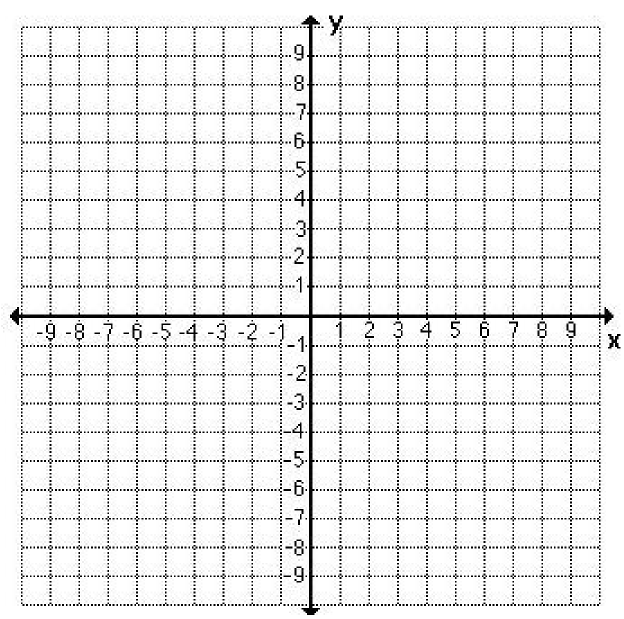
D) $f_{(x)} = \begin{cases} 3, & x < -1 \\ -x+2, & -1 \le x \le 2 \\ \frac{1}{3}(x-2)^2, & x > 2 \end{cases}$



E) $f_{(x)} = \begin{cases} \frac{3}{5}\left|x+4\right| - 2, & x < 1 \\ \sqrt{x-1} + 3, & x > 1 \end{cases}$

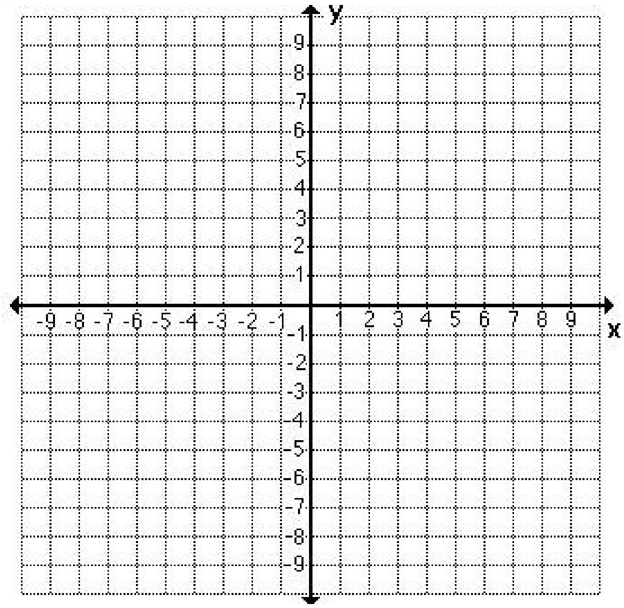


F) $f_{(x)} = \begin{cases} -\frac{2}{3}\left(x+4\right)^2 + 3, & -7 \le x < -4 \\ 3, & -4 \le x < 1 \\ -5x + 8, & 1 \le x < 2 \\ \frac{1}{3}\left(x-2\right)^2 - 2, & 2 \le x \le 5 \end{cases}$



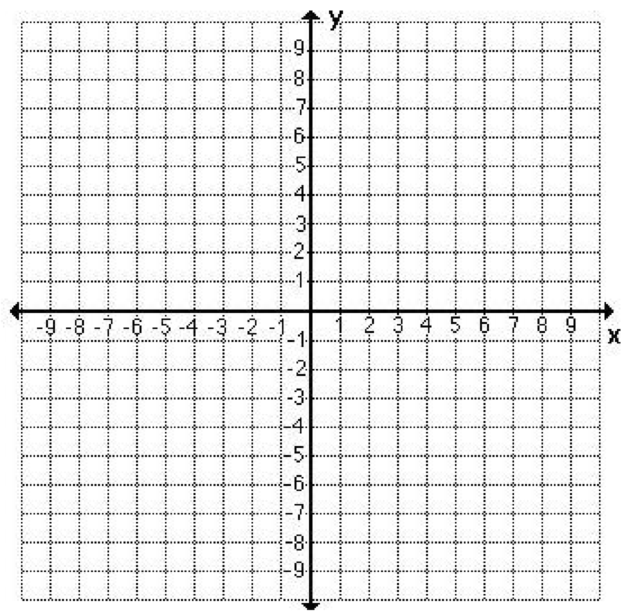
G)

