ALGEBRA II HONORS

WORKBOOK

First Semester

SOLUTION MANUAL

MR. RAYA'S CLASS

ALGEBRA II HONORS

WORKBOOK

FIRST SEMESTER

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NUMBERS

Objectives

The following is a list of objectives for this section of the workbook.

By the time the student is finished with this section of the workbook, he/she should be able to…

- ***Identify the symbols for each type of number.***
- ***Define the six types of numbers addressed in this section (Natural, Whole, Integer, Rational, Irrational and Real numbers).***
- ***Use the order of operations to simplify an algebraic expression.***
- ***Identify basic algebraic properties.***

Math Standards Addressed

The following state standards are addressed in this section of the workbook.

Algebra I

1.0 Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable:

Types of Numbers

What is the symbol for Natural Numbers? $\mathbb{N}$

What is the symbol for Whole Numbers? $\mathbb{W}$

What is the symbol for Integers? $\mathbb{Z}$

What is the symbol for Rational Numbers? $\mathbb{Q}$

What is the symbol for Irrational Numbers? $\mathbb{I}\mathbb{R}$

What is the symbol for Real Numbers? $\mathbb{R}$

Describe, in your own words, what Natural Numbers are.

Any whole number greater than zero. Also referred to as counting numbers

1, 2, 3, 4, 5 . . .

Describe, in your own words, what Whole Numbers are.

Any whole number including zero

0, 1, 2, 3, 4, 5 . . .

Describe, in your own words, what Integers are.

Any whole number that is positive, negative or zero.

. . . -3, -2, -1, 0, 1, 2, 3 . . .

Describe, in your own words, what Rational Numbers are.

Any number that can be written as the ratio of two intergers if you have a decimal value, it must repeat, or terminate.

Describe, in your own words, what Irrational Numbers are.

Any decimal value that goes on forever and has no repeating pattern.

$\sqrt{3}$, $\sqrt{2}$, $\sqrt{\ }$ . . .

What are Real Numbers?

All of the numbers that are natural, whole, or intergers rational or irrational. These are all real numbers.

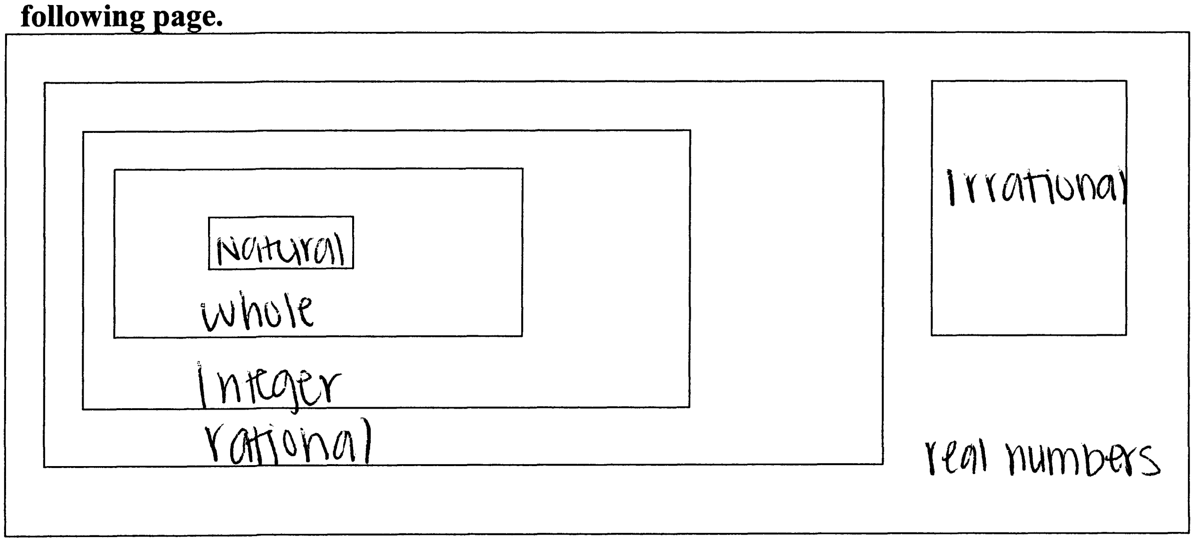
Check the appropriate box for each type of number listed.

	Natural	Whole	Integer	Rational	Irrational	Real
4	✓	✓	✓	✓		✓
-3			✓	✓		✓
3.5				✓		✓
$\sqrt{2}$					✓	✓
5.2323...				✓		✓
π					✓	✓
0		✓	✓	✓		✓
$\sqrt{9}$	✓	✓	✓	✓		✓
2.26483...					✓	✓
$-\frac{1}{3}$				✓		✓
$\frac{12}{3}$	✓	✓	✓	✓		✓

Based on your observations from the previous chart, what conclusions can you make about the nature of these types of numbers?

All ~~rea~~ are real numbers. once you check a box like whole numbers, you will check all others except irrational If a number is irrational The only other box that can be checked is real.

Complete the chart showing the relationship between the numbers. Write the appropriate number type in each box, and use this information to answer the for the questions on the following page.



Are all Natural Numbers Integers?

yes

Are all Rational Numbers Integers?

No

Are all Integers Whole Numbers?

No

Are all Whole Numbers Rational?

yes.

Are all Irrational Numbers Real?

yes.

Are all Real Numbers Whole Numbers?

No

A real number is rational **if it can be written as the ratio of two integers** $\frac{a}{b}$**, where** $b \neq 0$.

irrational **numbers have infinite non-repeating decimal representations.**

The distance between a point on the number line and the origin is the absolute value **of the real number.**

Numbers that can be written as the product of two or more prime numbers are called composite **numbers.**

The numerical factor of a variable term is the coefficient **of the variable term.**

Write the following list of numbers in descending order.

$$\frac{1}{2}, \frac{4}{5}, \frac{6}{7}, -\frac{5}{9}, -\frac{8}{15}, \frac{3}{4}$$

$\frac{6}{7}, \frac{4}{5}, \frac{3}{4}, \frac{1}{2}, -\frac{8}{15}, -\frac{5}{9}$

Write the following list of number in ascending order.

$$\pi, \frac{5}{7}, \frac{8}{3}, -1.06, -1.061, \sqrt{2}$$

$-1.061, -1.06, \frac{5}{7}, \sqrt{2}, \frac{8}{3}, \pi$

Order of Operations

What is the Order of Operations?

parenthesis Exponents Multiplication Division Addition Subtraction

What does "same level operations" mean?

This means these operations are of equal importance you do whichever comes first going left to right in the problem.

Using the Order of Operations, which operations are same level operations?

multiplication & division and Addition & Subtraction.

An algebraic expression may contain 4 elements. What are they?

Numbers
variables
operation signs
grouping symbols

Simplify each of the following algebraic expressions using the order of operations.

A) $3(x-4)-6\cdot 3+x(4-6)^2$

$7x-30$

B) $16-3\cdot 8\div 4\cdot 5-18$

-32

C) $4+3(2^3-3\cdot 5^2)$

-197

D) $2\left[(5-3)^2-3(2\cdot 8\div 4)\right]^2$

128

E) $2-(x-4)+3(3x-5)+4$

8x − 5

F) $\frac{3}{2}-\left(\frac{2}{3}\right)^3+\frac{5}{6}$

$\frac{55}{27}$

G) $4(3x-2)-2(5-x)+9(x+4)$

23x +18

H) $8+(2\cdot 5)\cdot 34\div 9$

$\frac{412}{9}$

I) $18\div 3+3\cdot 4-3$

15

J) $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{4}+\frac{1}{5}}$

$\frac{50}{27}$

K) $\dfrac{\frac{2}{3}+\frac{5}{6}-\frac{1}{2}}{\frac{1}{8}-\frac{1}{3}+\frac{1}{12}}$

−8

L) $\dfrac{2+2\cdot 5}{\frac{8}{2}+2}$

2

M) $\dfrac{\frac{-72}{(-4)(-3)}}{\left(-4-(-2)\right)^2}$

$-\frac{3}{2}$

N) $5^2-2\cdot 8\div 6+(4+1)^2+3^2\div 2+(5-2)^2-2^2\cdot 5+2^2$

$\frac{269}{6}$

Basic Algebraic Properties

	Property	Example
Commutative Property of Addition:	$a+b=b+a$	$5+6=6+5$
Commutative Property of Multiplication:	$ab=ba$	$3\cdot 4=4\cdot 3$
Associative Property of Addition:	$(a+b)+c=a+(b+c)$	$(5+2)+3=5+(2+3)$
Associative Property of Multiplication:	$(ab)c=a(bc)$	$(2\cdot 3)\cdot 5=2\cdot(3\cdot 5)$
Distributive Property:	$a(b+c)=ab+ac$	$3(2x+y)=3\cdot 2x+3y$
	$(a+b)c=ac+bc$	$(5x+z)2=5x\cdot 2+z\cdot 2$
Additive Identity Property:	$a+0=a$	$6+0=6$
Multiplicative Identity Property:	$a\cdot 1=a$	$12\cdot 1=12$
Additive Inverse Property:	$a+(-a)=0$	$3+(-3)=0$
Multiplicative Inverse Property:	$a\cdot\frac{1}{a}=1, \quad a\neq 0$	$2a\cdot\frac{1}{2a}=1$

What are the two things you look for when identifying the associative property that are dead giveaways?

grouping symbols, and the numbers <u>do not</u> move.

Remember, this chart of basic algebraic properties is given using the variables a, b and c, where each letter represents a real number, variable, or algebraic expression.

Identify each property that is illustrated, such as "associative property of addition," or "commutative property of multiplication."

$3x+(5y+12z)=(3x+5y)+12z$	associative property of addition
$(3x-6y)4=12x-24y$	distributive property
$\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)=1$	inverse property of multiplication
$(2x-7y)+3=3+(2x-7y)$	commutative property of addition
$(1)(x^2yz)=x^2yz$	identity property of multiplication
$(2a\cdot 3b)\cdot 5c=2a\cdot(3b\cdot 5c)$	associative property of multiplication
$25x+(-25x)=0$	additive inverse property
$6x\cdot(5y\cdot 3z)=(5y\cdot 3z)\cdot 6x$	commutative property of multiplication
$5a+12b=12b+5a$	commutative property of addition
$(x+2)(5y-3z)=5y(x+2)-3z(x+2)$	distributive property

<u>Checking Progress</u>

You have now completed the "Numbers" section of the workbook. The following is a checklist so that you may check your progress. Check off each of the objectives you have accomplished.

The student should be able to…

- ☐ ***Identify the symbols for each type of number.***
- ☐ ***Define the six types of numbers addressed in this section (Natural, Whole, Integer, Rational, Irrational and Real numbers).***
- ☐ ***Use the order of operations to simplify an algebraic expression.***
- ☐ ***Identify basic algebraic properties.***

ALGEBRAIC
EQUATIONS

ALGEBRAIC EQUATIONS

Objectives

The following is a list of objectives for this section of the workbook.

By the time the student is finished with this section of the workbook, he/she should be able to…

- ***List the five elements an algebraic equation may contain.***
- ***Solve basic algebraic equations by using inverse operations.***
- ***Solve absolute value equations.***
- ***Find the solution of an algebraic equation if the variables cancel leaving the student with a true or false statement.***
- ***Solve literal equations by isolating the required variable.***

Math Standards Addressed

The following state standards are addressed in this section of the workbook.

Algebra I

2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

3.0 Students solve equations and inequalities involving absolute values.

4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as 3(2x-5) + 4(x-2) = 12.

Algebra II

1.0 Students solve equations and inequalities involving absolute value.

Basic Algebraic Equations

An algebraic equation may contain five elements. What are they?

Numbers
Variables
Operation signs
grouping symbols
equal sign

When solving an equation, sometimes the variables will cancel out. In that case, you are left with either a true statement, or a false statement. What does that tell you in terms of your solution?

If left with a true statement your answer is all real numbers.

If left with a false statement your answer is no solutions.

Solve each of the following equations. Give exact solutions, no calculator estimations. Be sure to reduce your answers.

A) $3(2x+5)-4=6$

$x=-\frac{5}{6}$

B) $-2(x-3)+4=4(2-3x)$

$x=-\frac{1}{5}$

C) $-6+3x=-20$

$x=-\frac{14}{3}$

D) $-3(x-2)+6=5(3-2x)$

$x=\frac{3}{7}$

E) $5(3-4x)=7-(4-x)$

$x=\frac{4}{7}$

F) $-3x-4=11$

$x=-5$

G) $2(x-3)=5-3(x+6)$

$x=-\frac{7}{5}$

H) $5\left(2-\frac{1}{2}x\right)=4-\left(\frac{1}{4}x-1\right)$

$x=\frac{20}{9}$

I) $-2(x+17)=13-x$

$x=-47$

J) $2+3(x-1)=x-1$

$X=0$

K) $\frac{2}{3}x+5=-\frac{1}{3}x+17$

$X=12$

L) $\frac{x}{3}+\frac{1}{2}=\frac{7}{6}$

$X=2$

M) $\frac{1}{2}x+\frac{1}{4}=\frac{1}{4}(x-6)$

$X=-7$

N) $\frac{x}{3}-\frac{x-5}{5}=3$

$X=15$

O) $x-0.2x=72$

$X=90$

P) $4-6(2x-3)+1=3+2(5-x)$

$X=1$

Q) $0.03(x+200)+0.05x=86$

$X=1000$

R) $\frac{3}{4}-\frac{1}{3}\left(\frac{1}{2}x-2\right)=3\left(x-\frac{1}{4}\right)$

$X=\frac{13}{19}$

S) $1.3-0.2(6-3x)=0.1(0.2x+3)$

$X=\frac{16}{29}$

Absolute Value Equations

When solving absolute value equations, remember you are creating two separate problems to solve. Consider the statement $|?| = 2$; if this is true, then there must be a 2 or a -2 inside the absolute value symbols. The same thinking is used for absolute value equations. If you are told $|x+3| = 7$, you can conclude that $x+3$ must be equal to either 7 or -7. This would give you the desired result.

Example

$$2|x-7|+6=18$$

The first step to solving this equation is to isolate the absolute value.

$$2|x-7|+6=18$$

$$2|x-7|=12 \qquad \textit{Subtract 6 to both sides.}$$

$$|x-7|=6 \qquad \textit{Divide both sides by 2.}$$

Now you must create two separate problems to solve. Recall the sample above, if the absolute value of $x-7$ is equal to 6, then $x-7$ must be equal to either 6 or -6. Set up two problems showing this.

	$x-7=6$	***or***	$x-7=-6$	
Add 7 to both sides to solve.	$x=13$		$x=1$	***Add 7 to both sides to solve.***

$$x=\{1,13\} \qquad \textit{Our solution set is 1 and 13.}$$

Solve each of the following absolute value equations.

A) $|x|-11=14$

$x = \{\pm 25\}$

B) $|x|+7=4$

No solution

C) $|x-2|=7$

$x = \{-5, 9\}$

D) $|x+14|=20$

$x = \{-34, 6\}$

E) $-6|x-7|+50=14$

$x = \{1, 13\}$

F) $|x+7|-14=14$

$x = \{-35, 21\}$

G) $|7x+3|-4=14$

$x = \{-3, \frac{15}{7}\}$

H) $\frac{1}{2}|x+2|-3=2$

$x = \{-12, 8\}$

I) $4-3|2x+1|=-11$

$x = \{-3, 2\}$

J) $3-|5x-1|=8$

No solution

Literal Equations

For each problem, solve for the indicated variable.

A) Solve for l : $p = 2l + 2w$

$l = \frac{p-2w}{2}$

B) Solve for P: $A = P\left(1+\frac{r}{n}\right)^{nt}$

$P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$

C) Solve for C : $F = \frac{9}{5}C + 32$

$C = \frac{5}{9}(F-32)$

D) Solve for P: $A = P + Prt$

$P = \frac{A}{1+rt}$

E) Solve for r : $A = P + Prt$

$r = \frac{A-P}{Pt}$

F) Solve for b : $A = \frac{1}{2}bh$

$b = \frac{2A}{h}$

G) Solve for r : $V = \frac{1}{3}\pi r^2 h$

$r = \sqrt{\frac{3V}{\pi h}}$

H) Solve for r : $C = 2\pi r$

$r = \frac{C}{2\pi}$

I) Solve for L : $S = 2WL + 2LH + 2WH$

$L = \frac{S - 2WH}{2W + 2H}$

Checking Progress

You have now completed the "Algebraic Equations" section of the workbook. The following is a checklist so that you may check your progress. Check off each of the objectives you have accomplished.

The student should be able to…

- ☐ ***List the five elements an algebraic equation may contain.***
- ☐ ***Solve basic algebraic equations by using inverse operations.***
- ☐ ***Solve absolute value equations.***
- ☐ ***Find the solution of an algebraic equation if the variables cancel leaving the student with a true or false statement.***
- ☐ ***Solve literal equations by isolating the required variable.***

INEQUALITIES

Objectives

The following is a list of objectives for this section of the workbook.

By the time the student is finished with this section of the workbook, he/she should be able to…

- ***Describe a solution set using interval notation.***
- ***Tell the difference between inclusive and exclusive symbols and use each.***
- ***Solve simple inequalities in a single variable.***
- ***Solve compound inequalities.***
- ***Find the solution to inequalities using the word "and."***
- ***Find the solution to inequalities using the word "or."***
- ***Solve absolute value inequalities.***
- ***Find the domain of a radical function using inequalities.***

Math Standards Addressed

The following state standards are addressed in this section of the workbook.

Algebra I

3.0 Students solve equations and inequalities involving absolute values.

4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as 3(2x-5) + 4(x-2) = 12.

5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

Algebra II

1.0 Students solve equations and inequalities involving absolute value.

Interval Notation

Whenever you need to describe an interval, be sure to use interval notation. When using interval notation, the smaller number is always on the left. Pay particular attention to compound inequality problems. With compound inequality problems, it is sometimes necessary to rearrange the solution so that the smaller number is on the left.

When using interval notation, which symbols mean inclusive?

Brackets.

When writing in interval notation, which symbols mean exclusive?

Parenthesis

Practice writing each of the following solutions using interval notation.

A) $x \geq 4$

$[4, \infty)$

B) $x < \frac{1}{2}$

$(-\infty, \frac{1}{2})$

C) $x > -3$

$(-3, \infty)$

D) $x \leq -4$

$(-\infty, -4]$

E) $6 > x$

$x < 6$

$(-\infty, 6)$

F) $3 < x \leq 6$

$(3, 6]$

G) $-4 > x > 5$

$(-\infty, -4) \cup (5, \infty)$

H) $x \neq 3$

$(-\infty, 3) \cup (3, \infty)$

I) $3 \geq x > -4$

$-4 < x \leq 3$

$(-4, 3]$

J) $1 \geq x \geq 1$

$(-\infty, \infty)$

K) $\frac{3}{7} > x \geq \frac{5}{13}$

$\frac{5}{13} \leq x < \frac{3}{7}$

$[\frac{5}{13}, \frac{3}{7})$

L) $-4 \leq x \leq -6$

$-6 \geq x \geq -4$

$(-\infty, -6] \cup [-4, \infty)$

M) $x > 4$ and $x > 7$

$(7, \infty)$

N) $x < 4$ or $x > -3$

$(-\infty, \infty)$

Simple Inequalities

Remember, when graphing inequalities, we have replaced the open circle and solid dot usually used, with parenthesis and brackets when appropriate.

