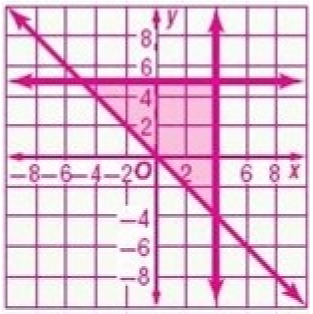
3-3 Optimization with Linear Programming

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

1. $y \le 5$
$x \le 4$
$y \ge -x$
$f(x, y) = 5x - 2y$

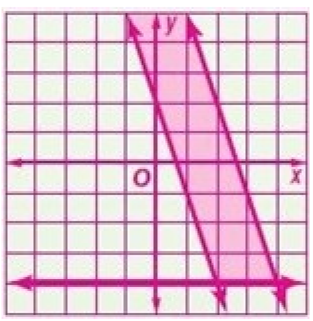
ANSWER:



$(4,5),(4,-4),(-5,5)$; max $= 28$, min $= -35$

3. $y \ge -3x + 2$
$9x + 3y \le 24$
$y \ge -4$
$f(x, y) = 2x + 14y$

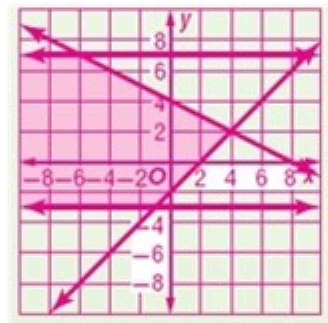
ANSWER:



(2, –4), (4, –4); max does not exist, min = –52

5. $-3 \le y \le 7$
$4y \ge 4x - 8$
$6y + 3x \le 12$
$f(x, y) = -12x + 9y$

ANSWER:



(4, 2), (–1, –3), (–6, 7); max does not exist; min = –26

7. **CCSS PRECISION** The total number of workers' hours per day available for production in a skateboard factory is 85 hours. There are 40 hours available for finishing decks and quality control each day. The table shows the number of hours needed in each department for two different types of skateboards.

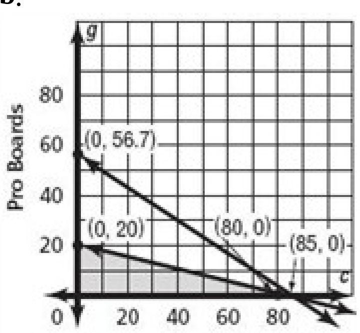
Skateboard Manufacturing Time		
Board Type	Production Time	Deck Finishing/Quality control
Pro Boards	1.5 hours	2 hours
Specialty Boards	1 hour	0.5 hour

a. Write a system of inequalities to represent the situation.
b. Draw the graph showing the feasible region.
c. List the coordinates of the vertices of the feasible region.
d. If the profit on a pro board is $50 and the profit on a specialty board is $65, write a function for the total profit on the skateboards.
e. Determine the number of each type of skateboard that needs to be made to have a maximum profit. What is the maximum profit?

ANSWER:

a.
$$g \geq 0$$
$$c \geq 0$$
$$1.5g + c \leq 85$$
$$2g + 0.5c \leq 40$$

b.



c. (0, 0), (0, 20), (80, 0)

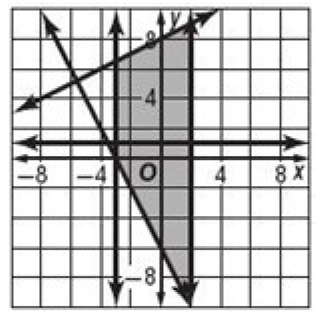
d. $f(c, g) = 65c + 50g$

e. 80 specialty boards, 0 pro boards; $5200

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

9.
$$2 \geq x \geq -3$$
$$y \geq -2x - 6$$
$$4y \leq 2x + 32$$
$$f(x, y) = -4x - 9y$$