$$f_{(x)} = \begin{cases} 5, & -7 \le x \le -4 \\ -(x+3)^2 + 9, & -4 < x < 0 \\ \frac{2}{3}x + 1, & 0 \le x \le 4 \end{cases}$$



H)

$$f_{(x)} = \begin{cases} \log_2(-x), & -8 \le x < -1 \\ |x| - 1, & -1 \le x \le 1 \\ \log_3 x, & 1 < x \le 9 \end{cases}$$

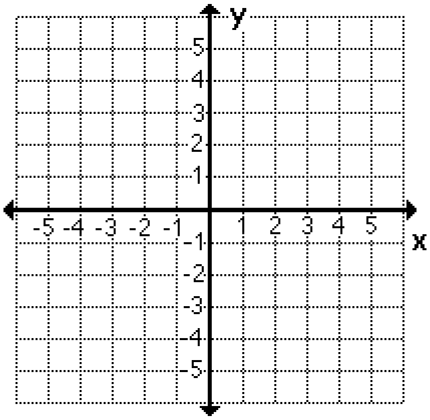


Which if any of the piece-wise functions you just graphed are discontinuous?

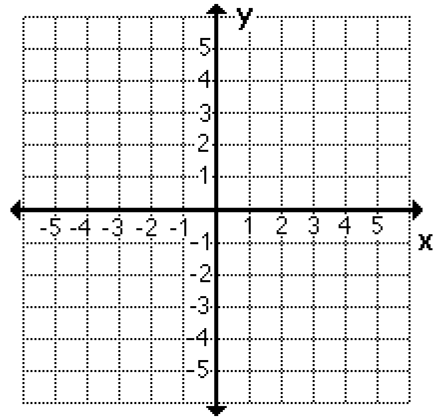
Functions Review

Graph each of the following functions. ***You need only label the key point or vertex for each. Do not worry about anything else. These problems are meant to quiz you on your knowledge of the parent functions and the translations thereof.***