Solve each inequality, sketch the graph for each and write the solution interval using interval notation.

A) $2x+5 \ge 2-(x-9)$

$2x+5 \ge 2-x+9$

$2x+5 \ge -x+11$

$+x \quad +x$

$3x+5 \ge 11$

$-5 \quad -5$

$3x \ge 6$

$x \ge 2$

1 2 3

B) $3x-5 < x+2(x+7)$

$3x-5 < x+2x+14$

$3x-5 < 3x+14$

$-3x \quad -3x$

$-5 < 14$

True

-1 0 1

$(-\infty, \infty)$

C) $15 > -3x+6$

$+3x \quad +3x$

$3x+15 > 6$

$-15 \quad -15$

$\frac{3x}{3} > \frac{-9}{3}$

$x > -3$

-4 -3 -2

$(-3, \infty)$

D) $5x-4 \le 3(x-7)$

$5x-4 \le 3x-21$

$-3x+4 \quad -3x+4$

$\frac{2x}{2} \le \frac{-17}{2}$

$x \le -\frac{17}{2}$

-9 $\frac{-17}{2}$ -8

$(-\infty, -\frac{17}{2}]$

E) $5(2x+3) \ge 7x-3(4-x)+32$

$10x+15 \ge 7x-12+3x+32$

$10x+15 \ge 10x+20$

$-10x \quad -10x$

$15 \ge 20$

false

No solution

F) $3(x+1) > 1+5x$

$3x+3 > 1+5x$

$-5x-3 \quad -3-5x$

$\frac{-2x}{-2} > \frac{-2}{-2}$

$x < 1$

0 1 2

$(-\infty, 1)$

G) $3-2x \le 5+3x$

$-3x \quad -3x$

$3-5x \le 5$

$-3 \quad -3$

$\frac{-5x}{-5} \le \frac{2}{-5}$

$x \ge -\frac{2}{5}$ $[-\frac{2}{5}, \infty)$

-1 $-\frac{2}{5}$ 0

H) $2(x-5)-8x < -7x-14$

$2x-10-8x < -7x-14$

$+10 \quad +10$

$-6x < -7x-4$

$+7x \quad +7x$

$x < -4$

-5 -4 -3

$(-\infty, -4)$

I) $2x-14 > 4x+4$

$-4x \quad -4x$

$-2x-14 > 4$

$+14 \quad +14$

$\frac{-2x}{-2} > \frac{18}{-2}$

$x < -9$

-10 -9 -8

$(-\infty, -9)$

J) $\frac{42}{6}-\frac{6(8+5x)}{6} > \frac{114}{6}$

$7-(8+5x) > 19$

$-7 \quad -7$

$-8-5x > 12$

$+8 \quad +8$

$-5x > 20$

$x < -4$

-5 -4 -3 $(-\infty, -4)$

K) $-7-8(5x+3) < 9$

$-7-40x-24 < 9$

$-31-40x < 9$

$+31 \quad +31$

$\frac{-40x}{-40} < \frac{40}{-40}$

$x > -1$

-2 -1 0

$(-1, \infty)$

Compound Inequalities

Conjunction problems use the word and**, and disjunction problems use the word** or**.**

For conjunction problems, we are looking for an overlap **of the graphs to determine the solution interval.**

For disjunction problems, there is no overlap **needed. Any area shaded is part of the solution interval.**

Solve each inequality, sketch the graph for each and write the solution interval using interval notation.

A) $5 < 2x - 3 < 11$

$+3 \quad +3 \quad +3$

$\frac{8}{2} < \frac{2x}{2} < \frac{14}{2}$

$4 < x < 7$

4 7

$(4,7)$

B) $-1 < 5 - 3x \le 14$

$-5 \quad -5 \quad -5$

$\frac{-6}{-3} < \frac{-3x}{-3} \le \frac{9}{-3}$

$2 > x \ge -3$

$-3 \le x < 2$

-3 2

$[-3,2)$

C) $-2 > 3x + 1 > 10$

$-1 \quad -1 \quad -1$

$\frac{-3}{3} > \frac{3x}{3} > \frac{9}{3}$

$-1 > x > 3$

-1 3

$(-\infty,-1) \cup (3,\infty)$

D) $3 \le 3 - 5(x-3) \le 8$

$-3 \quad -3 \quad -3$

$0 \le -5x + 15 \le 5$

$-15 \quad -15 \quad -15$

$\frac{-15}{-5} \le \frac{-5x}{-5} \le \frac{-10}{-5}$

$3 \ge x \ge 2$

$2 \le x \le 3$

2 3

$[2,3]$

E) $2 \ge 4 - \frac{1}{2}(x-8) \ge 10$

$-4 \quad -4 \quad -4$

$-2 \ge -\frac{1}{2}x + 4 \ge 6$

$-4 \quad -4 \quad -4$

$(-2) \cdot -6 \ge -\frac{1}{2}x \ge 2(-2)$

(mult. by -2) $12 \le x \le -4$

$-4 \ge x \ge 12$

-4 12

$(-\infty,-4] \cup [12,\infty)$

F) $-2 > \frac{1-3x}{-2} \ge 7$

$(-2)(-2) > -2\left(\frac{1-3x}{-2}\right) \ge 7(-2)$

$4 < 1 - 3x \le -14$

$-1 \quad -1$

$\frac{3}{-3} < \frac{-3x}{-3} \le \frac{-15}{-3}$

$-1 > x \ge 5$

-1 5

$(-\infty,-1) \cup [5,\infty)$

G) $-1.2 \ge 1 - 0.02(x - 6.1) > 1.4$

$-1 \quad -1 \quad -1$

$-2.2 \ge -0.02x + 0.122 > 0.4$

$-0.122 \quad -0.122 \quad -0.122$

$\frac{-2.322}{-0.02} \ge \frac{-0.02x}{-0.02} > \frac{0.278}{-0.02}$

$116.1 \le x < -13.9$

$-13.9 > x \ge 116.1$

-13.9 116.1

$(-\infty,-13.9) \cup [116.1,\infty)$

H) $x < 5$ and $x > -3$

$x < 5$

$x > -3$

-3 5

$(-3,5)$

I) $x \le 6$ or $x > -2$

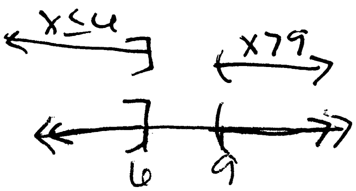
$x > -2$

$x \le 6$

-2 6

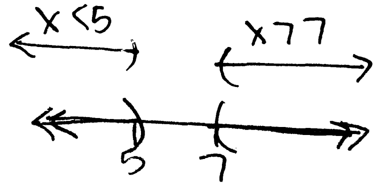
$(-\infty,\infty)$

J) $x \le 6$ or $x > 9$



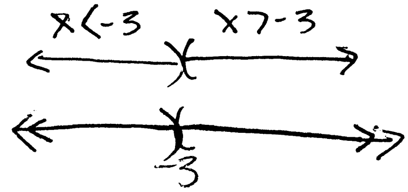
$(-\infty, 6] \cup (9, \infty)$

K) $x > 7$ or $x < 5$



$(-\infty, 5) \cup (7, \infty)$

L) $x > -3$ or $x < -3$

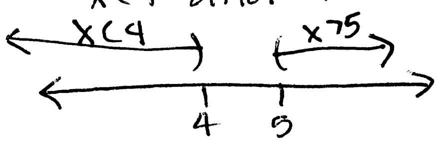


M) $0.5x < 2$ and $\dfrac{-0.6x}{-0.6} < \dfrac{-3}{-0.6}$

$\frac{1}{2}x < 2$

$(2)(\frac{1}{2}x) < 2(2)$ $\quad x > 5$

$x < 4$ and $x > 5$



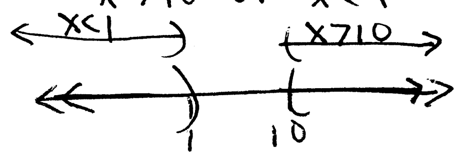
No solution ∅

N) $x - 3 > 7$ or $3 - x > 2$

$+3 \quad +3 \qquad -3 \quad -3$

$-x > -1$

$x > 10$ or $x < 1$



$(-\infty, 1) \cup (10, \infty)$

O) $5 + 4(x-1) < 3$ and $1 + 3(x-1) < 1$

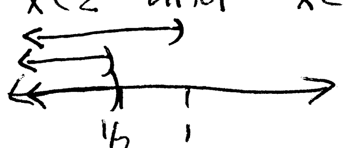
$-5 \quad -5 \qquad -1 \quad -1$

$4x - 4 < -2$ $\qquad 3x - 3 < 0$

$+4 \quad +4 \qquad +3 \quad +3$

$\frac{4x}{4} < \frac{2}{4}$ $\qquad \frac{3x}{3} < \frac{3}{3}$

$x < \frac{1}{2}$ and $x < 1$



$(-\infty, \frac{1}{2})$

P) $\frac{1}{4}x - \frac{1}{3} > -\frac{1}{5}$ and $\frac{1}{2}x < 2$

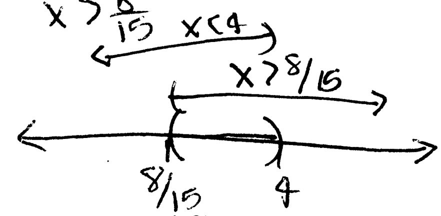
$60(\frac{1}{4}x - \frac{1}{3}) > 60(-\frac{1}{5})$ $\qquad 2(\frac{1}{2}x) < 2(2)$

$15x - 20 > -12$ $\qquad x < 4$

$+20 \quad +20$

$\frac{15x}{15} > \frac{8}{15}$

$x > \frac{8}{15}$



$(8/15, 4)$

Absolute Value Inequalities

For absolute value inequalities, you will create a compound inequality.

$$|x-6| + 3 \le 10$$

$$|x-6| \le 7$$ ***Isolate the absolute value first.***

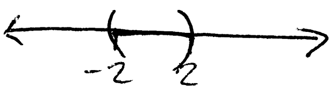
At this point you will write a compound inequality. Remember to use the same sign that is used in the original problem. Once this is done, the absolute value symbols are gone. Solve just as you would a normal compound inequality.

$$-7 \le x - 6 \le 7$$

Solve each inequality, sketch the graph for each and write the solution interval using interval notation.

A) $|x| < 2$

$-2 < x < 2$



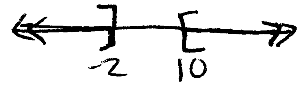
$(-2, 2)$

B) $|x-4| \geq 6$

$-6 \geq x-4 \geq 6$

$+4 \quad +4 \quad +4$

$-2 \geq x \geq 10$



$(-\infty, -2] \cup [10, \infty)$

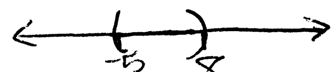
C) $|2x-3| < 13$

$-13 < 2x - 3 < 13$

$+3 \quad +3 \quad +3$

$\frac{-10}{2} < \frac{2x}{2} < \frac{16}{2}$

$-5 < x < 8$



$(-5, 8)$

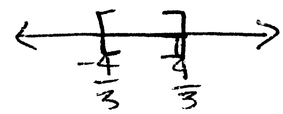
D) $3|x| + 1 \leq 5$

$-1 \quad -1$

$\frac{3|x|}{3} \leq \frac{4}{3}$

$|x| \leq \frac{4}{3}$

$-\frac{4}{3} \leq x \leq \frac{4}{3}$



$\left[-\frac{4}{3}, \frac{4}{3}\right]$

E) $\frac{-2|5-x|}{-2} \geq \frac{-14}{-2}$

$|5-x| \leq 7$

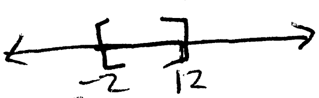
$-7 \leq 5 - x \leq 7$

$-5 \quad -5 \quad -5$

$\frac{-12}{-1} \leq \frac{-x}{-1} \leq \frac{2}{-1}$

$12 \geq x \geq -2$

$-2 \leq x \leq 12$



$[-2, 12]$

F) $2|3-2x| - 6 \geq 18$

$+6 \quad +6$

$\frac{2|3-2x|}{2} \geq \frac{24}{2}$

$|3-2x| \geq 12$

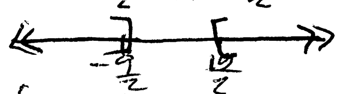
$-12 \geq 3 - 2x \geq 12$

$-3 \quad -3 \quad -3$

$\frac{-15}{-2} \leq \frac{-2x}{-2} \leq \frac{-9}{-2}$

$\frac{15}{2} \leq x \leq -\frac{9}{2}$

$-\frac{9}{2} \geq x \geq \frac{15}{2}$



$(-\infty, -\frac{9}{2}] \cup [\frac{15}{2}, \infty)$

G) $\left|\frac{2x+3}{2}\right| > 1$

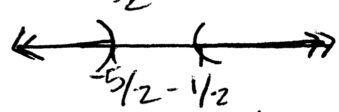
$-1 > \frac{2x+3}{2} > 1$

$-2 > 2x + 3 > 2$

$-3 \quad -3 \quad -3$

$\frac{-5}{2} > \frac{2x}{2} > \frac{-1}{2}$

$-\frac{5}{2} > x > -\frac{1}{2}$



$(-\infty, -\frac{5}{2}) \cup (-\frac{1}{2}, \infty)$

H) $2|5-2x| - 15 \geq 5$

$+15 \quad +15$

$\frac{2|5-2x|}{2} \geq \frac{20}{2}$

$|5-2x| \geq 10$

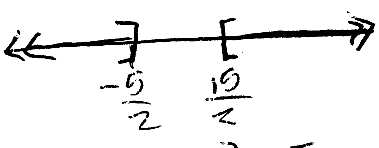
$-10 \geq 5 - 2x \geq 10$

$-5 \quad -5 \quad -5$

$\frac{-15}{-2} \geq \frac{-2x}{-2} \geq \frac{5}{-2}$

$\frac{15}{2} \leq x \leq -\frac{5}{2}$

$-\frac{5}{2} \geq x \geq \frac{15}{2}$



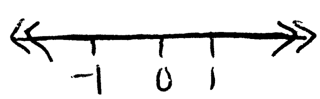
$(-\infty, -\frac{5}{2}] \cup [\frac{15}{2}, \infty)$

I) $3 + |7-2x| \geq 3$

$-3 \quad -3$

$|7-2x| \geq 0$

All real numbers

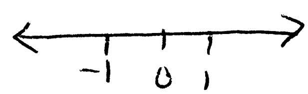


J) $|3x-5| + 16 < 12$

$-16 \quad -16$

$|3x-5| < -4$

No solution



Practice Using Intervals

Write each union or intersection of intervals as a single interval if possible. It may be helpful to graph them first.

A) $(-\infty,5) \cap (-\infty,9)$

$(-\infty,5)$

B) $(-3,\infty) \cup (-6,\infty)$

$(-6,\infty)$

C) $(2,\infty) \cup (4,\infty)$

$(2,\infty)$

D) $(-\infty,4] \cap [2,\infty)$

$[2,4]$

E) $(-\infty,5) \cup [-3,\infty)$

$(-\infty,\infty)$

F) $(3,\infty) \cap (-\infty,3]$

$\emptyset$

G) $(3,5) \cap [4,8)$

$[4,5)$

H) $(-\infty,-2) \cup (-\infty,3]$

$(-\infty,3]$

I) $(-\infty,2] \cup (2,\infty)$

$(-\infty,\infty)$

In the above problems, what does the symbol $\cap$ tell you to look for?

An overlap in the two inequalities
the overlap is the solution area

In the above problems, what does the symbol $\cup$ tell you to look for?

Any area shaded is the solution area.

Word Problems

Which inequality symbol is used for the phrase "at most"?

$\leq$

Which inequality symbol is used for the phrase "at least"?

$\geq$

Which inequality symbol is used for the phrase "no less than"?

$\geq$

Which inequality symbol is used for the phrase "no more than"?

$\leq$

Write the algebraic inequality that represents each of the following.

A) Each teacher in this school has at least 100 students. Write an inequality showing how many students each teacher has.

$S \geq 100$

B) Write the inequality showing all passing scores on a 100 point exam. Assume the minimum passing score is 60%. ***It could be a simple inequality, or compound inequality.***

$S \geq 60$ or $60 \leq S \leq 100$ or $S > 59$ or $59 < S \leq 100$

C) The temperature in this classroom has never been greater than 87 degrees. Write the inequality showing the possible temperatures in this classroom.

$t \leq 87°$

D) A brand new pencil is six inches long. You discard the pencil once it reaches two inches in length. Write the compound inequality showing all possible usable lengths of the pencil.

$2 < x < 6$

Solve each of the following word problems using inequalities.

A) Find three consecutive even integers whose sum is between 58 and 73.

$58 < x + (x+2) + (x+4) < 73$

$58 < 3x + 6 < 73$

$-6 \quad -6 \quad -6$

$\frac{52}{3} < \frac{3x}{3} < \frac{67}{3}$

$17\frac{1}{3} < x < 22\frac{1}{3}$

$\{18, 20, 22\}$

$\{20, 22, 24\}$

$\{22, 24, 26\}$

B) Find three consecutive odd integers whose sum is between 65 and 83.

$65 < 3x + 6 < 83$

$-6 \quad -6 \quad -6$

$\frac{59}{3} < \frac{3x}{3} < \frac{77}{3}$

$19\frac{2}{3} < x < 25\frac{2}{3}$

$\{21, 23, 25\}$

$\{23, 25, 27\}$

$\{25, 27, 29\}$

C) Sally is taking her fifth test in her algebra class. On the previous 4 exams, she scored 74, 68, 92 and 85 respectively. What must Sally score on the next exam in order to have an average test score of 78%?

$\frac{74 + 68 + 92 + 85 + x}{5} \geq 78$

$319 + x \geq 390$

$-319 \quad -319$

$x \geq 71$

must score at least 71%

D) Frank is shopping for a new truck in a city with an 8% sales tax. There is also an $84 title and license fee to pay. He wants to get a good truck, and he plans to spend at least $12,000 but not more than $15,000. What is the price range for the truck to the nearest dollar?

$12000 \leq 1.08x + 84 \leq 15000$

$-84 \quad -84 \quad -84$

$\frac{11916}{1.08} \leq \frac{1.08x}{1.08} \leq \frac{14916}{1.08}$

$11033.33 \leq x \leq 13811.11$

price must be between $11,033 and $13,811

E) Jennifer is shopping for a new car. In the addition to the price of the car, there is an 8% sales tax and a $172 title and license fee. If Jennifer decides that she will spend less than $12,000 total, then what is the price range of the car?

$1.08 + 172 < 12000$

$-172 \quad -172$

$\frac{1.08x}{1.08} < \frac{11828}{1.08}$

The car must cost no more than $10,951.85

$x < \$10951.85$

Using Inequalities to Find Domain

Sometimes, it is necessary to find the domain of a function using inequalities. This typically happens when you are dealing with a radical function. Since the square root of a negative number is not real, the numbers inside a radical must greater than or equal to zero.

Example: Find the domain of the function $f_{(x)} = \sqrt{5x-9}$

To find the domain of this function, you must evaluate $5x - 9 \geq 0$.

Find the domain of each of the following functions by setting the radicand greater than or equal to zero. Write the domain of the function in interval notation.