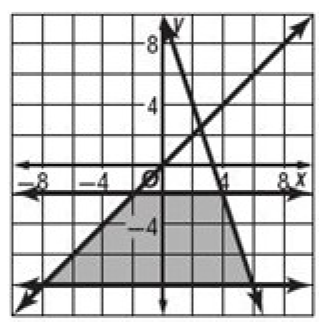
ANSWER:



(2, –10), (–3, 0), (–3, 6.5), (2, 9); max = 82, min = –89

11.
$$-8 \leq y \leq -2$$
$$y \leq x$$
$$y \leq -3x + 10$$
$$f(x, y) = 5x + 14y$$

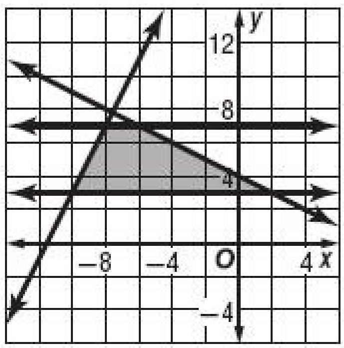
ANSWER:



(6, –8), (4, –2), (–2, –2), (–8, –8);max = –8, min = –152

13.
$$3 \le y \le 7$$
$$2y + x \le 8$$
$$y - 2x \le 23$$
$$f(x, y) = -3x + 5y$$

ANSWER:

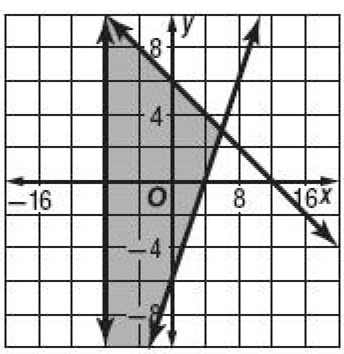


(–10, 3), (2, 3), (–6, 7), (–8, 7); max = 59, min = 9

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

15.
$$x \ge -8$$
$$3x + 6y \le 36$$
$$2y + 12 \ge 3x$$
$$f(x, y) = 10x - 6y$$

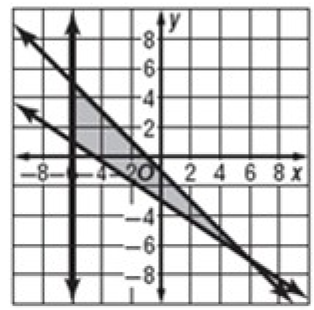
ANSWER:



(6, 3), (–8, 10), (–8, –18); max = 42, min = –140

17.
$$x \ge -6$$
$$y + x \le -1$$
$$2x + 3y \ge -9$$
$$f(x, y) = -10x - 12y$$

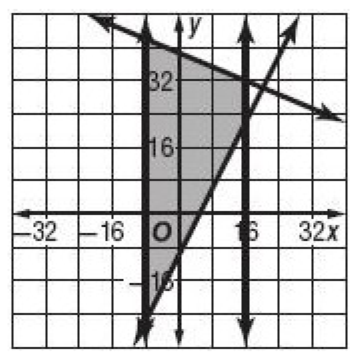
ANSWER:



(–6, 1), (6, –7), (–6, 5); max = 48, min = 0

19.
$$-8 \le x \le 16$$
$$y \ge 2x - 10$$
$$2y + x \le 80$$
$$f(x, y) = 12x + 15y$$

ANSWER:

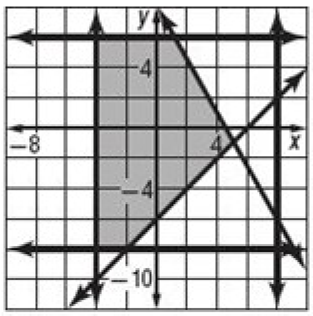


(–8, 44), (16, 32), (–8, –26), (16, 22); max = 672, min = –486

$-4 \le x \le 8$
$-8 \le y \le 6$

21. $y \ge x - 6$
$4y + 7x \le 31$
$f(x, y) = 12x + 8y$

ANSWER:



(5, –1), (1, 6), (–2, –8), (–4, –8), (–4, 6), max = 60, min = –112

23. **COOKING** Jenny's Bakery makes two types of birthday cakes: yellow cake, which sells for $25, and strawberry cake, which sells for $35. Both cakes are the same size, but the decorating and assembly time required for the yellow cake is 2 hours, while the time is 3 hours for the strawberry cake. There are 450 hours of labor available for production. How many of each type of cake should be made to maximize revenue?