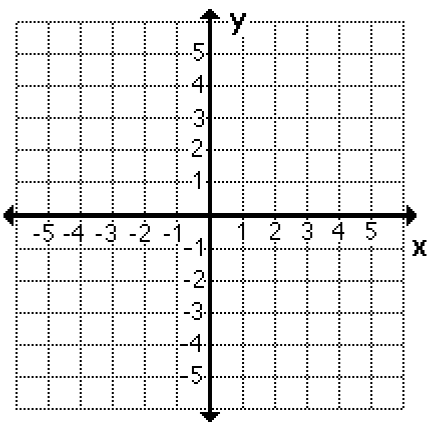
1. $f_{(x)} = -\sqrt{x-2} + 3$



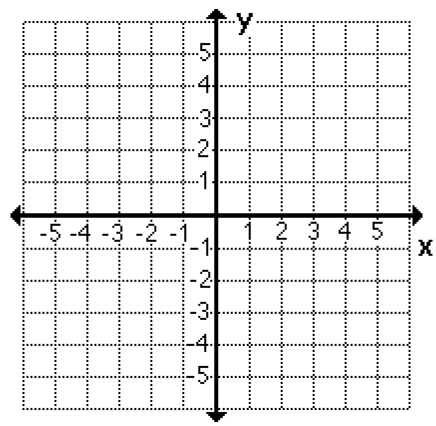
2. $f_{(x)} = |x-3| + 2$



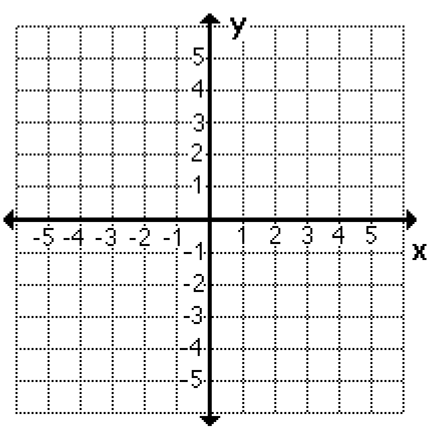
3. $f_{(x)} = x^2 + 2$



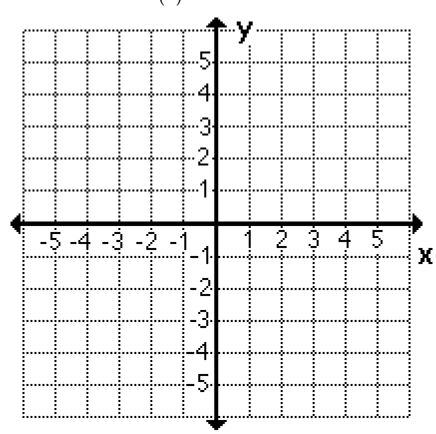
4. $f_{(x)} = \left(\frac{1}{2}\right)^x - 4$