A) $f_{(x)} = \sqrt{x}$

$x \geq 0$

$[0, \infty)$

B) $f_{(x)} = \sqrt{x-7}$

$x - 7 \geq 0$

$x \geq 7$

$[7, \infty)$

C) $f_{(x)} = \sqrt{\frac{1}{2}x - 3}$

$\frac{1}{2}x - 3 \geq 0$

$\frac{1}{2}x \geq 3$

$x \geq 6$

$[6, \infty)$

D) $f_{(x)} = \sqrt{2x-12}$

$2x - 12 \geq 0$

$2x \geq 12$

$x \geq 6$

$[6, \infty)$

E) $f_{(x)} = \sqrt{-x}$

$-x \geq 0$

$x \leq 0$

$(-\infty, 0]$

F) $f_{(x)} = \sqrt{2x-5}$

$2x - 5 \geq 0$

$2x \geq 5$

$x \geq \frac{5}{2}$

$[\frac{5}{2}, \infty)$

G) $f_{(x)} = \sqrt{5-x}$

$5 - x \geq 0$

$5 \geq x$

$x \leq 5$

$(-\infty, 5]$

H) $f_{(x)} = \sqrt{3-2x}$

$3 - 2x \geq 0$

$-2x \geq -3$

$x \leq \frac{3}{2}$

$(-\infty, \frac{3}{2}]$

I) $f_{(x)} = \sqrt{x^2}$

$x^2 \geq 0$

$\mathbb{R}$

$(-\infty, \infty)$

Checking Progress

You have now completed the "Inequalities" section of the workbook. The following is a checklist so that you may check your progress. Check off each of the objectives you have accomplished.

The student should be able to…

- ☐ ***Describe a solution set using interval notation.***
- ☐ ***Tell the difference between inclusive and exclusive symbols and use each.***
- ☐ ***Solve simple inequalities in a single variable.***
- ☐ ***Solve compound inequalities.***
- ☐ ***Find the solution to inequalities using the word "and."***
- ☐ ***Find the solution to inequalities using the word "or."***
- ☐ ***Solve absolute value inequalities.***
- ☐ ***Find the domain of a radical function using inequalities.***

LINEAR FUNCTIONS

Objectives

The following is a list of objectives for this section of the workbook.

By the time the student is finished with this section of the workbook, he/she should be able to…

- ***Identify whether a relation determines a function.***
- ***Verify a point exists on a line.***
- ***Find the slope of a line.***
- ***Write the equation of a line in standard form.***
- ***Find the equation of a line given the slope and y intercept.***
- ***Find the equation of a line given a point on the line and the slope of the line.***
- ***Find the equation of a line given two points on the line.***
- ***Find the slope of parallel and perpendicular lines.***
- ***Find the distance between two points on a line segment.***
- ***Find the midpoint of a line segment.***
- ***Find the x and y intercepts of a line.***
- ***Graph a line using a table.***
- ***Graph a line by finding intercepts.***
- ***Graph a line using the slope intercept method.***
- ***Solve linear systems of equations in two variables by graphing.***
- ***Solve linear systems of equations in two variables using the substitution method.***
- ***Solve linear systems of equations in two variables using the linear combination method.***
- ***Determine whether or not an ordered pair is a solution to a system.***
- ***Graph linear inequalities in two variables.***
- ***Graph systems of inequalities.***
- ***Solve systems of equations in three variables.***
- ***Perform basic function operations including finding composite functions.***

Math Standards Addressed

The following state standards are addressed in this section of the workbook.

Algebra I

5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

6.0 Students graph a linear equation and compute the *x*- and *y*- intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).

7.0 Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.

8.0 Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point.

9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

16.0 Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.

17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.

18.0 Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion.

Algebra II

2.0 Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices.

24.0 Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions.

25.0 Students use properties from number systems to justify steps in combining and simplifying functions.

Definition of a Function

Formal Definition of a Function

A function is a rule that produces a correspondence between two sets of elements such that to each element in the first set there corresponds one and only one element in the second set.

Set Form of the Definition of a Function

A function is a set of ordered pairs with the property that no two ordered pairs have the same first component and different second components.

The set of all first components in a function, or x values, is called the domain of the function. The set of all of the second components in a function, or y values, is called the range of the function.

Simply put, each x value of an ordered pair can have only one y value. You cannot have two sets of ordered pairs with the same x value and different y values. This would cause the two points to line up vertically, which means the particular graph would fail a vertical line test meaning it is not a function.

Theorem: Vertical Line Test for a Function

An equation defines a function if each vertical line in the rectangular coordinate system passes through at most one point on the graph of the equation.

If any vertical line passes through two or more points on the graph of an equation, then the equation does not define a function.

Indicate whether each set defines a function.

A) $\{(2,4),\ (3,6),\ (4,8),\ (5,10)\}$

function

B) $\{(0,1),\ (1,1),\ (3,1),\ (1,2)\}$

Not a function

C) $\{(10,3),\ (5,-5),\ (0,0),\ (5,5),\ (10,10)\}$

Not a function

D) $\{(0,1),\ (1,1),\ (2,1),\ (4,2)\}$

function

You must also be able to identify the dependent and independent variables in a linear function.

In the function: $y = 6x + 3$

y is the dependent variable, and x is the independent variable. The variable that attempts to stand alone is dependant.

Verifying a point exists on a line

In order to tell whether or not a point exists on a line, you must substitute the x and y values given into the function. If the resultant statement is true, then the point exists on the line. If false, then the point does not exist on that line. This may also be asked in the form "Is (x,y) a solution to the equation?"

What is an ordered pair?

a set of cor-dinates located on the cartesian plane

Determine if the given ordered pair is a solution to the function: $3x-2y=8$

A) $(-3,4)$

$3x-2y=8$

$3(-3)-2(4)=8$

$-9-8=8$

$-17\neq 8$

Not a solution

B) $(-4,-10)$

$3x-2y=8$

$3(-4)-2(-10)=8$

$-12+20=8$

$8=8$

solution

C) $\left(\frac{2}{5},-\frac{17}{5}\right)$

$3x-2y=8$

$3(\frac{2}{5})-2(\frac{17}{5})=8$

$\frac{6}{5}+\frac{34}{5}=8$

$\frac{40}{5}=8$

solution

D) $\left(\frac{1}{8},-\frac{15}{4}\right)$

$3x-2y=8$

$3(\frac{1}{8})-2(-\frac{15}{4})=8$

$\frac{3}{8}+\frac{30}{4}\ \frac{60}{8}=8$

$\frac{63}{8}\neq 8$ Not a solution.

Complete the given ordered pairs so that each ordered pair satisfies the given equation.

A) $(2, 0), (3, -3)$, $y=-3x+6$

$y=-3(2)+6$

$y=0$

$y=3x+6$

$(-3)=-3x+6$

$-9=-3x$

$x=3$

B) $(-4, -33), (22, 6)$, $\frac{1}{2}x-\frac{1}{3}y=9$

$\frac{1}{2}x-\frac{1}{3}y=9$

$\frac{1}{2}x-\frac{1}{3}(6)=9$

$\frac{1}{2}x-2=9$

$\frac{1}{2}x=11$

$x=22$

$\frac{1}{2}(-4)-\frac{1}{3}y=9$

$-2-\frac{1}{3}y=9$

$-\frac{1}{3}y=11$

$y=-33$

Since ordered pairs are not always given in terms of the variables x and y, how do you know what order the variables go in if they are different?

They go in alphabetical order.

Find the value of k **so that the ordered pair satisfies the equation.**

A) $3x-y=k;\quad(-2,5)$

$3(-2)-5=k$

$-6-5=k$

$k=-11$

B) $kx-4y=12;\quad(2,3)$

$2k-4(3)=12$

$2k-12=12$

$2k=24$

$k=12$

C) $5x-ky=k;\quad(3,3)$

$5(3)-3k=k$

$15-3k=k$

$15=4k$

$k=\frac{15}{4}$

The Slope of a Line

What is the formula for finding the slope of a line?

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

What is the one thing you can never do when using the formula to find the slope?

$$\frac{y_2 - y_1}{x_1 - x_2} \quad \text{OR} \quad \frac{y_1 - y_2}{x_2 - x_1}$$

The slope of a vertical line is undefined **.**

The slope of a horizontal line is zero **.**

The slope of the y axis is undefined**, and the slope of the x axis is** zero **.**

Find the slope of the line containing the following points.

A) $(-3,2),\ (7,12)$

$m = \frac{12-2}{7-(-3)} = \frac{10}{10}$

$m = 1$

B) $(4,-2),\ (-3,5)$

$m = \frac{5-(-2)}{-3-4} = \frac{7}{-7}$

$m = -1$

C) $(-3,2),\ (-3,-2)$

$m = \frac{-2-2}{-3-(-3)} = \frac{-4}{0}$

undefined

D) $(0,-3),\ (7,-8)$

$m = \frac{-8-(-3)}{7-0} = \frac{-5}{7}$

$m = -\frac{5}{7}$

E) $(-2,5),\ (3,5)$

$m = \frac{5-5}{3-(-2)} = \frac{0}{5}$

$m = 0$

F) $(3,-1),\ (-3,1)$

$m = \frac{1-(-1)}{-3-3} = \frac{2}{-6}$

$m = -\frac{1}{3}$

G) $\left(\frac{1}{2},-\frac{2}{3}\right),\ \left(\frac{3}{2},6\right)$

$m = \frac{\frac{18}{3}\ 6 - \left(-\frac{2}{3}\right)}{\frac{3}{2} - \frac{1}{2}} = \frac{\frac{20}{3}}{\frac{2}{2}1}$

$m = \frac{20}{3}$

H) $(a,b),\ (-b,-a)\ \ (a \neq -b)$

$m = \frac{-a-b}{-b-a} = \frac{-a-b}{-a-b}$

$m = 1$

I) $\left(4,\frac{1}{2}\right),\ \left(-3,\frac{1}{2}\right)$

$m = \frac{\frac{1}{2} - \frac{1}{2}}{-3-4} = \frac{0}{-7}$

$m = 0$

The slope of a line can also be found using the formula $Ax + By = C$ ***where*** $(B \neq 0)$. ***If you were to solve this equation for y, you get*** $y = -\frac{A}{B}x + \frac{C}{B}$. ***Therefore, the slope of a line can be identified by evaluating*** $m = -\frac{A}{B}$.

Find the slope of each of the following lines.

A) $3x - 2y = 6$

$-3x \quad -3x$

$\frac{-2y}{-2} = \frac{-3x + 6}{-2}$

$y = \frac{3}{2}x - 3$

$m = \frac{3}{2}$

B) $-4x - 7y = -12$

$+4x \quad +4x$

$\frac{-7y}{-7} = \frac{4x - 12}{-7}$

$y = -\frac{4}{7}x + \frac{12}{7}$

$m = -\frac{4}{7}$

C) $\left(\frac{x}{4} + \frac{y}{3} = 24\right)12$

$3x + 4y = 288$

$m = -\frac{A}{B} = \frac{-3}{4}$

$m = -\frac{3}{4}$

D) $y = 4$

$m = 0$

E) $\left(\frac{x}{3} - \frac{y}{-2} = 4\right)6$

$2x + 3y = 24$

$\frac{3y}{3} = \frac{-2x + 24}{3}$

$y = -\frac{2}{3}x + 8$

$m = -\frac{2}{3}$

F) $\left(\frac{1}{2}x + \frac{2}{3}y = 7\right)6$

$3x + 4y = 42$

$\frac{4y}{4} = \frac{-3x + 42}{4}$

$y = -\frac{3}{4}x + \frac{42}{4}$

$m = -\frac{3}{4}$

G) $x = 4y + 2$

$-2 \quad -2$

$\frac{x - 2}{4} = \frac{4y}{4}$

$\frac{1}{4}x - \frac{1}{2} = y$

$y = \frac{1}{4}x - \frac{1}{2}$

$m = \frac{1}{4}$

H) $5x = 3$

m = undefined

You'll notice that there was a problem with letter H above. You were not able to find the slope, because $B = 0$. **What does that tell you about the slope of the line?**

it must be undefined

What can you say about the slopes of parallel lines?

They are equal

The slope of line 1 is $\frac{2}{3}$. **Line 2 is perpendicular to line 1. What is the slope of line 2?**

$-\frac{3}{2}$

How are the slopes of perpendicular lines related?

slopes of perpendicular lines are opposite reciprocals of each other.

Equations of Lines

What is the Slope Intercept Form of a line?

$y = mx + b$

What is the Standard Form of a Line?

$Ax + By = C$

What is the Point-Slope form of a line?

$y - y_1 = m(x - x_1)$

When writing the equation of a line in standard form, what two restrictions must be satisfied?

- A, B, and C must be integers
- A must be positive

Find the equation of the line, in standard form, that has the slope m and contains the point P.

A) $P = (-3, 2),\ m = \frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{2}[x - (-3)]$

$y - 2 = \frac{1}{2}x + \frac{3}{2}$

$+2 \qquad +2$

$(2)\ y = \frac{1}{2}x + \frac{7}{2}(2)$

$2y = x + 7$

$-x \quad -x$

$(x + 2y = 7)\ {-1} = x - 2y = -7$

B) $P = (-3, -4),\ m = 2$

$y - y_1 = m(x - x_1)$

$y - (-4) = 2(x - (-3))$

$y + 4 = 2x + 6$

$-4 \qquad -4$

$y = 2x + 2$

$-2x \ -2x$

$(-2x + y = 2)\ {-1}$

$2x - y = -2$

C) $P = (4, 0),\ m = \frac{2}{3}$

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{2}{3}(x - 4)$

$(y = \frac{2}{3}x - \frac{8}{3})3$

$3y = 2x - 8$

$-2x \quad -2x$

38 $(-2x + 3y = -8)\ {-1}$

$2x - 3y = 8$

D) $P = (4, -3),\ m = \frac{1}{8}$

$y - y_1 = m(x - x_1)$

$y - (-3) = \frac{1}{8}(x - 4)$

$y + 3 = \frac{1}{8}(x - 4)$

$y + 3 = \frac{1}{8}x - \frac{1}{2}$

$-3 \qquad -3$

$(y = \frac{1}{8}x - \frac{7}{2})8$

$8y = x - 28$

$-x \quad -x$

$(-x + 8y = -28)\ {-1} = x - 8y = 28$

E) $P=(2,5),\ m=0$

$y-y_1=m(x-x_1)$
$y-5=0(x-2)$
$y-5=0$
$y=5$

F) $P=(-3,2),\ m=-\frac{1}{2}$

$y-y_1=m(x-x_1)$
$y-2=-\frac{1}{2}(x-(-3))$
$y-2=-\frac{1}{2}x-\frac{3}{2}$
$+2 \qquad +2$
$(y=-\frac{1}{2}x+\frac{1}{2})2$
$2y=-x+1$
$+x \qquad +x$
$x+2y=1$

G) $P=(-3,-2),\ m=-\frac{4}{5}$

$y-y_1=m(x-x_1)$
$y-(-2)=-\frac{4}{5}(x-(-3))$
$y+2=-\frac{4}{5}x-\frac{12}{5}$
$(y=-\frac{4}{5}x-\frac{22}{5})5$
$5y=-4x-22$
$+4x \qquad +4x$
$4x+5y=-22$

H) $P=(-1,3),\ m=undefined$

$x=-1$

Find the equation of a line, in standard form that passes through the points…

A) $(-3,2),\ (7,12)$

$m=\frac{12-2}{7-(-3)}=\frac{10}{10}=1$
$y-y_1=m(x-x_1)$
$y-2=1(x-(-3))$
$y-2=x+3$
$+2 \qquad +2$
$y=x+5$
$-x \qquad -x$
$(-x+y=5)-1 \quad x-y=-5$

B) $(0,-3),\ (7,-8)$

$m=\frac{8-(-3)}{7-0}=\frac{-5}{7}$
$y-y_1=m(x-x_1)$
$y-(-3)=-\frac{5}{7}(x-0)$
$(y+3=-\frac{5}{7}x)7$
$7y+21=-5x$
$+5x-21 \qquad +5x-21$
$5x+7y=-21$

C) $(-3,2),\ (-3,-2)$

$m=\frac{-2-2}{-3-(-3)}=\frac{-4}{0}$
undefined
$x=-3$

D) $(3,-2),\ (-6,12)$

$m=\frac{12-(-2)}{-6-3}=\frac{14}{-9}$
$y-y_1=m(x-x_1)$
$y-(-2)=-\frac{14}{9}(x-3)$
$[y+2=-\frac{14}{9}(x-3)]9$
$9y+18=-14x+42$
$-18 \qquad -18$
$9y=-14x+24$
$14x+9y=24$

E) $(-3,-4),\ (5,-4)$

$m=\frac{-4-(-4)}{5-(-3)}=\frac{0}{8}=0$
$y=-4$

F) $(3,-2),\ (-2,3)$

$m=\frac{3-(-2)}{-2-3}=\frac{5}{-5}=-1$
$y-y_1=m(x-x_1)$
$y-(-2)=-1(x-3)$
$y+2=-x+3$
$+x-2 \qquad +x-2$
$x+y=1$

G) $\left(\frac{3}{2},-\frac{1}{2}\right),\ \left(-\frac{1}{2},\frac{5}{2}\right)$

$m=\frac{\frac{5}{2}-(-\frac{1}{2})}{-\frac{1}{2}-\frac{3}{2}}=\frac{\frac{6}{2}}{-\frac{4}{2}}=\frac{3}{-2}$
$y-y_1=m(x-x_1)$
$4[y-(-\frac{1}{2})]=[-\frac{3}{2}(x-\frac{3}{2})]4$
$4y+2=-6(x-\frac{3}{2})$
$4y+2=-6x+9$
$6x+4y=7$

H) $\left(\frac{3}{4},\frac{4}{5}\right),\ \left(-\frac{1}{4},\frac{1}{2}\right)$

$m=\frac{\frac{1}{2}-\frac{4}{5}}{-\frac{1}{4}-\frac{3}{4}}=\frac{3}{10}$
$y-y_1=m(x-x_1)$
$10(y-\frac{4}{5})=[\frac{3}{10}(x-\frac{3}{4})]10$
$4(10y-8)=(3x-\frac{9}{4})4$
$40y-32=12x-9$
$12x-40y=-23$

I) $(3,-1),\ (-2,5)$

$m=\frac{5-(-1)}{-2-3}=\frac{6}{-5}$
$y-y_1=m(x-x_1)$
$5(y-5)=[\frac{6}{-5}(x-(-2))]5$
$5y+25=-6x-12$
$-25 \qquad -25$
$6x+5y=13$

Find the equation of the line that is parallel to $y=\frac{1}{4}x+3$, and contains the point (-4, 2).

$y-2=\frac{1}{4}(x+4)$

$4y-8=x+4$

$x-4y=-12$

$y-2=\frac{1}{4}x+1$

$y=\frac{1}{4}x+3$

Slope-Int Form:

$y=\frac{1}{4}x+3$

Standard Form:

$x-4y=-12$

Find the equation of the line that is parallel to $y=-\frac{2}{3}x-2$, and contains the point (5, 8).