ANSWER:

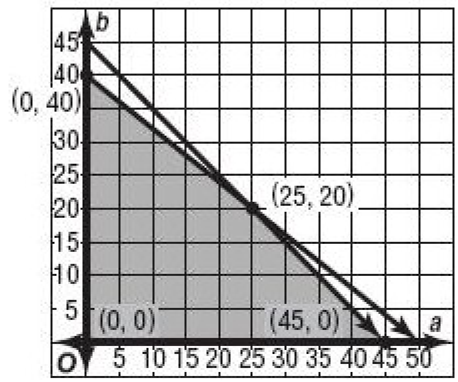
225 yellow cakes, 0 strawberry cakes

25. **CCSS PRECISION** Sean has 20 days to paint play houses and sheds. The sheds can be painted at a rate of 2.5 per day, and the play houses can be painted at a rate of 2 per day. He has 45 structures that need to be painted.
a. Write a system of inequalities to represent the possible ways Sean can paint the structures.
b. Draw a graph showing the feasible region and list the coordinates of the vertices of the feasible region.
c. If the profit is $26 per shed and $30 per play house, how many of each should he paint?
d. What is the maximum profit?

ANSWER:

a. $a \ge 0,\ b \ge 0,\ a + b \le 45,\ 4a + 5b \le 200$

b.



c. 25 sheds, 20 play houses
d. $1250

27. **BUSINESS** Each car on a freight train can hold 4200 pounds of cargo and has a capacity of 480 cubic feet. The freight service handles two types of packages: small, which weigh 25 pounds and are 3 cubic feet each, and large, which are 50 pounds and are 5 cubic feet each. The freight service charges $5 for each small package and $8 for each large package.
a. Find the number of each type of package that should be placed on a train car to maximize revenue.
b. What is the maximum revenue per train car?
c. In this situation, is maximizing the revenue necessarily the best thing for the company to do? Explain.

ANSWER:

a. 160 small packages, 0 large packages
b. $800
c. No; if revenue is maximized, the company will not deliver any large packages, and customers with large packages to ship will probably choose another carrier.

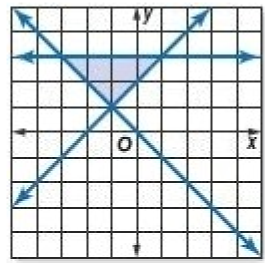
29. **OPEN ENDED** Create a set of inequalities that forms a bounded region with an area of 20 units^2 and lies only in the fourth quadrant.

ANSWER:

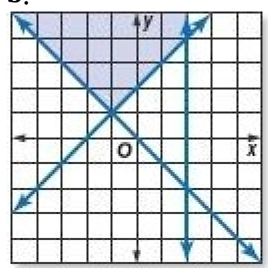
Sample answer: $-2 \geq y \geq -6,\ 4 \leq x \leq 9$

31. **CCSS ARGUMENTS** Identify the system of inequalities that is not the same as the other three. Explain your reasoning.

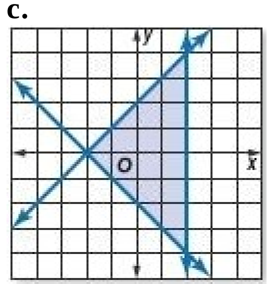
a.



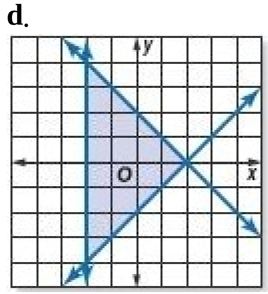
b.



c.



d.