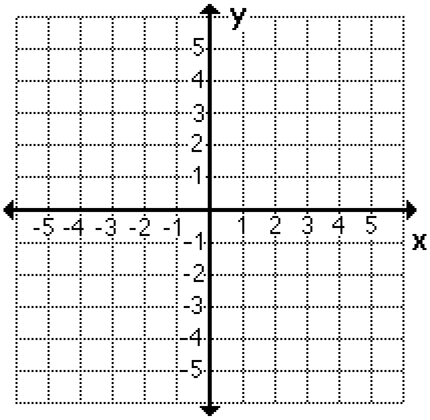
5. $f_{(x)} = -\log_3 x$



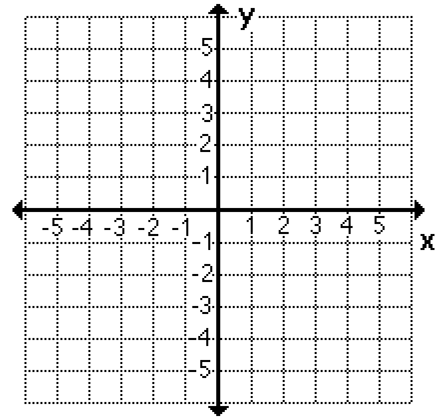
6. $f_{(x)} = \sqrt[3]{x+3} + 1$



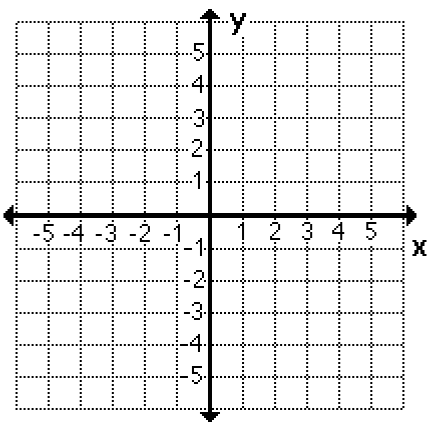
7. $f_{(x)} = (x+2)^3 - 4$



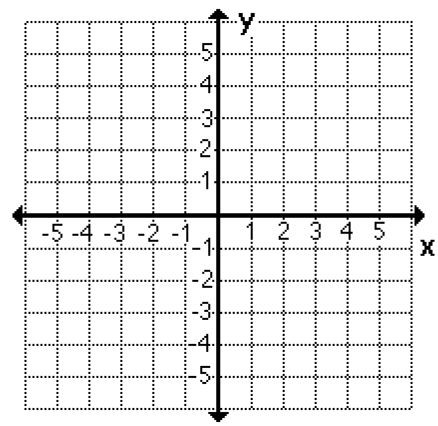
8. $f_{(x)} = \sqrt{-x+3} + 1$



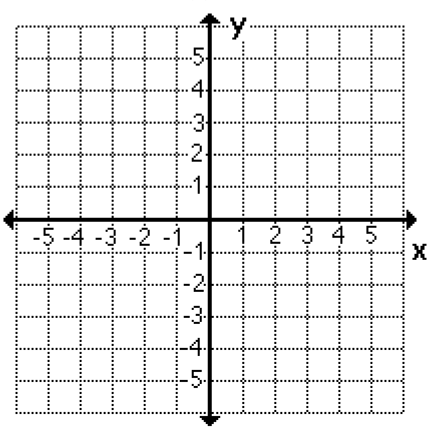
9. $f_{(x)} = -(x+3)^2 + 2$



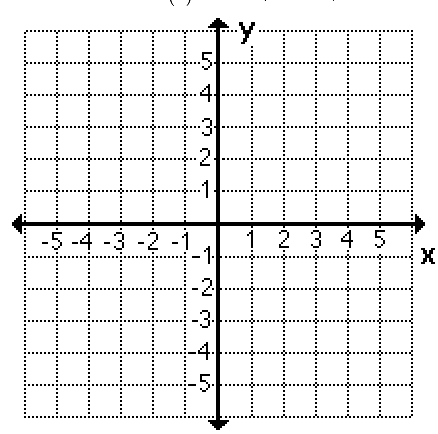
10. $f_{(x)} = -\sqrt[3]{x-3} - 2$



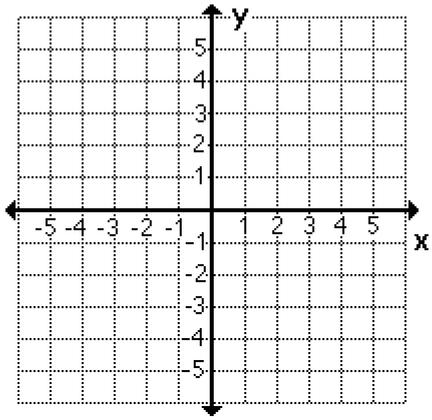
11. $f_{(x)} = x + 3$



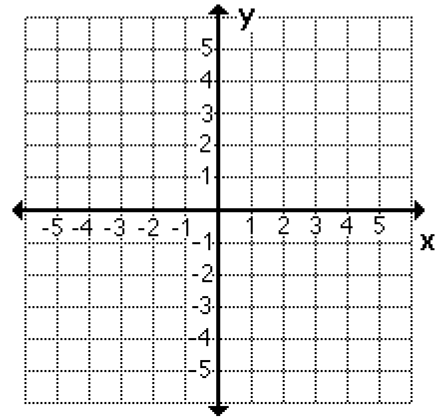
12. $f_{(x)} = \ln(x-2)$