$3(y-8)=3[-\frac{2}{3}(x-5)]$

$3y-24=-2x+10$

$+2x \quad +24 \quad +2x \quad +24$

$2x+3y=34$

$3y-24=-2x+10$

$3y=-2x+34$

$y=-\frac{2}{3}x+\frac{34}{3}$

Slope-Int Form:

$y=-\frac{2}{3}x+\frac{34}{3}$

Standard Form:

$2x+3y=34$

Find the equation of the line that is parallel to $y=7$, and contains the point (-3, 4).

Equation for a horizontal line containing(-3,4)

Slope-Int Form:

$y=4$

Standard Form:

$y=4$

Find the equation of the line that is parallel to $y=-\frac{1}{5}x-\frac{2}{3}$, and contains the point $\left(\frac{1}{2},3\right)$.

$(y-3)=-\frac{1}{5}(x-\frac{1}{2})$

$5(y-3)=5[-\frac{1}{5}(x-\frac{1}{2})]$

$2(5y-15)=2(-x+\frac{1}{2})$

$10y-30=-2x+1$

$2x+10y=31$

$5y-15=-x+\frac{1}{2}$

$+15 \quad +15$

$5y=-x+\frac{31}{2}$

$y=-\frac{1}{5}x+\frac{31}{10}$

Slope-Int Form:

$y=-\frac{1}{5}x+\frac{31}{10}$

Standard Form:

$2x+10y=31$

A line contains the points $(-7,-2)$ and $(4,7)$. Find the equation of a line that is parallel to this, and contains the point $(6,8)$.

$m=\frac{7+2}{4+7}=\frac{9}{11}$

$y-8=\frac{9}{11}(x-6)$

$11(y-8)=11[\frac{9}{11}(x-6)]$

$11y-88=9x-54 \rightarrow \frac{11y}{11}=\frac{9x}{11}+\frac{34}{11}$

$-9x \quad +88 \quad -9x \quad +88$

$-9x+11y=34$

$9x-11y=-34$

$y=\frac{9}{11}x+\frac{34}{11}$

Slope-Int Form:

$y=\frac{9}{11}x+\frac{34}{11}$

Standard Form:

$9x-11y=-34$

A line contains the points $\left(-\frac{1}{2},\frac{2}{3}\right)$ and $\left(\frac{1}{4},2\right)$. Find the equation of a line that is parallel to this, and contains the point $(-4,-5)$.

$m=\left[2-\frac{2}{3}\right]\div\left[\frac{1}{4}+\frac{1}{2}\right]=$

$\left[\frac{6}{3}-\frac{2}{3}\right]\div\left[\frac{1}{4}+\frac{2}{4}\right]$

$\frac{4}{3}\div\frac{3}{4}$

$\frac{4}{3}\cdot\frac{4}{3}$

$\frac{16}{9}$

$9(y+5)=9\left[\frac{16}{9}(x+4)\right]$

$9y+45=16x+64$

$-9y-64 \quad -64-9y$

$16x-9y=-19$

$\frac{9y}{9}=\frac{16x}{9}+\frac{19}{9}$

$y=\frac{16}{9}\cancel{x}+\frac{19}{9}$

Slope-Int Form:

$y=\frac{16}{9}+\frac{19}{9}$

Standard Form:

$16x-9y=-19$

Find the equation of the line that is perpendicular to $y=\frac{1}{4}x+3$, and contains the point $(-3,4)$.

$m=-4$

$y-4=-4(x+3)$

$y=-4x-8$

$y-4=-4x-12$

$4x+y=-8$

Slope-Int Form:

$y=-4x-8$

Standard Form:

$4x+y=-8$

Find the equation of the line that is perpendicular to $y=-\frac{2}{3}x+7$, and contains the point $(-2,5)$.

$m=\frac{3}{2}$

$2[y-5]\cancel{=}2\left[\frac{3}{2}(x+2)\right]$

$2y-10=3x+6$

$2y=3x+16$

$3x-2y=-16$

$y=\frac{3}{2}x+8$

Slope-Int Form:

$y=\frac{3}{2}x+8$

Standard Form:

$3x-2y=-16$

Find the equation of the line that is perpendicular to $y=4$, and contains the point $(-3,4)$.

Equation of a vertical line passing through (-3,4)

Slope-Int Form:

$x=-3$

Standard Form:

$x=-3$

A line contains the points $(-1,4)$ and $(7,2)$. Find the equation of a line that is perpendicular to this, and contains the point $(4,8)$.

$m = \frac{2-4}{7+1} = -\frac{2}{8} = -\frac{1}{4}$

$m = 4$

$y-8 = 4(x-4)$

$y-8 = 4x-16$

$4x-y=8$

$y = 4x-8$

Slope-Int Form:

$y=4x-8$

Standard Form:

$4x-y=8$

Find the equation of the vertical line passing through (2,-4).

$x=2$

Find the equation of the horizontal line passing through (8,6).

$y=6$

Find the equation of the vertical and horizontal lines that intersect at (-3,5).

$x=-3$

$y=5$

Where do the lines $x=4$ and $y=-2$ intersect?

$(4,-2)$

What is the equation of the y axis?

$x=0$

What is the equation of the x axis?

$y=0$

The equation of a line that is parallel to $y=-\frac{2}{3}x+\frac{7}{5}$ has a slope of $-\frac{2}{3}$ **.**

The equation of a line that is perpendicular to $y=7x+5$ has a slope of $-\frac{1}{7}$ **.**

Distance Formula

What is the distance formula?

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

If you ever forget the distance formula, what other method could you use to find the distance between two points?

The pythagorean Theorem

Find the distance of the line segment that has the following endpoints.

A) $(-2,-3),\ (4,3)$

$d=\sqrt{(4+2)^2+(3+3)^2}$
$d=\sqrt{6^2+6^2}$
$d=\sqrt{36+36}$
$d=\sqrt{2(36)}$
$d=6\sqrt{2}$

B) $(-2,4),\ (5,-4)$

$d=\sqrt{(5+2)^2+(-4-4)^2}$
$d=\sqrt{7^2+(-8)^2}$
$d=\sqrt{49+64}$
$d=\sqrt{113}$

C) $(4,3),\ (12,5)$

$d=\sqrt{(12-4)^2+(5-3)^2}$
$d=\sqrt{(8)^2+(2)^2}$
$d=\sqrt{64+4}$
$d=\sqrt{68}$
$d=2\sqrt{17}$

D) $\left(\frac{1}{2},4\right),\ \left(\frac{7}{2},-3\right)$

$d=\sqrt{(\frac{7}{2}-\frac{1}{2})^2+(-3-4)^2}$
$d=\sqrt{3^2+(-7)^2}$
$d=\sqrt{9+49}$
$d=\sqrt{58}$

E) $(6,2),\ (-3,8)$

$d=\sqrt{(-3-6)^2+(8-2)^2}$
$d=\sqrt{(-9)^2+(6)^2}$
$d=\sqrt{81+36}$
$d=\sqrt{117}$
$d=3\sqrt{13}$

F) $(5,3),\ (5,7)$

$d=\sqrt{(5-5)^2+(7-3)^2}$
$d=\sqrt{0+4^2}$
$d=\sqrt{16}$
$d=4$

G) $\left(\frac{1}{2},-1\right),\ (-1,1)$

$d=\sqrt{(-1-\frac{1}{2})^2+(1+1)^2}$
$d=\sqrt{(-\frac{3}{2})^2+2^2}$
$d=\sqrt{\frac{9}{4}+4}$
$d=\sqrt{\frac{25}{4}}=\frac{5}{2}$

H) $(\sqrt{2},1),\ (-\sqrt{2},0)$

$d=\sqrt{(-\sqrt{2}-\sqrt{2})^2+(0-1)^2}$
$d=\sqrt{(-2\sqrt{2})^2+(-1)^2}$
$d=\sqrt{8+1}$
$d=\sqrt{9}$
$d=3$

I) $(5,\sqrt{5}),\ (3,-\sqrt{5})$

$d=\sqrt{(3-5)^2+(-\sqrt{5}-\sqrt{5})^2}$
$d=\sqrt{(-2)^2+(-2\sqrt{5})^2}$
$d=\sqrt{4+20}$
$d=\sqrt{24}$
$d=2\sqrt{6}$

Find x so that the distance between the points is 13.

A) $(1,2),\ (x,-10)$

$\sqrt{(x-1)^2+(-10-2)^2}=13$
$(\sqrt{x^2-2x+1+144})^2=(13)^2$
$x^2-2x+145=169$
$x^2-2x-24=0$
$(x-6)(x+4)=0$
$x=6 \quad x=-4$

B) $(-8,0),\ (x,5)$

$\sqrt{(x+8)^2+5^2}=13$
$\sqrt{x^2+16x+64+25}=(13)^2$
$x^2+16x+89=169$
$x^2+16x-80=0$
$(x+20)(x-4)=0$
$x=-20 \quad x=4$

Find y so that the distance between the points is 17.

A) $(0,0),\ (8,y)$

$\sqrt{8^2+y^2}=17$
$(\sqrt{y^2+64})^2=(17)^2$
$y^2+64=289$
$-64 \quad -64$
$y^2=225$
$y=\pm 15$

B) $(-8,4),\ (7,y)$

$\sqrt{(7+8)^2+(y-4)^2}=17$
$\sqrt{(15)^2+(y^2-8y+16)}=17$
$(\sqrt{225+y^2-8y+16})^2=(17)^2$
$y^2-8y+24=289$
$y^2-8y-48=0$
$(y-12)(y+4)=0$
$y=-4$
$y=12$

Midpoint formula

What is the midpoint formula?

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Find the midpoint of the line segment that has the following endpoints.

A) $(-2,-3), (4,3)$

$\left(\frac{-2+4}{2}, \frac{-3+3}{2}\right)$

$(1,0)$

B) $(-2,4), (5,-4)$

$\left(\frac{-2+5}{2}, \frac{4-4}{2}\right)$

$\left(\frac{3}{2},0\right)$

C) $(4,3), (12,5)$

$\left(\frac{4+12}{2}, \frac{3+5}{2}\right)$

$(8,4)$

D) $\left(\frac{1}{2},4\right), \left(\frac{7}{2},-3\right)$

$\left(\frac{\frac{1}{2}+\frac{7}{2}}{2}, \frac{4-3}{2}\right)$

$\left(2,\frac{1}{2}\right)$

E) $(6,2), (-3,8)$

$\left(\frac{6-3}{2}, \frac{2+8}{2}\right)$

$\left(\frac{3}{2}, \frac{10}{2}\right)$

$\left(\frac{3}{2},5\right)$

F) $(5,3), (5,7)$

$\left(\frac{5+5}{2}, \frac{3+7}{2}\right)$

$\left(\frac{10}{2}, \frac{10}{2}\right)$

$(5,5)$

G) $\left(\frac{1}{2},-1\right), (-1,1)$

$\left(\frac{\frac{1}{2}-1}{2}, \frac{-1+1}{2}\right)$

$\left(-\frac{1}{2}\div 2, \frac{0}{2}\right)$

$\left(-\frac{1}{4},0\right)$

H) $(\sqrt{2},1), (-\sqrt{2},0)$

$\left(\frac{\sqrt{2}-\sqrt{2}}{2}, \frac{1+0}{2}\right)$

$\left(0,\frac{1}{2}\right)$

I) $(5,\sqrt{5}), (3,-\sqrt{5})$

$\left(\frac{5+3}{2}, \frac{\sqrt{5}-\sqrt{5}}{2}\right)$

$\left(\frac{8}{2}, \frac{0}{2}\right)$

$(4,0)$

Finding X and Y Intercepts

Find the x and y intercepts of each of the following linear functions.

A) $3x-4y=24$

x	y
0	-6
8	0

$(0,-6)(8,0)$

B) $12x+16y=-4$

x	y
0	-1/4
-1/3	0

$\left(-\frac{1}{3},0\right) \left(0,-\frac{1}{4}\right)$

C) $2x-7y=18$

x	y
0	-18/7
9	0

$(9,0)(0,-18/7)$

$0=\frac{2}{3}x-3$

$\frac{2}{3}x=3$

$x=\frac{9}{2}$

D) $y=\frac{2}{3}x-3$

x	y
0	-3
9/2	0

$\left(\frac{9}{2},0\right)(0,-3)$

$0=-\frac{1}{4}x+12$

$\frac{1}{4}x=12$

$x=48$

E) $y=-\frac{1}{4}x+12$

x	y
0	12
48	0

$(0,12)(48,0)$

F) $y=3$

x	y
0	3

$(0,3)$

No x-int.

G) $4x-5y=15$

x	y
0	-3
15/4	0

$(0,-3)$

$\left(\frac{15}{4},0\right)$

H) $x=-2$

x	y
-2	0

$(-2,0)$

No y-int.

$0=3x-19$

$3x=19$

$x=\frac{19}{3}$

I) $y=3x-19$

x	y
0	-19
19/3	0

$(0,-19)$

$\left(\frac{19}{3},0\right)$

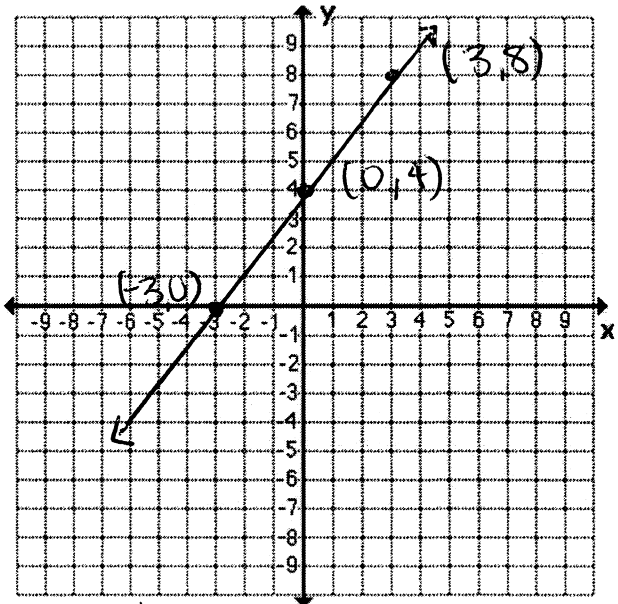
Sometimes you will be asked for only the x intercept or y intercept rather than of both.
Why would this type of question be asked?

To make sure we know the difference between the two. To make sure we know what a x-intercept and what a y-intercept look like.

Graphing Linear Functions

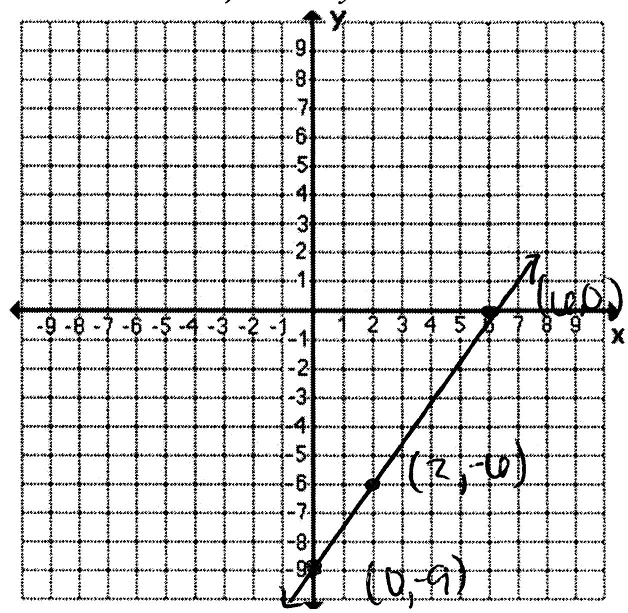
Graph each of the following linear functions. Be sure to label the x and y intercepts.

A) $4x-3y=-12$



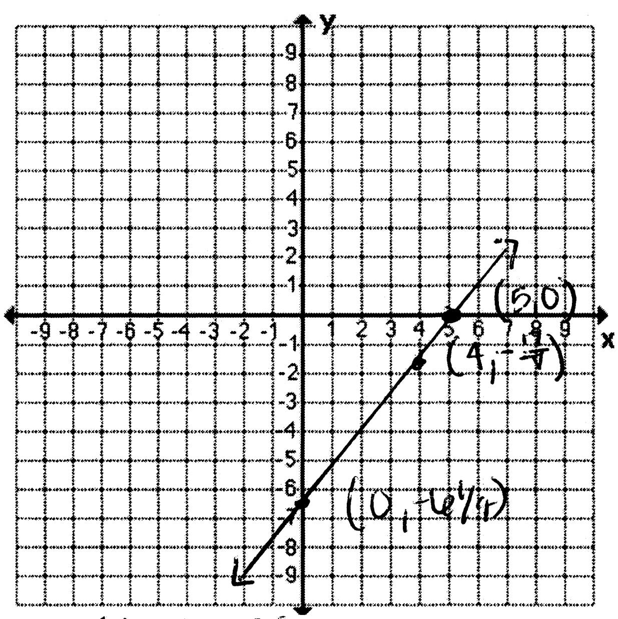
$4x-3y=-12$
$-4x \qquad -4x$
$\frac{-3y}{-3}=\frac{-4x-12}{-3}$
$y=\frac{4}{3}x+4$

B) $3x-2y=18$



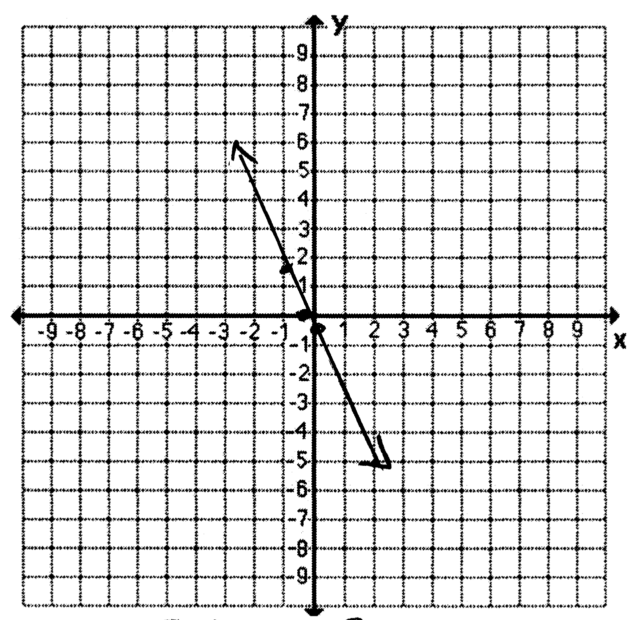
$3x-2y=18$
$-3x \qquad -3x$
$\frac{-2y}{-2}=\frac{-3x+18}{-2}$
$y=\frac{3}{2}x-9$

C) $5x-4y=25$



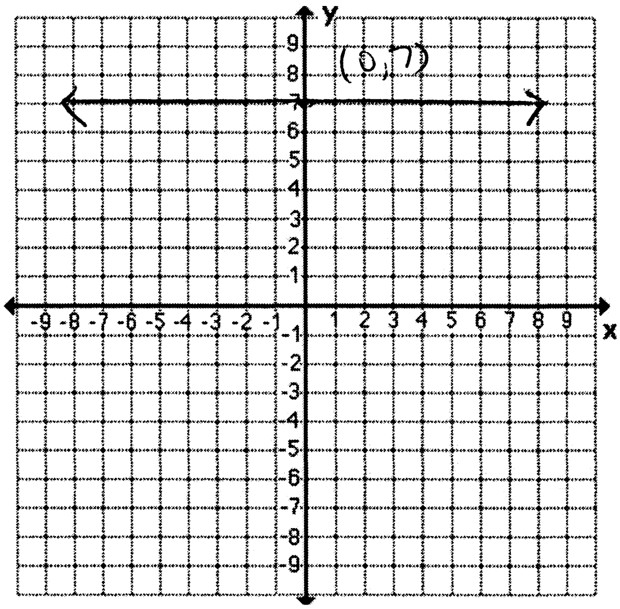
$5x-4y=25$
$-5x \qquad -5x$
$\frac{-4y}{-4}=\frac{-5x+25}{-4}$
$y=\frac{5}{4}x-\frac{25}{4}$