ANSWER:

b; The feasible region of Graph b is unbounded while the other three are bounded.

33. **WRITING IN MATH** Upon determining a bounded feasible region, Ayumi noticed that vertices $A(-3, 4)$ and $B(5, 2)$ yielded the same maximum value for $f(x, y) = 16y + 4x$. Kelvin confirmed that her constraints were graphed correctly and her vertices were correct. Then he said that those two points were not the only maximum values in the feasible region. Explain how this could have happened.

ANSWER:

Sample answer: Even though the region is bounded, multiple maximums occur at A and B and all of the points on the boundary of the feasible region containing both A and B. This happened because that boundary of the region has the same slope as the function.

35. **SHORT RESPONSE** A family of four went out to dinner. Their bill, including tax, was $60. They left a 17% tip on the total cost of their bill. What is the total cost of the dinner including tip?

ANSWER:

$70.20

37. **GEOMETRY** Which of the following best describes the graphs of $y = 3x - 5$ and $4y = 12x + 16$?

A The lines have the same y-intercept.
B The lines have the same x-intercept.
C The lines are perpendicular.
D The lines are parallel.

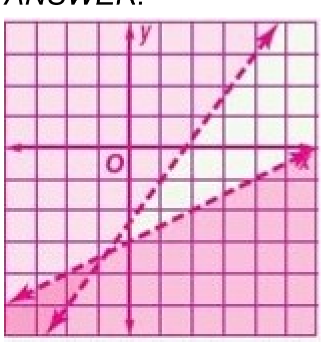
ANSWER:

D

Solve each system of inequalities by graphing.

39. $4x - 3y < 7$
$2y - x < -6$

ANSWER:



41. **BUSINESS** Last year the chess team paid $7 per hat and $15 per shirt for a total purchase of $330. This year they spent $360 to buy the same number of shirts and hats because the hats now cost $8 and the shirts cost $16. Write and solve a system of two equations that represents the number of hats and shirts bought each year.

ANSWER:

$7x + 15y = 330, 8x + 16y = 360$; hats: 15, shirts: 15

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

43. passes through $(-3, 5)$ and $(3, 2)$

ANSWER:

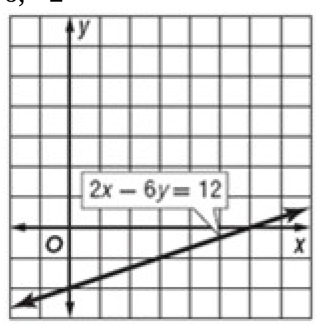
$y = -\frac{1}{2}x + \frac{7}{2}$

Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation.

45. $2x - 6y = 12$

ANSWER:

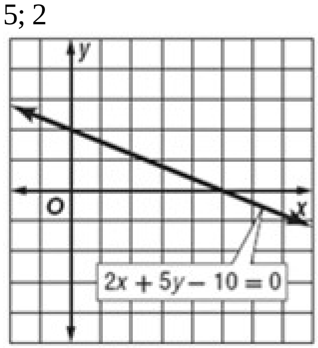
6; –2



47. $2x + 5y - 10 = 0$

ANSWER:

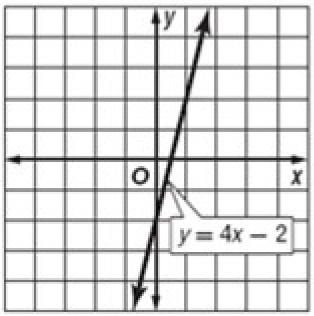
5; 2



49. $y = 4x - 2$

ANSWER:

$\frac{1}{2}; -2$



Evaluate each expression if *x* = –1, *y* = 3, and *z* = 7.

51. $2x - y + 2z$

ANSWER:

9

53. $4x + 2y - z$

ANSWER:

–5

55. $-3x - 3y + 3z$

ANSWER:

15