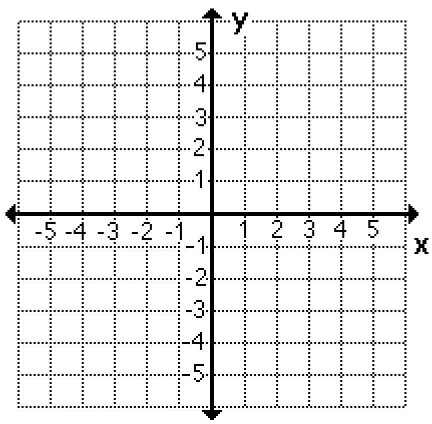
13. $f_{(x)} = -3^{x+2} - 1$



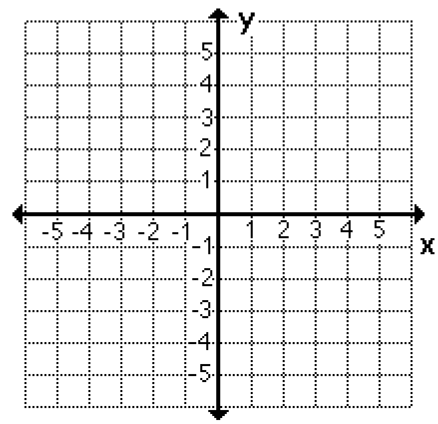
14. $f_{(x)} = -\left(x+2\right)^3$



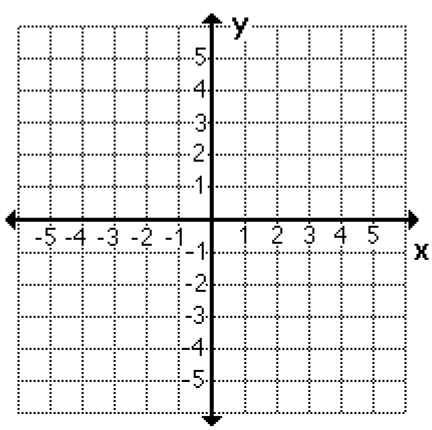
15. $f_{(x)} = -\log_2 x + 2$



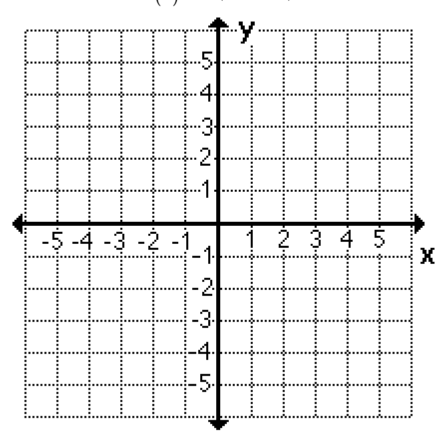
16. $f_{(x)} = -\left|x-3\right| + 2$



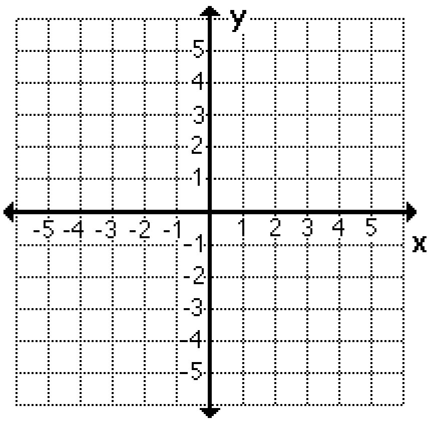
17. $f_{(x)} = e^{x+3} + 2$



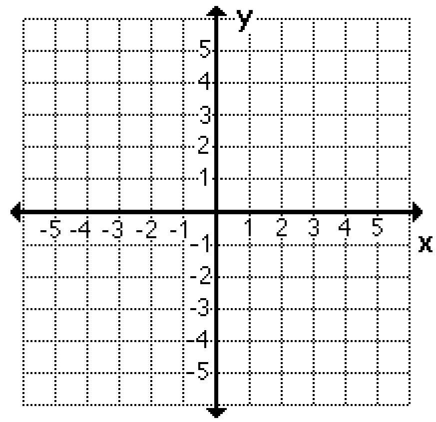
18. $f_{(x)} = \left(x+3\right)^2 - 4$



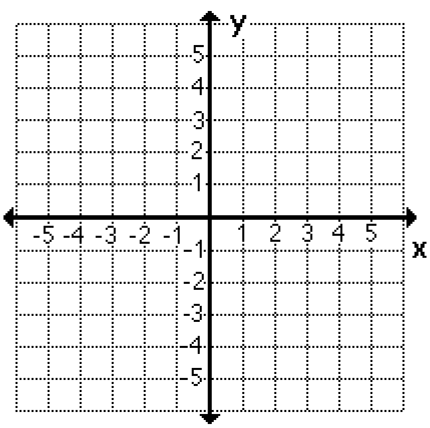
19. $f_{(x)} = -\left(\frac{1}{2}\right)^{x+2} + 3$