D) $8x+4y=-2$

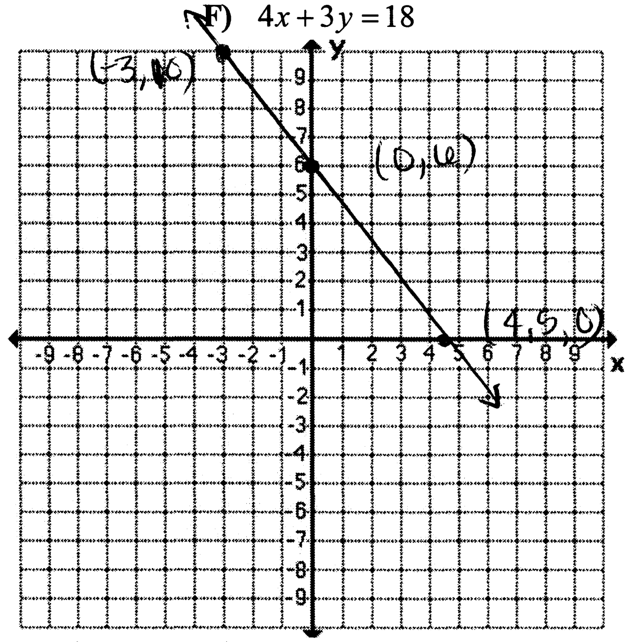


$8x+4y=-2$
$-8x \qquad -8x$
$\frac{4y}{4}=\frac{-8x-2}{4}$
$y=-2x-\frac{1}{2}$

E) $y = 7$

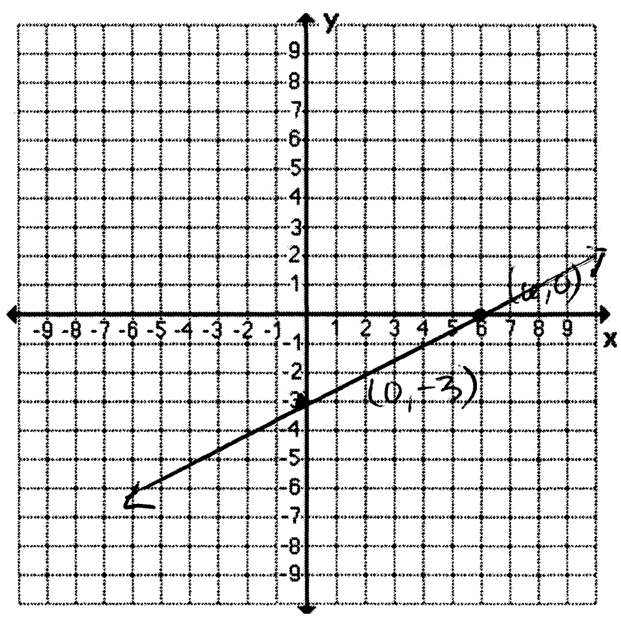


F) $4x + 3y = 18$

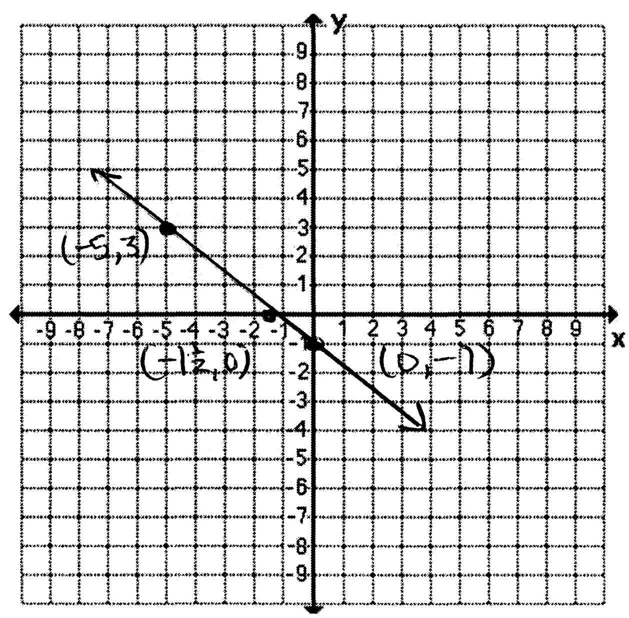


$4x + 3y = 18$
$-4x \quad\quad -4x$
$\frac{3y}{3} = \frac{-4x + 18}{3}$
$y = -\frac{4}{3}x + 6$

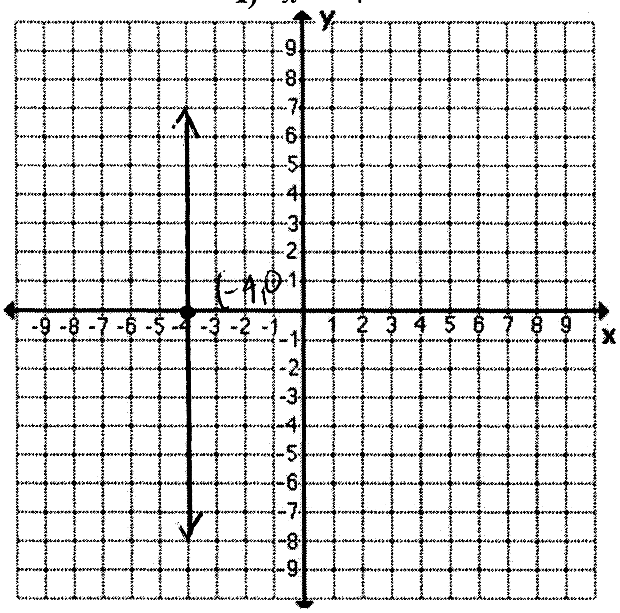
G) $y = \frac{1}{2}x - 3$



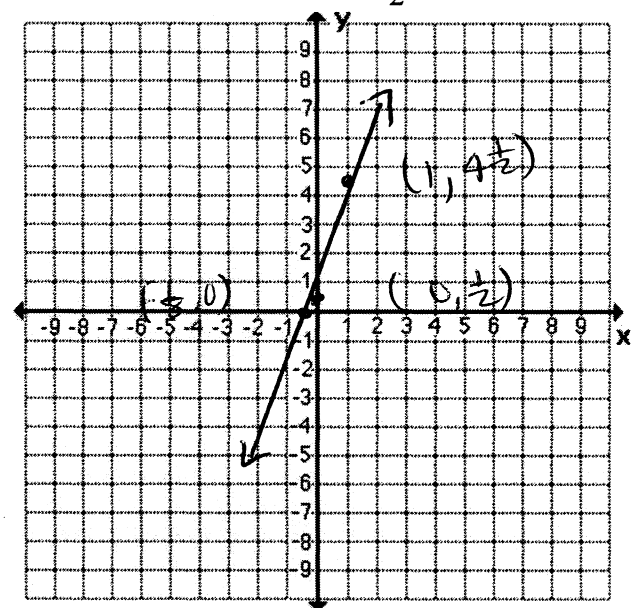
H) $y = -\frac{4}{5}x - 1$



I) $x = -4$



J) $y = 4x + \frac{1}{2}$



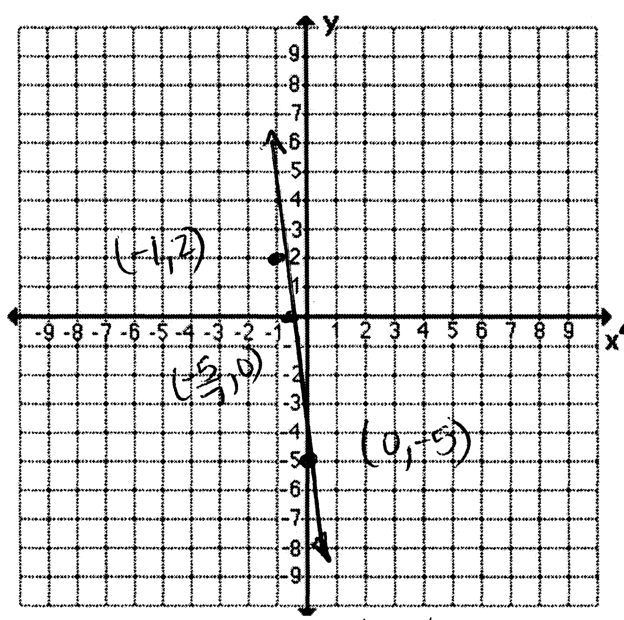
x	y
0	1/2
-1/8	0

$0 = 4x + \frac{1}{2}$

$4x = -\frac{1}{2}$

$x = -\frac{1}{8}$

K) $y = -7x - 5$



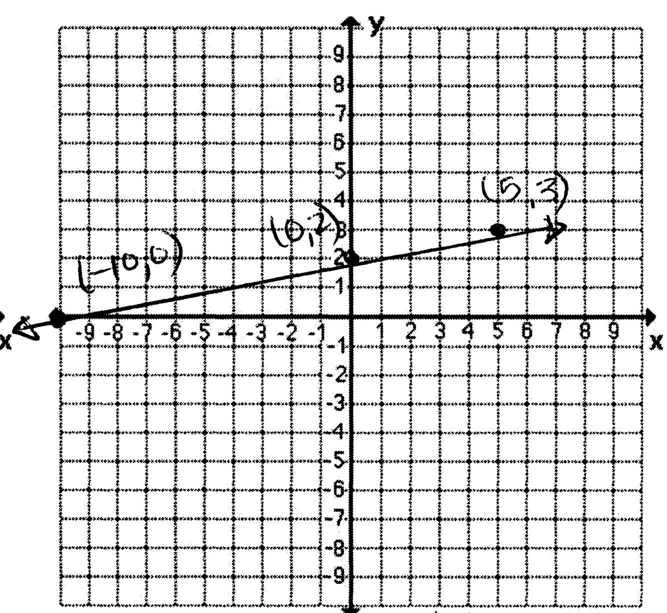
$0 = -7x - 5$

$-7x = 5$

$x = -\frac{5}{7}$

x	y
0	-5
-5/7	0

L) $y = \frac{1}{5}x + 2$



$0 = \frac{1}{5}x + 2$

$-\frac{1}{5}x = 2$

$x = -10$

x	y
0	2
-10	0

Graphing Linear Inequalities

When graphing a linear inequality, what is the difference between $\geq$ and $>$?

for ≥ use a solid line. For > use a broken line.

How do you determine whether or not a given ordered pair satisfies an inequality?

whether its true or false. If true, then it satisfies. if false, then it does not satisfy.

Determine whether or not the given ordered pair satisfies the inequality $y < 4x + 3$.

A) $(3,-2)$

$-2 < 4(3)+3$

$-2 < 12+3$

$-2 < 15$

True

B) $(-4,6)$

$6 < 4(-4)+3$

$6 < -16+3$

$6 < -13$

false

C) $\left(\frac{1}{2},5\right)$

$5 < 4(\frac{1}{2})+3$

$5 < 2+3$

$5 < 5$

false

D) $\left(\frac{3}{4},\frac{21}{4}\right)$

$\frac{21}{4} < 4(\frac{3}{4})+3$

$\frac{21}{4} < 3+3$

$\frac{21}{4} < 6$

$5\frac{1}{4} < 6$

true.

Determine whether or not the given ordered pair satisfies the inequality $y \leq -\frac{2}{3}x + 2$.

A) $(-6,4)$

$4 \leq -\frac{2}{3}(-6)+2$

$4 \leq 4+2$

$4 \leq 6$

true

B) $(7,4)$

$4 \leq -\frac{2}{3}(7)+2$

$4 \leq -\frac{14}{3}+\frac{6}{3}$

$4 \leq -\frac{8}{3}$

$4 \leq -2\frac{2}{3}$

false

C) $\left(\frac{1}{2},\frac{5}{4}\right)$

$\frac{5}{4} \leq -\frac{2}{3}(\frac{1}{2})+2$

$\frac{5}{4} \leq -\frac{1}{3}+\frac{6}{3}$

$\frac{5}{4} \leq \frac{5}{3}$

$1\frac{1}{4} \leq 1\frac{2}{3}$

true

D) $(18,-2)$

$-2 \leq -\frac{2}{3}(18)^{6}+2$

$-2 \leq -12+2$

$-2 \leq -10$

false

Determine whether or not the given ordered pair satisfies the inequality $3x - 5y > 6$.

A) $(2,3)$

$3(2)-5(3) > 6$

$6-15 > 6$

$-9 > 6$

false

B) $(-3,-7)$

$3(-3)-5(-7) > 6$

$-9+35 > 6$

$26 > 6$

true

C) $(-2,-1)$

$3(-2)-5(-1) > 6$

$-6+5 > 6$

$-1 > 6$

false

D) $(7,3)$

$3(7)-5(3) > 6$

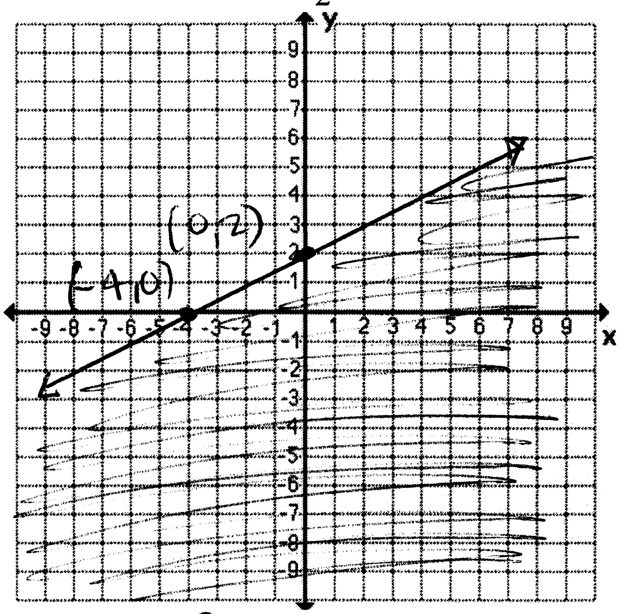
$21-15 > 6$

$6 > 6$

false

Graph each of the following linear inequalities. Label all x and y intercepts.