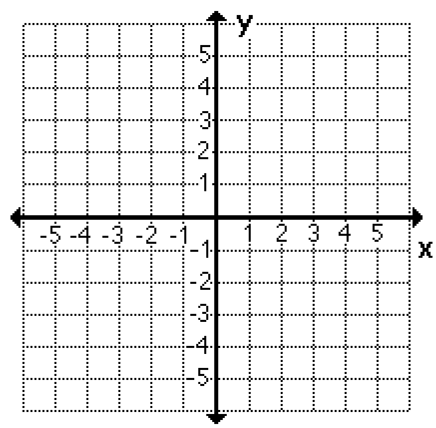
20. $f_{(x)} = -x^2 + 4$



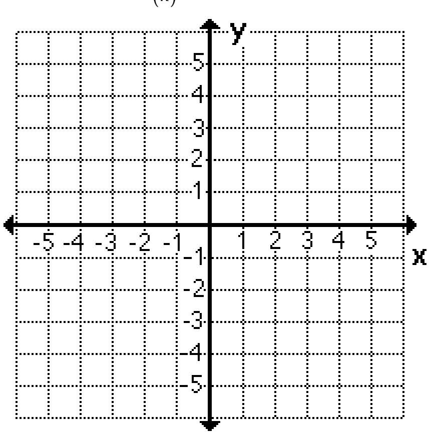
21. $f_{(x)} = 3$



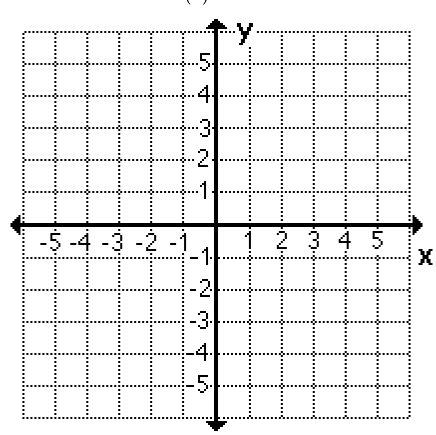
22. $f_{(x)} = (x-2)^3$



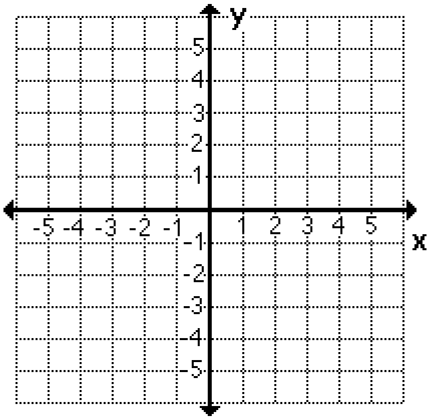
23. $f_{(x)} = \sqrt[3]{x-2} + 3$



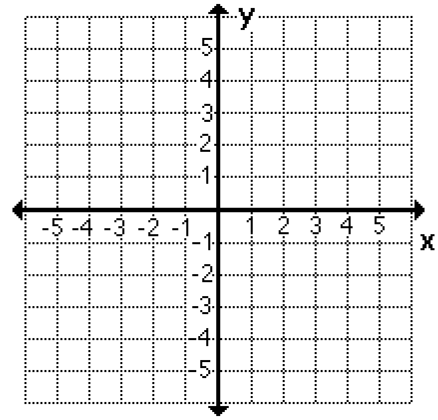
24. $f_{(x)} = -x + 3$



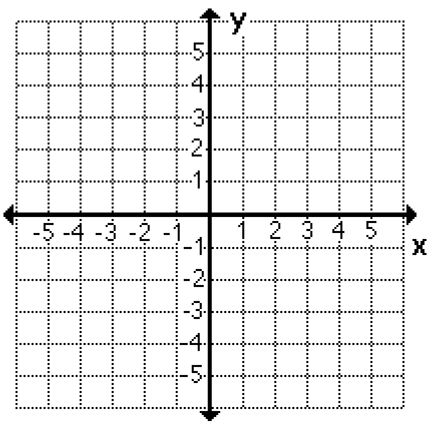
25. $f_{(x)} = -e^{x+3}$



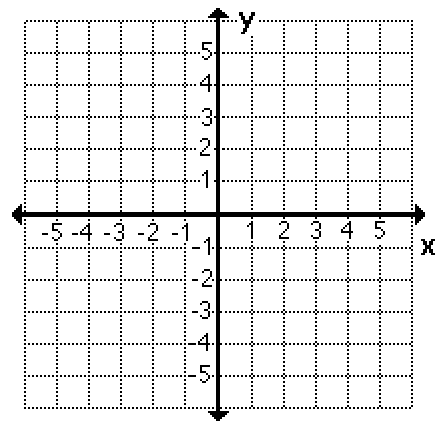
26. $f_{(x)} = \ln(x+3)+2$



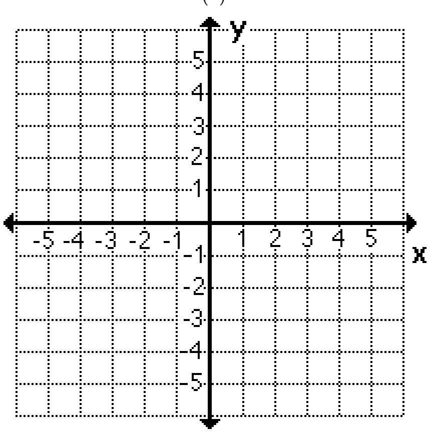
27. $f_{(x)} = -|x-2|-1$



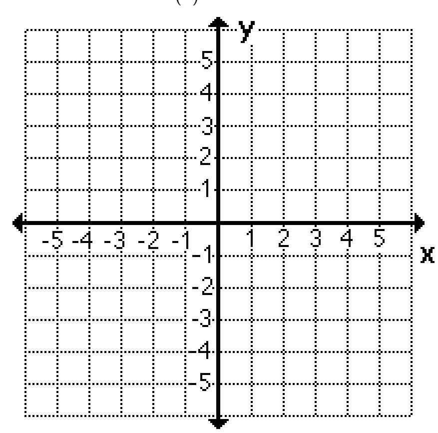
28. $f_{(x)} = \sqrt{x-3}+2$



29. $f_{(x)} = 3^{-x}$



30. $f_{(x)} = -2^{x-1}-2$



Checking Progress

You have now completed the "Functions" section of the workbook. The following is a checklist so that you may check your progress. Check off each of the objectives you have accomplished.

The student should be able to…

QUADRATIC FUNCTIONS

- ☐ ***Determine the properties of a quadratic function in standard form.***
- ☐ ***Find the x and y intercepts of a quadratic function.***
- ☐ ***Find the range and domain of a quadratic function.***
- ☐ ***Find the vertex of a quadratic function in standard form.***
- ☐ ***Graph a quadratic function.***

ABSOLUTE VALUE FUNCTIONS

- ☐ ***Determine the properties of an absolute value function in standard form.***
- ☐ ***Find the x and y intercepts of an absolute value function.***
- ☐ ***Find the range and domain of an absolute value function.***
- ☐ ***Find the vertex of an absolute value function.***