A) $y \le \frac{1}{2}x + 2$



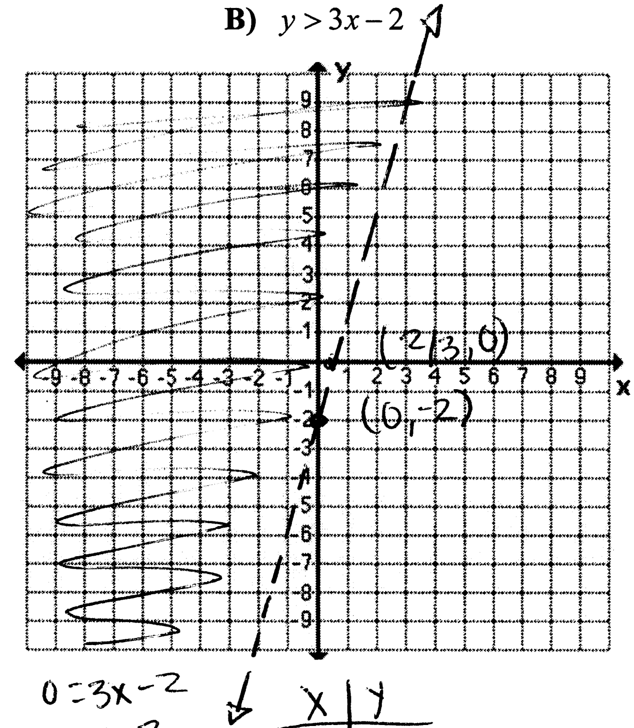
$0 = \frac{1}{2}x + 2$

$\frac{1}{2}x = 2$

$x = -4$

x	y
0	2
-4	0

B) $y > 3x - 2$



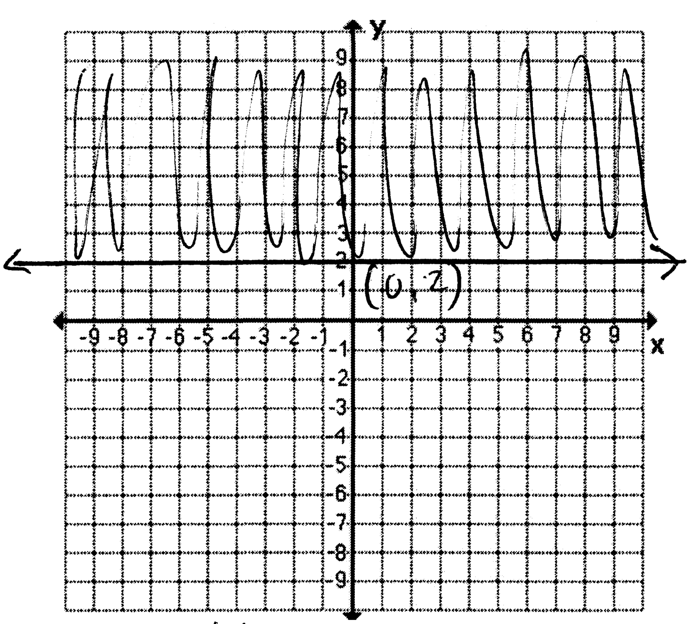
$0 = 3x - 2$

$3x = 2$

$x = \frac{2}{3}$

x	y
0	-2
$\frac{2}{3}$	0

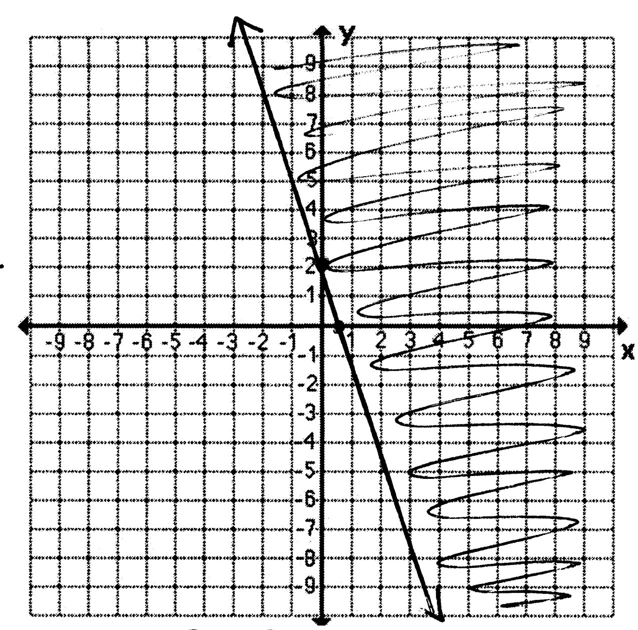
C) $y \ge 2$



x	y
0	2

No x intercept

D) $y \ge -3x + 2$



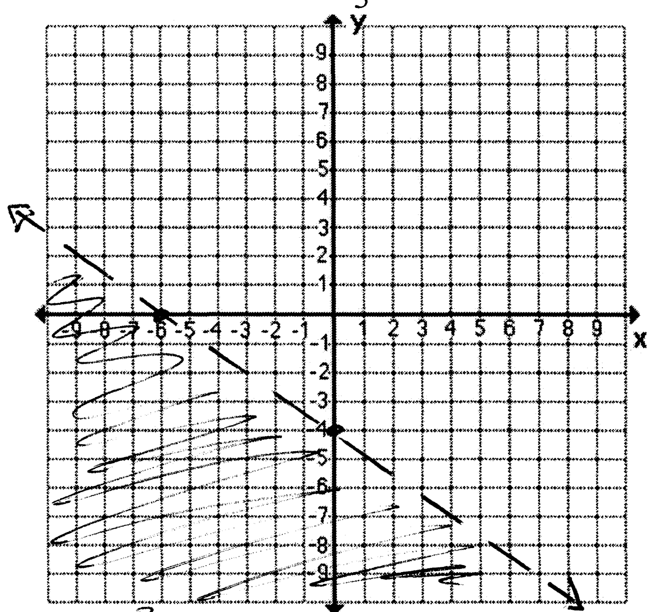
$0 = -3x + 2$

$3x = 2$

$x = \frac{2}{3}$

x	y
0	2
2/3	0

E) $y < -\frac{2}{3}x - 4$



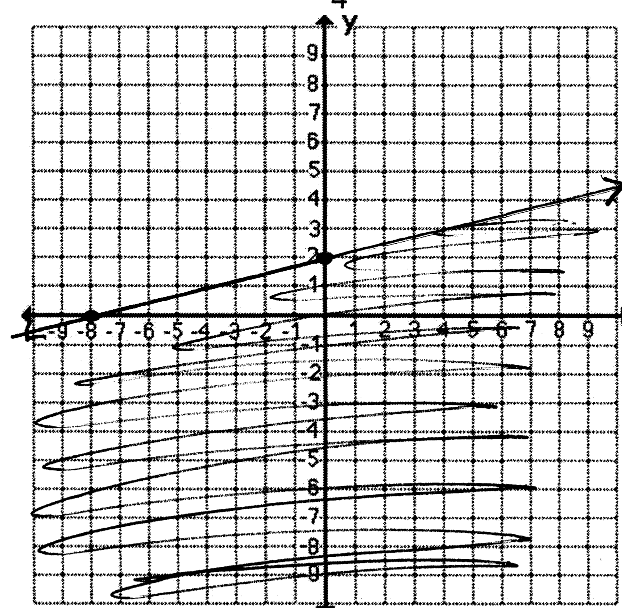
$0 = -\frac{2}{3}x - 4$

$\left(\frac{3}{2}\right)\frac{2}{3}x = -4\left(\frac{3}{2}\right)$

$x = -6$

x	y
0	-4
-6	0

F) $y \le \frac{1}{4}x + 2$



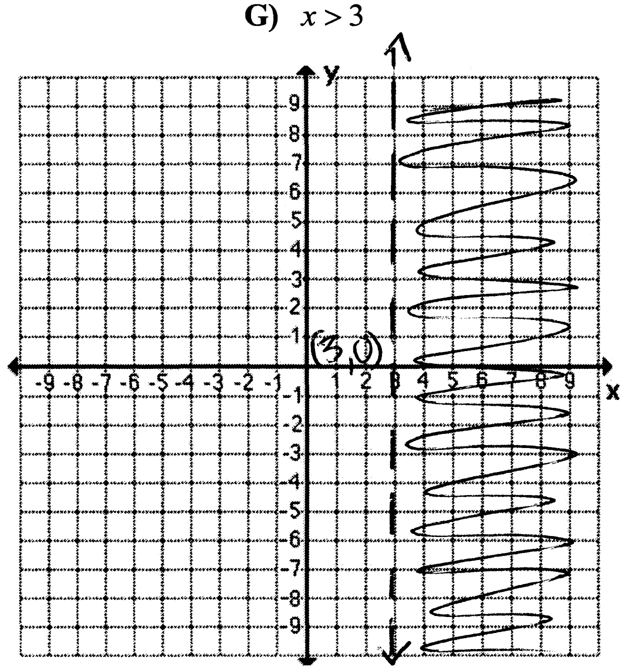
$0 = \frac{1}{4}x + 2$

$\frac{x}{4} = -2$

$x = -8$

x	y
-8	0
0	2

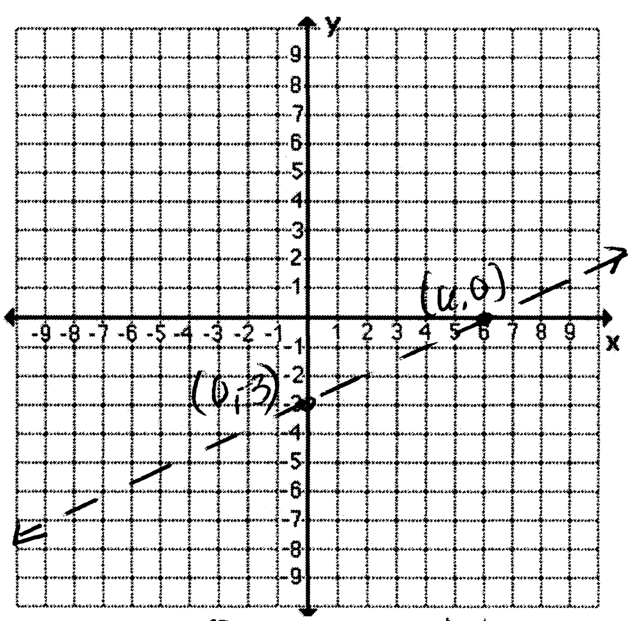
G) $x > 3$



x	y
3	0

No y-int

H) $2x - 4y > 12$



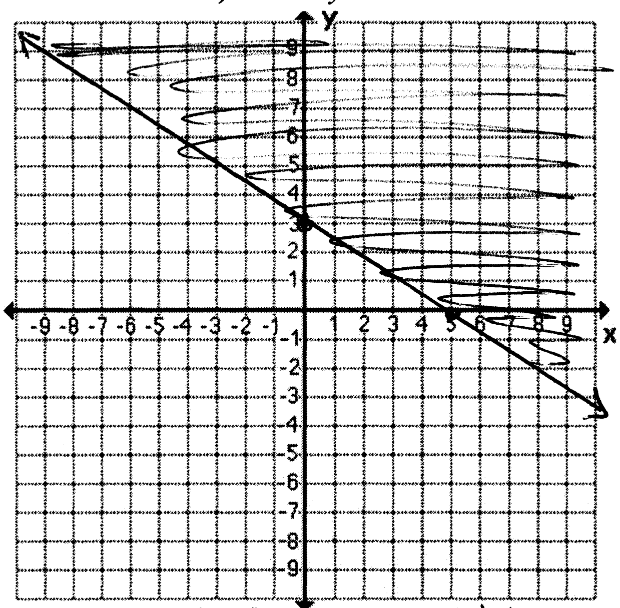
$2x - 4y > 12$

$\frac{-4y}{-4} > \frac{-2x + 12}{-4}$

$y < \frac{1}{2}x - 3$

x	y
0	-3
6	0

E) $3x+5y \ge 15$



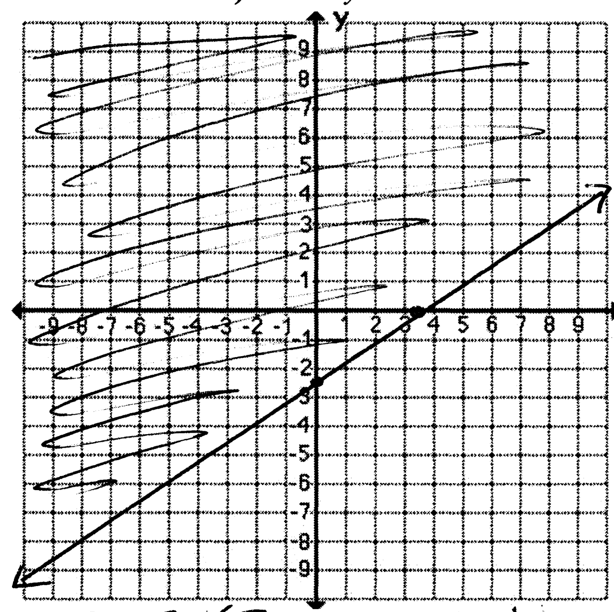
$3x+5y \ge 15$

$5y \ge -3x+15$

$y \ge -\frac{3}{5}x+3$

x	y
0	3
5	0

F) $2x-3y \le 7$



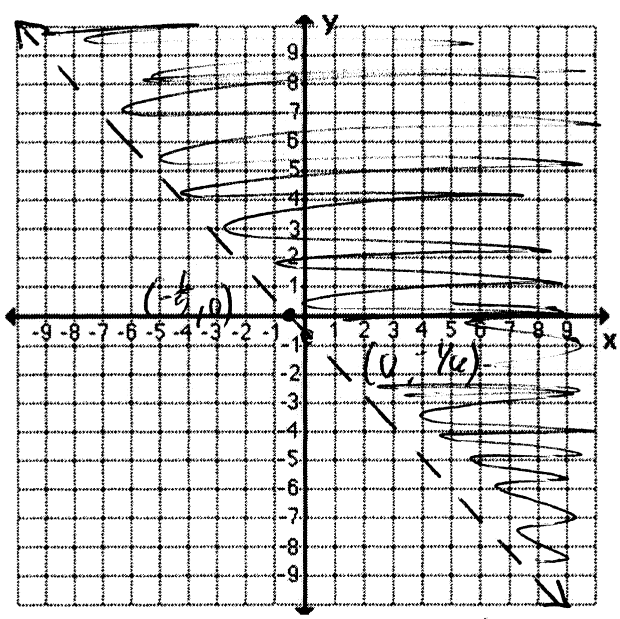
$2x-3y \le 7$

$\frac{-3y}{-3} \le \frac{-2x+7}{-3}$

$y \ge \frac{2}{3}x-\frac{7}{3}$

x	y
0	$-\frac{7}{3}$
$\frac{7}{2}$	0

G) $5x+6y > -1$



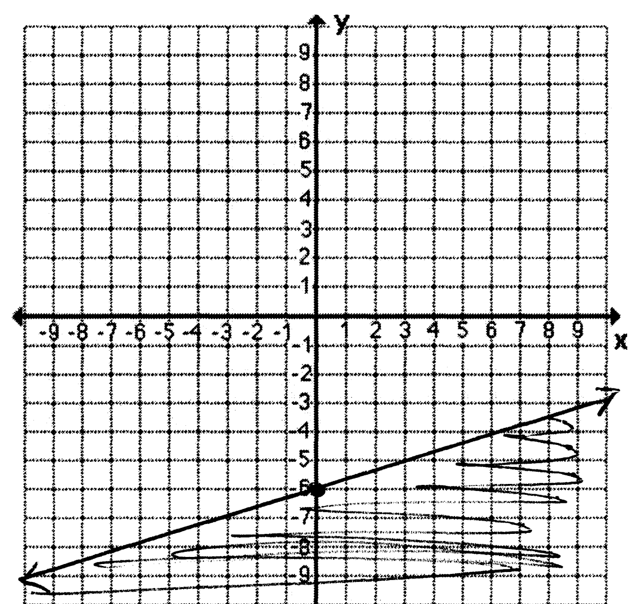
$5x+6y > -1$

$\frac{6y}{6} > \frac{-5x-1}{6}$

$y > -\frac{5}{6}x-\frac{1}{6}$

x	y
0	$-\frac{1}{6}$
$-\frac{1}{5}$	0

H) $x-3y \ge 18$



$x-3y \ge 18$

$\frac{-3y}{-3} \ge \frac{-x}{-3}\frac{+18}{-3}$

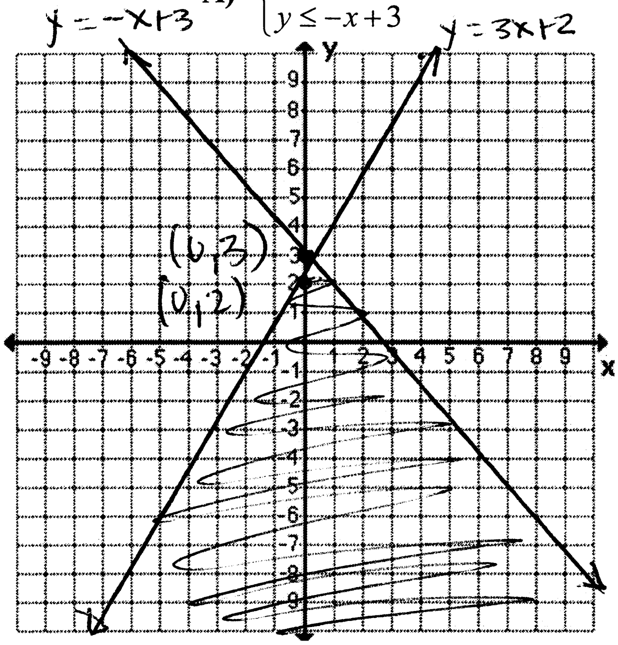
$y \le \frac{1}{3}x-6$

x	y
0	-6
18	0

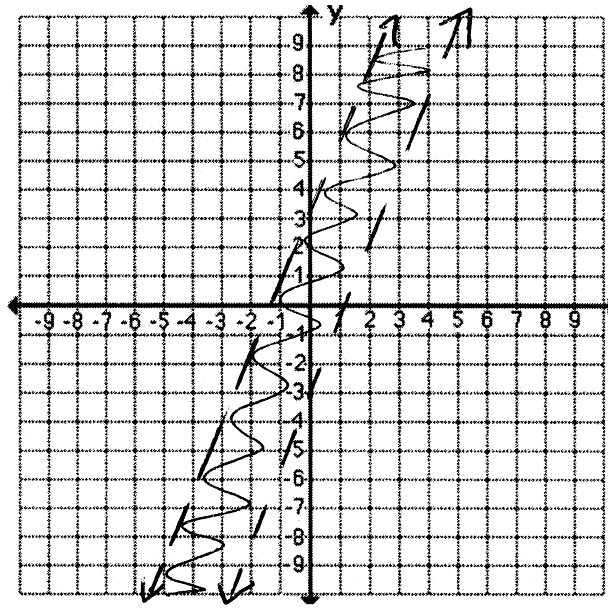
Graphing Systems of Inequalities

Graph each of the following systems of inequalities

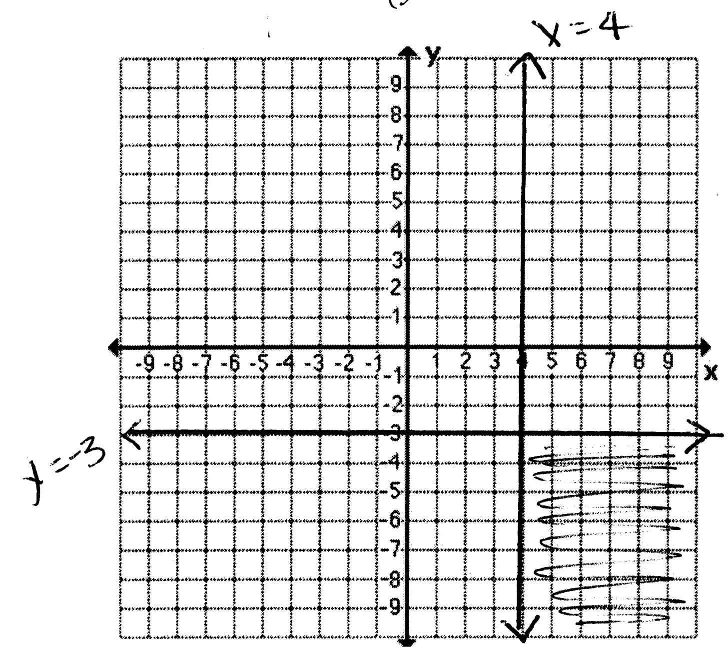
A) $\begin{cases} y \le 3x+2 \\ y \le -x+3 \end{cases}$



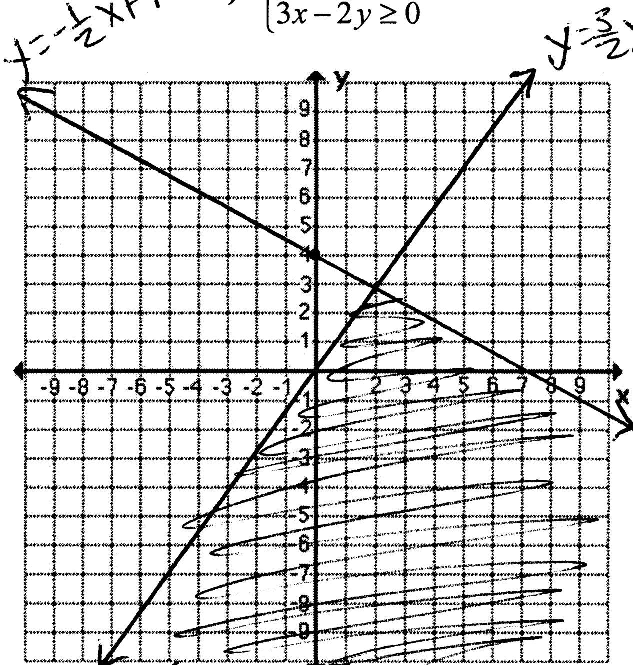
B) $\begin{cases} y > 4x-3 \\ y < 4x+3 \end{cases}$