- ☐ ***Graph an absolute value function.***

RADICAL FUNCTIONS

- ☐ ***Determine the properties of a radical function in standard form.***
- ☐ ***Find the x and y intercepts of a radical function.***
- ☐ ***Find the range and domain of a radical function.***
- ☐ ***Find the point of origin of a radical function***
- ☐ ***Graph a radical function.***

EXPONENTIAL FUNCTIONS

- ☐ ***Determine the properties of an exponential function in standard form.***
- ☐ ***Find the x and y intercepts of an exponential function.***
- ☐ ***Find the range and domain of an exponential function.***
- ☐ ***Find the key point of an exponential function.***
- ☐ ***Graph an exponential function.***

LOGARITHMIC FUNCTIONS

- ☐ ***Determine the properties of a logarithmic function in standard form.***
- ☐ ***Find the x and y intercepts of a logarithmic function.***
- ☐ ***Find the range and domain of a logarithmic function.***
- ☐ ***Find the key point of a logarithmic function.***
- ☐ ***Graph a logarithmic function.***

Checklist continued.

The student should be able to…

CUBIC FUNCTIONS

- ☐ ***Determine the properties of a cubic function in standard form.***
- ☐ ***Find the x and y intercepts of a cubic function.***
- ☐ ***Find the range and domain of a cubic function.***
- ☐ ***Find the vertex of a cubic function.***
- ☐ ***Graph a cubic function.***

PIECEWISE FUNCTIONS

- ☐ ***Shift the graph of a function without actually knowing the equation, i.e. graphing*** $f_{(x+2)}$.
- ☐ ***Graph piece-wise functions.***