C) $\begin{cases} x \ge 4 \\ y \le -3 \end{cases}$



D) $\begin{cases} x+2y \le 8 \\ 3x-2y \ge 0 \end{cases}$



$x+2y \le 8$

$2y \le -x+8$

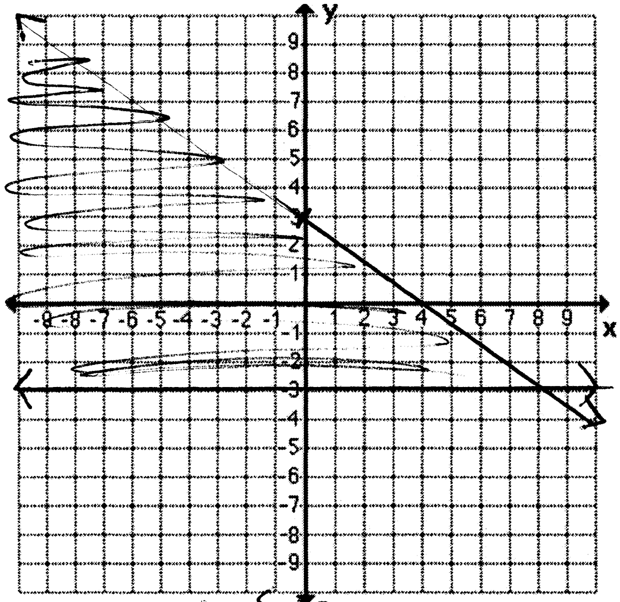
$y \le -\frac{1}{2}x+4$

$3x-2y \ge 0$

$\frac{-2y}{-2} \ge \frac{-3x}{-2}$

$y \le \frac{3}{2}x$

E) $\begin{cases} 3x+4y \le 12 \\ y \ge -3 \end{cases}$

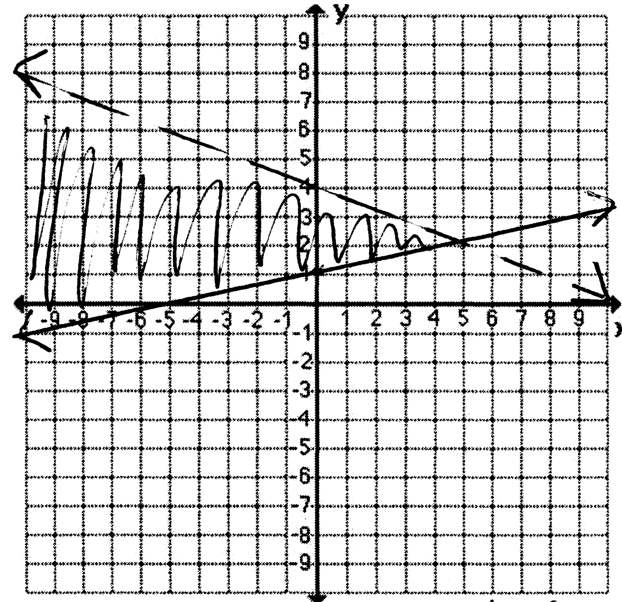


$3x+4y \le 12$

$4y \le -3x+12$

$y \le -\frac{3}{4}x+3$

F) $\begin{cases} 2x+5y < 20 \\ x-5y \le -5 \end{cases}$



$2x+5y < 20$

$5y < -2x+20$

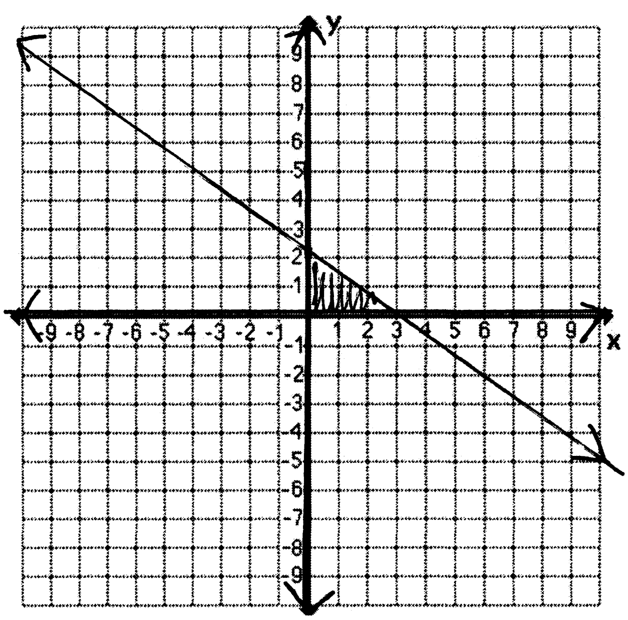
$y = -\frac{2}{5}x+4$

$x-5y \le -5$

$-5y \le -x-5$

$y \ge \frac{1}{5}x+1$

G) $\begin{cases} 2x+3y \le 6 \\ x \ge 0 \\ y \ge 0 \end{cases}$

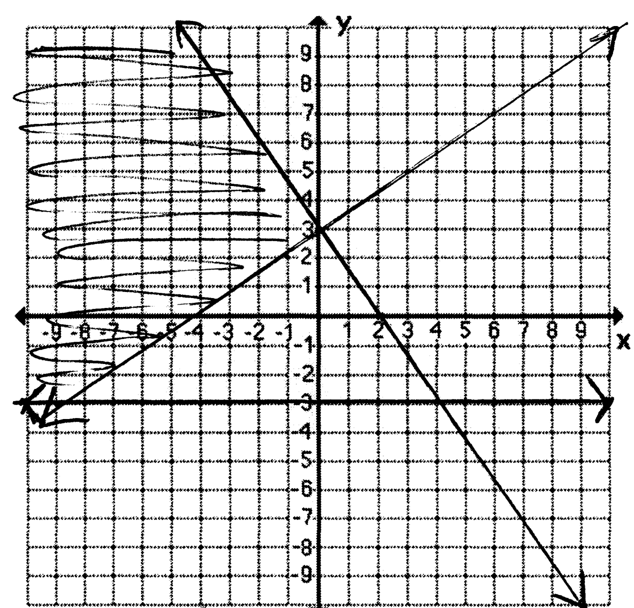


$2x+3y \le 6$

$3y \le 2x+6$

$y \le -\frac{2}{3}x+2$

H) $\begin{cases} 2x-3y \le -9 \\ 3x+2y \le 6 \\ y \ge -3 \end{cases}$



$2x-3y \le -9$

$-3y \le -2x-9$

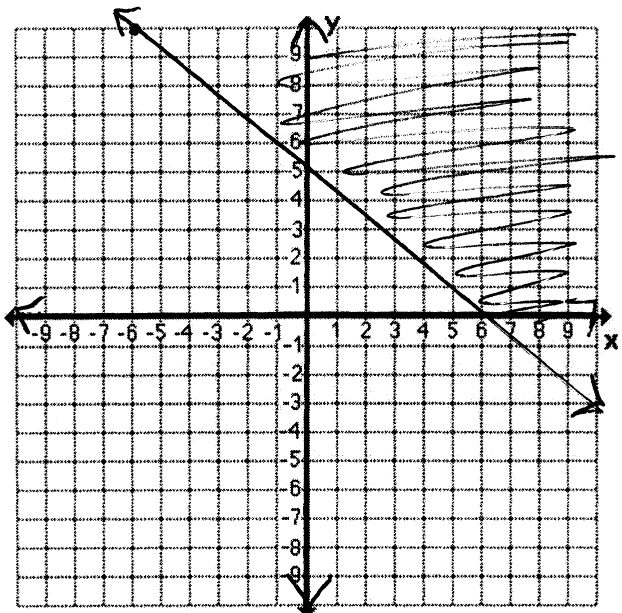
$y \ge \frac{2}{3}x+3$

$3x+2y \le 6$

$2y \le -3x+6$

$y \le -\frac{3}{2}x+3$

I) $\begin{cases} 5x+6y \geq 30 \\ x \geq 0 \\ y \geq 0 \end{cases}$

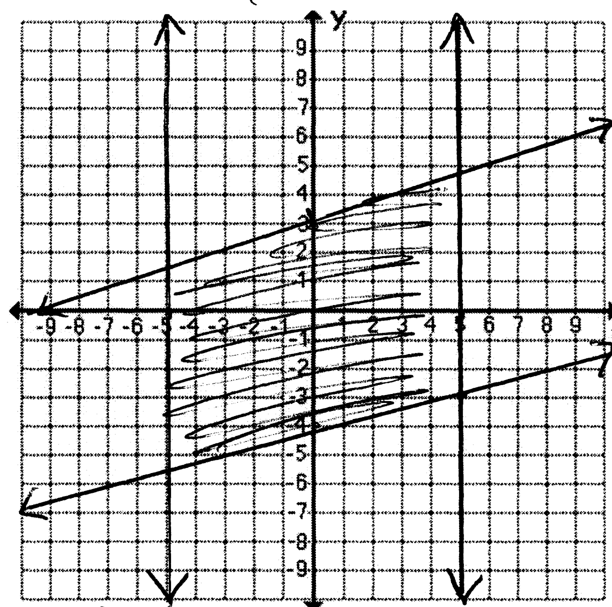


$5x+6y \geq 30$

$6y \geq -5x+30$

$y \geq -\frac{5}{6}x+5$

J) $\begin{cases} x-5y \leq 20 \\ x-5y \geq -15 \\ x \geq -5 \\ x \leq 5 \end{cases}$



$x-5y \leq 20$

$-5y \leq -x+20$

$y \geq \frac{1}{5}x-4$

$x-5y \geq -15$

$-5y \geq -x-15$

$y \leq \frac{1}{5}x+3$

K) $\begin{cases} 2x+y \leq 8 \\ x+3y \leq 12 \\ x \geq 0 \\ y \geq 0 \end{cases}$

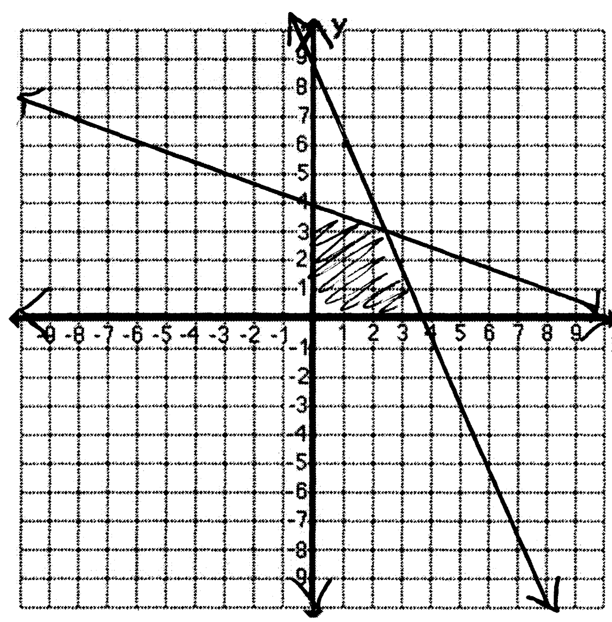
$2x+y \leq 8$

$y \leq -2x+8$

$x+3y \leq 12$

$3y \leq -x+12$

$y \leq -1/3x+4$



Solving Systems of Equations in 2 Variables Graphically

Basically, what is a system of linear equations?

two lines on the same coordinate plane.

What is the solution to a system of linear equations?

where the lines intersect

When solving a system of linear equations, you will have one solution, infinite solutions, or no solutions. Describe the circumstances in which you could have each of these types of solutions.

if the slopes are different => one solution

if the ~~s~~lopes are the same
and y-intercepts are the same => Infinite solutions (same line)

if the slopes are the same
but the y-intercepts are different => No solution
(parallel lines)

Once I put the two equations of a linear system into slope-intercept form, I can tell which of these types of answers I will get. How can I tell what type of answer I will get without actually graphing the lines? What two things do you look at?

the slope and the y-intercept

What is the difference between a consistent and an inconsistent system of equations?

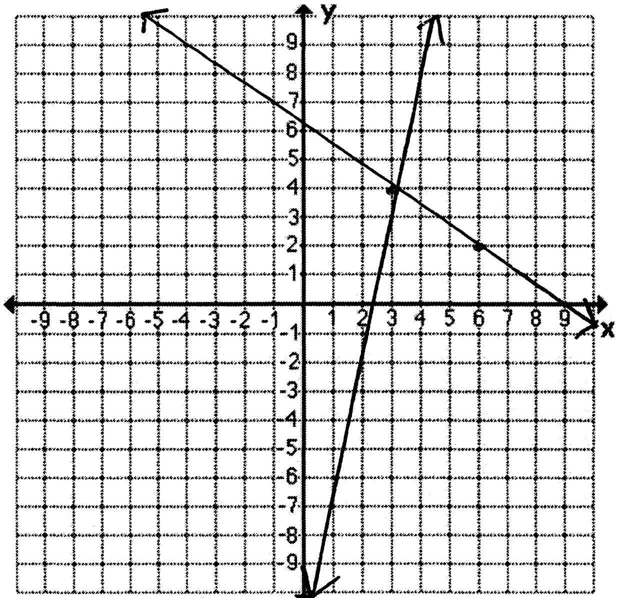
A consistant system has at least one solution
An inconsistent system is a system that has no solutions.

What are the three methods used to solve a system of equations in two variables?

graphing
substitution
Linear combination

Solve each of the following linear systems by graphing.

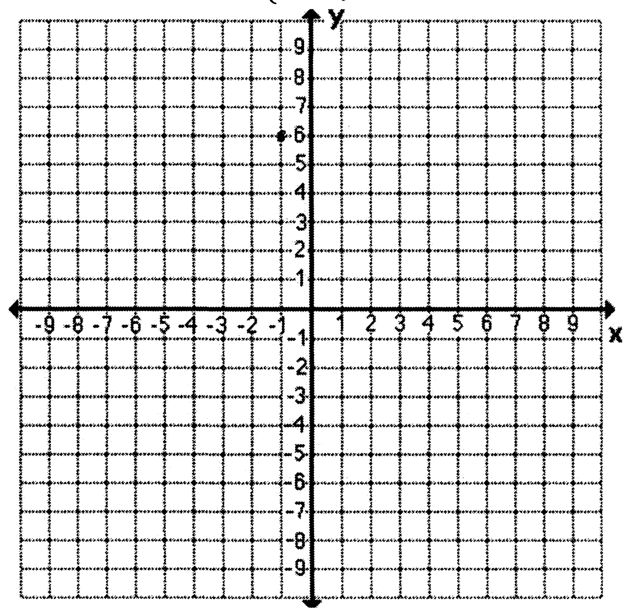
A) $\begin{cases} 2x+3y=18 \\ 5x-\ \ y=11 \end{cases}$



$2x+3y=18$
$3y=-2x+18$
$y=-\frac{2}{3}x+6$

$5x-y=11$
$-y=-5x+11$
$y=5x-11$

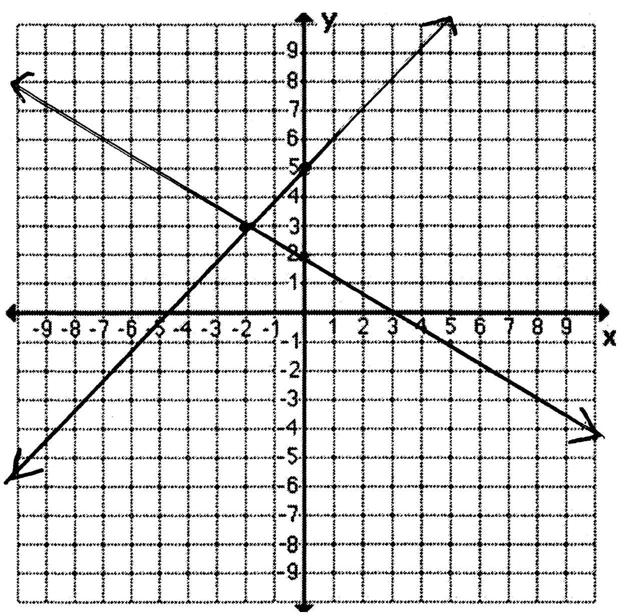
B) $\begin{cases} 3x+y=3 \\ 2x+y=4 \end{cases}$



$3x+y=3$
$y=-3x+3$

$2x+y=4$
$y=-2x+4$

C) $\begin{cases} x-\ \ y=-5 \\ x+2y=\ \ 4 \end{cases}$

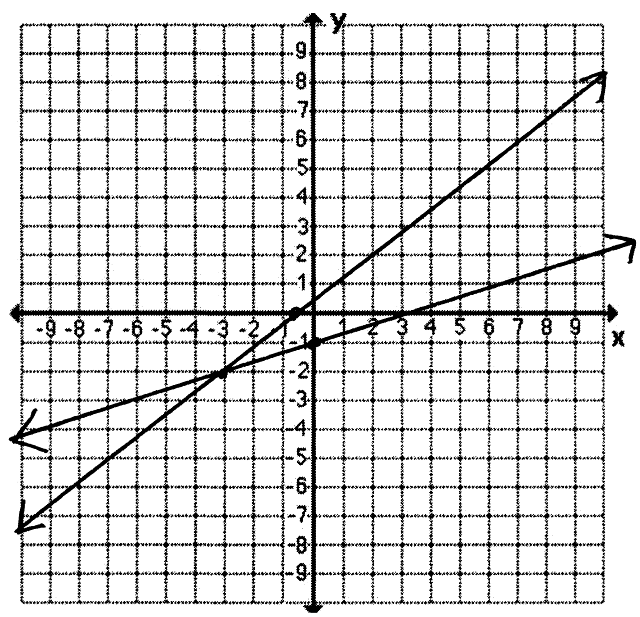


$x-y=-5$
$-y=-x-5$
$y=x+5$

$x+2y=4$

x	y
0	2
4	0

D) $\begin{cases} 4x-5y=-2 \\ x-3y=\ \ 3 \end{cases}$



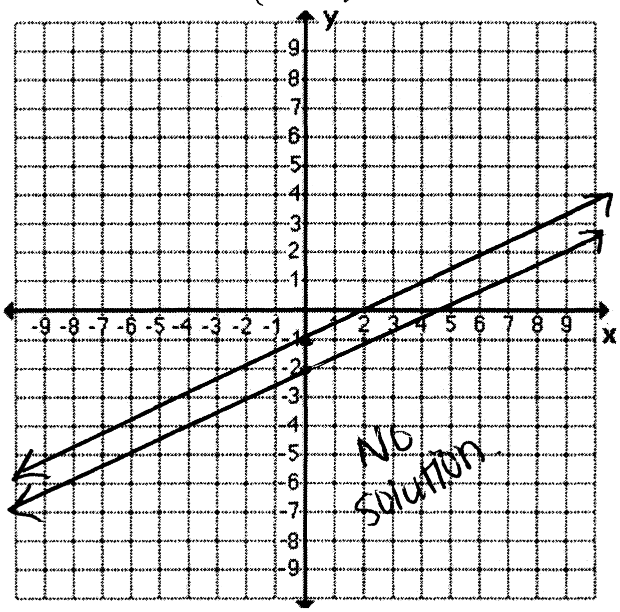
$4x-5y=-2$

x	y
0	$\frac{2}{5}$
$-\frac{1}{2}$	0

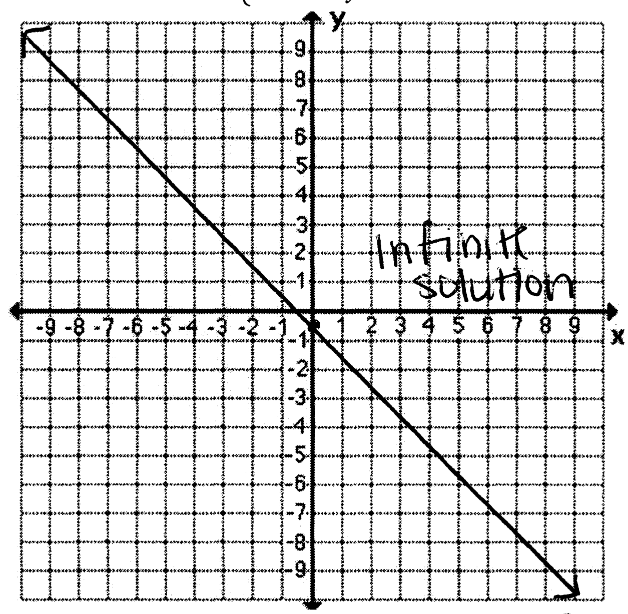
$x-3y=3$

x	y
0	-1
3	0

E) $\begin{cases} 2x-3y=6 \\ 4x-6y=6 \end{cases}$



F) $\begin{cases} x+3y=-2 \\ 4x+12y=-8 \end{cases}$



$2x-3y=6$
$-3y=-2x+6$
$y=\frac{2}{3}x-2$

$4x-6y=6$
$-6y=-4x+6$
$y=\frac{2}{3}x-1$

$x+3y=-2$

x	y
0	-2/3
-2	0

$4x+12y=-8$

x	y
0	-2/3
-2	0

The two remaining algebraic methods used to solve a system of equations are substitution, and the linear combination method. Remember, solving a linear system graphically is the least reliable way to solve a system of equations.

Before we begin working on solving systems of equations algebraically, let's practice verifying the solutions to a system. To verify a solution to a system of linear equations, substitute the x and y values into both equations. If both statements are true, the given ordered pair is a solution to the system of equations. If only one statement is true, the ordered pair is not a solution.

When an ordered pair is substituted into a linear equation, and the resultant statement comes out true, what was just proven regarding the point and the line?

That the point exists on the line.

Why is it necessary to substitute the values of an ordered pair into both equations?

to verify that the point exsits on both lines.

Decide whether the ordered pair is a solution of the linear system.

$(5,-2)$

A) $\begin{cases} 5x - y = 27 \\ -3x + 4y = -23 \end{cases}$

$5(5)-(-2)=27$

$25+2=27$

$27=27$

$-3(5)+4(-2)=-23$

$-15-8=-23$

$-23=-23$

yes, is a solution.

$(4,0)$

B) $\begin{cases} -x + 6y = 4 \\ 3x + y = 12 \end{cases}$

$-(4)+6(0)=4$

$-4 \neq 4$

No, not a solution

$(9,1)$

C) $\begin{cases} 2x - 9y = 9 \\ -x + 8y = 17 \end{cases}$

$2(9)-9(1)=9$

$18-9=9$

$9=9$

$-(9)+8(1)=17$

$-9+8=17$

$-1 \neq 17$

no, not a solution

$(-3,3)$

D) $\begin{cases} 2x - 4y = -18 \\ 4x - 2y = -18 \end{cases}$

$2(-3)-4(3)=-18$

$-6-12=18$

$-18=-18$

$4(-3)-2(3)=-18$

$-12-6=-18$

$-18=-18$

yes, is a solution

$(-2,-7)$

E) $\begin{cases} 4x - 3y = 13 \\ 7x - y = -7 \end{cases}$

$4(-2)-3(-7)=13$

$-8+21=13$

$13=13$

$7(-2)-(-7)=-7$

$-14+7=-7$

$-7=-7$

yes, is a solution.

$(8,9)$

F) $\begin{cases} -2x + 4y = -20 \\ 3x + y = 33 \end{cases}$

$-2(8)+4(9)=-20$

$-16+36=-20$

$20 \neq -20$

no, not a solution.

$(0,3)$

G) $\begin{cases} x + 2y = 8 \\ 2x + y = -3 \end{cases}$

$(0)+2(3)=8$

$6 \neq 8$

no, not a solution.

$\left(\frac{2}{3},0\right)$

F) $\begin{cases} -3x + 2y = 2 \\ 3x + 2y = -2 \end{cases}$

$-3(\frac{2}{3})+2(0)=2$

$-2 \neq 2$

no, not a solution.

Solving Systems of Equations in 2 Variables Algebraically

Solve each of the following systems of equations using the substitution method.

A) $\begin{cases} 2x + y = 9 \\ 3x - 4y = 8 \end{cases}$ $y = -2x + 9$

$3x - 4(-2x + 9) = 8$

$3x + 8x - 36 = 8$

$11x = 44$

$x = 4$

$y = -2(4) + 9$

$y = -8 + 9$

$y = 1$

$(4, 1)$

B) $\begin{cases} 3x + 5y = 12 \\ x + 4y = 11 \end{cases}$ $x = -4y + 11$

$3(-4y + 11) + 5y = 12$

$-12y + 33 + 5y = 12$

$-7y = -21$

$y = 3$

$x = -4(3) + 11$

$x = -12 + 11$

$x = -1$

$(-1, 3)$

C) $\begin{cases} x - 9y = 25 \\ 6x - 5y = 3 \end{cases}$ $x = 9y + 25$

$6(9y + 25) - 5y = 3$

$54y + 150 - 5y = 3$

$49y = -147$

$y = -3$

$x = 9(-3) + 25$

$x = -27 + 25$

$x = -2$

$(-2, -3)$

D) $\begin{cases} 2x + 3y = -6 \\ 3x + 2y = 25 \end{cases}$ $x = \frac{3}{2}y - 3$

$3(\frac{-3}{2}y - 3) + 2y = 25$

$-\frac{9}{2}y - 9 + \frac{4}{2}y = 25$

$(-\frac{2}{5}) - \frac{5}{2}y = 34(-\frac{2}{5})$

$y = -\frac{68}{5}$

$x = -\frac{3}{2}(-\frac{\cancel{68}^{34}}{5}) - 3$

$x = \frac{102}{5} - \cancel{3}\frac{15}{5}$

$x = \frac{87}{5}$

$(\frac{87}{5}, -\frac{68}{5})$

E) $\begin{cases} 5x + 16y = 15 \\ -2x - 4y = 1 \end{cases}$ $\frac{-2x}{-2} = \frac{4y}{-2} + \frac{1}{-2}$

$x = -2y - \frac{1}{2}$

$5(-2y - \frac{1}{2}) + 16y = 15$

$-10y - \frac{5}{2} + 16y = 15$

$6y = \frac{30}{2} + \frac{5}{2}$

$(\frac{1}{6})6y = \frac{35}{2}(\frac{1}{6})$

$y = \frac{35}{12}$

$x = -2(\frac{35}{12}) - \frac{1}{2}$

$x = -\frac{35}{6} - \frac{1}{2}$

$x = -\frac{35}{6} - \frac{3}{6}$

$x = \frac{-38}{6} = -\frac{19}{3}$

$(-\frac{19}{3}, \frac{35}{12})$

F) $\begin{cases} 2x - 5y = 8 \\ 4x - 10y = 20 \end{cases}$ $2x = 5y + 8$

$x = \frac{5}{2}y + 4$

$4(\frac{5}{2}y + 4) - 10y = 20$

$10y + 16 - 10y = 20$

$16 \neq 20$

no solution

G) $\begin{cases} x - 3y = 12 \\ 2x - 6y = 24 \end{cases}$ $x = 3y + 12$

$2(3y + 12) - 6y = 24$

$6y + 24 - 6y = 24$

$24 = 24$

infinite solutions.

H) $\begin{cases} 2x + 5y = 16 \\ y = 4 \end{cases}$

$2x + 5(4) = 16$

$2x + 20 = 16$

$2x = -4$

$x = -2$

$(-2, 4)$

I) $\begin{cases} -2x + y = 8 \\ 3x + y = -2 \end{cases}$ $y = 2x + 8$

$3x + (2x + 8) = -2$

$5x + 8 = -2$

$5x = -10$

$x = -2$

$y = 2(-2) + 8$

$y = -4 + 8$

$y = 4$

$(-2, 4)$

Solve each of the following systems of equations using the linear combination method.

A) $\begin{cases} x + 3y = -2 \\ 4x + 12y = -8 \end{cases}$

$-4(x+3y=-2)$

$-4x-12y=8$

$4x+12y=-8$

$0=0$

Infinite solutions.

B) $\begin{cases} 5x - 3y = 12 \\ 2x - y = 4 \end{cases}$

$-3(2x-y=4)$

$5x-3y=12$

$-6x+3y=-12$

$-x=0$

$x=0$

$2(0)-y=4$

$-y=4$

$y=-4$

$(0,-4)$

C) $\begin{cases} 2x - 3y = 6 \\ 4x - 6y = 6 \end{cases}$

$-2(2x-3y=6)$

$-4x+6y=-12$

$4x-6y=6$

$0=-6$

no solution.

D) $\begin{cases} 4x + 5y = 6 \\ 2x - 3y = 8 \end{cases}$

$3(4x+5y=6)$

$5(2x-3y=8)$

$12x+15y=18$

$10x-15y=40$

$22x=68$

$\frac{22x}{22} = \frac{68}{22}$

$x=\frac{29}{11}$

$-2(2x-3y=8)$

$4x+5y=6$

$-4x+6y=-16$

$4x+5y=6$

$11y=-10$

$y=-\frac{10}{11}$

$\left(\frac{29}{11}, -\frac{10}{11}\right)$

E) $\begin{cases} 8x - 3y = 4 \\ 16x - 6y = 8 \end{cases}$

$-2(8x-3y=4)$

$16x-6y=8$

$-16x+6y=-8$

$16x-6y=8$

$0=0$

Infinite solutions.

F) $\begin{cases} 7x + 20y = 11 \\ 3x + 10y = 5 \end{cases}$

$-2(3x+10y=5)$

$7x+20y=11$

$-6x-20y=-10$

$7x+20y=11$

$x=1$

$7(1)+20y=11$

$-7 \quad -7$

$20y=4$

$y=\frac{4}{20}$

$y=\frac{1}{5}$

$\left(1, \frac{1}{5}\right)$

G) $\begin{cases} 2x + 3y = -6 \\ 3x + 2y = 25 \end{cases}$

$-2(2x+3y=-6)$

$3(3x+2y=25)$

$-4x-6y=12$

$4x+6y=75$

$5x=87$

$x=\frac{87}{5}$

$-3(2x+3y=-6)$

$2(3x+2y=26)$

$-6x-9y=18$

$6x+4y=50$

$-5y=68$

$y=\frac{-68}{5}$

$\left(\frac{87}{5}, -\frac{68}{5}\right)$

H) $\begin{cases} 5x + 16y = 15 \\ -2x - 4y = 1 \end{cases}$

$4(-2x-4y=1)$

$5x+16y=15$

$-8x-16y=4$

$5x+16y=15$

$-3x=19$

$x=-\frac{19}{3}$

$2(5x+16y=15)$

$5(-2x-4y=1)$

$10x+32y=30$

$-10x-20y=5$

$\frac{12y}{12}=\frac{35}{12}$

$y=\frac{35}{12}$

$\left(-\frac{19}{3}, \frac{35}{12}\right)$

I) $\begin{cases} 2x - 3y = 6 \\ 5x - 7y = 10 \end{cases}$

$-5(2x-3y=6)$

$2(5x-9y=10)$

$-10x+15y=-30$

$10x-14y=20$

$y=-10$

$2x-3(-10)=6$

$2x+30=6$

$2x=-24$

$x=-12$

$(-12,-10)$

When solving systems of equations using either the substitution or linear combination method, there is a chance the variables will all cancel out. This leaves you with a true or false statement regarding the values of the remaining numbers. What is the significance of each of these statements, be it true or false, in terms of your solution to the system?

If the statement is true, your answer is infinite solutions.

If the resultant statement is false, there is no solution.

Linear Word Problems

When solving linear word problems, it is useful to put the problem in slope-intercept form. When doing this, remember the slope of the line is a rate of change. This can be easily identified by the words: per, for each, for every, etc. The b term in slope-intercept will be some constant. For systems of equations, it may be necessary to set both equations up in standard form.

Solve each of the following word problems by either setting up one linear equation to solve the problem, or two equations to solve the system.

A) Paul works for a company and earns a base salary of $2500 a month. If he decides to work overtime, he is paid at a rate of $32.50 per hour. How much will Paul make in a month if he decides to work 12 hours of overtime?

$P = 32.50x + 2500$
$P = 32.50(12) + 2500$
$P = 390 + 2500$
$P = 2890$

let P = total pay
let x = # overtime hours.

Paul made $2,890.

B) The measure of the two acute angles of a right triangle differ by 24°. What are their measures?

let angle 1 = x
let angle 2 = x+24

$x + (x+24) = 90$
$2x = 66$
$x = 33$
$x + 24 = 57$

33° and 57° respectively

C) Your hourly wage at a grocery store is greater after 5:00 p.m. than during the day. One week you work 18 daytime hours and 22 evening hours, and earn $419. The next week you work 30 daytime hours and 10 evening hours, and earn $365. What is the daytime hourly rate? What is the evening hourly rate?

$18D + 22E = 419$
$30D + 10E = 365$
$\frac{10E}{10} = \frac{-30D + 365}{10}$
$E = -3D + 36.5$
$E = -3(8) + 36.5$
$E = -24 + 36.5 = 12.50$

let E = Evening rate
let D = Daytime rate

$18D + 22(-3D + 36.5) = 419$
$18D - 66D + 803 = 419$
$-48D = -384$
$D = 8$

Daytime rate is - $8.00

Evening rate is - $12.50

D) Paul wants to invest \$10,000, some in stocks earning 15% annually and the rest in bonds earning 6% annually. How much should be invested at each rate to get a return of \$1,140 annually from the two investments?

let x = amount at 15%

let y = amount at 6%

$-6(x+y=10\,000)$

$100(.15x+.06y=1140)$

$-6x-6y=-60\,000$

$15x+6y=114\,000$

$9x=54\,000$

$\frac{9x}{9}=\frac{54\,000}{9}$

$x=6\,000$

$x+y=10\,000$

$6000+y=10\,000$

$y=4\,000$

$\boxed{\$6000 \text{ at } 15\%,\ \$4000 \text{ at } 6\%}$

E) How many liters of a 20% solution of Boric Acid should be mixed with a 50% solution to obtain a 60% solution of Boric Acid?

let x = #liters at 20%

let y = #liters at 50%

$\boxed{x} + \boxed{y} = \boxed{60}$

20% 50% 30%

$-2(x+y=60)$

$10(.2x+.5y=18)$

$x+y=60$

$.2x+.5y=18$

$-2x-2y=-120$

$2x+5y=180$

$3y=60$

$y=20$

$x+y=60$

$x+20=60$

$x=40$

$\boxed{\text{40 liters at 20\%, 20 liters at 50\%}}$

F) How many liters of a 30% solution of sodium Chlorite must be added to 50 liters of a 10% solution in order to obtain a solution that is 25% Sodium Chlorite?

$\boxed{x} + \boxed{50} = \boxed{x+50}$

30% 10% 25%

$.3x+.1(50)=.25(x+50)$

multiply by 100

$30x+10(50)=25(x+50)$

$30x+500=25x+1250$

$-25x\ -500\ -25x\ -500$

$5x=750$

$x=150$

$\boxed{\text{150L of a 30\% solution must be added}}$

G) How much pure antifreeze must be added to a 10% solution to obtain 30 liters of a 40% solution?

let x = # leters pure

let y = # liters 10%

$\boxed{x} + \boxed{y} = \boxed{30}$

100% 10% 40%

$x+y=30$

$x+.1y=12$

$-1(x+y=30)$

$10(x+.1y=12)$

$-x-y=-30$

$10x+y=120$

$9x=90$

$x=10$

$\boxed{\text{10L of pure antifreeze must be added}}$

H) Paul and Jason decide to take their respective families to a ball game. While they are at the game, Paul buys 3 hot dogs and 2 sodas for his family and pays \$28.00. Jason buys 7 hot dogs and 4 sodas for his family and pays \$62.50. How much does a hot dog cost?

let H = price of hot dog
let S = price of soda

$3H + 2S = 28$
$7H + 4S = 62.50$

$-2(3H + 2S = 28) \longrightarrow -6H - 4S = -56.00$
$7H + 4S = 62.50 \longrightarrow 7H + 4S = 62.50$

$H = 6.50$

A hot dog costs \$6.50

I) In 1985, a home was purchased for \$180,000. In the year 2006, that same home sold for \$520,000. By what approximate amount did the value of the home increase per year?

$$\frac{520\,000 - 180\,000}{2006 - 1985} = \frac{340\,000}{21} = \$16{,}190.48$$

the value of the house increased by approximately \$16,190.48 per year

J) In 1970, the population of Tinytown was 200 people. In 1975, that population had increased to 315 people. Assuming the population of the town increases at a constant rate, what would the population of Tinytown be in the year 2008?

(1970, 200)
(1975, 315)

$$\frac{315 - 200}{1975 - 1970} = \frac{115}{5} = 23$$

let t = 0 represent 1970

$p = 23t + b$ ← variation of slope-int form

$315 = 23(5) + b$ ← if t=0 is 1970, t=5 is 1975

$315 = 115 + b$

$b = 200$

$P = 23t + 200$
$P = 23(38) + 200$
$P = 874 + 200$
$P = 1074$

The populations of tinytown in 2008 would be 1074

K) Last year, the principal of a school earned a salary of \$86,520. In four years, his salary will be \$108,312. How much would his salary increase by per year for this to happen?

$$\frac{108\,312 - 86\,520}{4} = \frac{21\,792}{4} = 5{,}448$$

His salary would increase by \$5,448 per year

L) Bill works at a local store. His monthly salary is \$3,250. If Bill works overtime, he is paid at a rate of \$32 per hour. How much overtime did Bill work if is paycheck last month was \$3,826?

let x = # hours overtime

let p = total pay

$p = 32x + 3250$

$3{,}826 = 32x + 3250$

$-3{,}250 \qquad -3{,}250$

$\frac{576}{32} = \frac{32x}{32}$ $x = 18$

$\boxed{\text{18 hours of overtime}}$

Solving Systems of Equations in 3 Variables

Solving a system of equations is 3 variables isn't much more complicated than a system of equations in two variables.

$$\begin{aligned} Eq_1 &\quad a + b + c = 0 \\ Eq_2 &\quad 4a + 2b + c = -1 \\ Eq_3 &\quad 9a + 3b + c = 0 \end{aligned}$$

Here you have a system of equations in 3 variables. Each equation has been labeled as being equation 1, equation2 and equation 3 to make it easier to follow.

$$\begin{aligned} Eq_1 &\quad a + b + c = 0 \\ Eq_2 &\quad 4a + 2b + c = -1 \end{aligned} \qquad \begin{aligned} Eq_1 &\quad a + b + c = 0 \\ Eq_3 &\quad 9a + 3b + c = 0 \end{aligned}$$

One of these equations will be used twice. It doesn't matter which. In this case, equation 1 will be used twice.

$$\begin{array}{ll} -Eq_1 & -a - b - c = 0 \\ +Eq_2 & 4a + 2b + c = -1 \\ \hline Eq_4 & 3a + b = -1 \end{array} \qquad \begin{array}{ll} -Eq_1 & -a - b - c = 0 \\ +Eq_3 & 9a + 3b + c = 0 \\ \hline Eq_5 & 8a + 2b = 0 \end{array}$$

Multiplying equation 1 by -1 and combining the equations yielded two new equations. You <u>must</u> get rid of the same variable each time.

$$\begin{aligned} Eq_4 &\quad 3a + b = -1 \\ Eq_5 &\quad 8a + 2b = 0 \end{aligned}$$

Now we have a system of equations in two variables.

$$\begin{array}{ll} -2Eq_4 & -6a - 2b = 2 \\ +\ Eq_5 & 8a + 2b = 0 \\ \hline & 2a = 2 \\ & a = 1 \end{array}$$

Multiply equation 4 by -2, and add the result to equation 5. This yields a numerical value for the variable a.

$a = 1$

Substitute into equation 5

$$\begin{aligned} 8(1) + 2b &= 0 \\ 2b &= -8 \\ b &= -4 \end{aligned}$$

$a = 1 \quad b = -4$

Substitute into equation 1

$$\begin{aligned} (1) + (-4) + c &= 0 \\ -3 + c &= 0 \\ c &= 3 \end{aligned}$$

Once the value of the first variable is found, substitute that number, in this case 1, into either equation 4 or 5 and solve for the remaining variable. Now that the value of two of the variables is known, go back to equation 1, substitute and find the value of the third variable.

$a = 1 \quad b = -4 \quad c = 3$

$(1, -4, 3)$

The value of all three variables has now been found.

Write your solution in (x,y,z) format.

Solve each of the following systems of equations.

A) $\begin{cases} x + 4y + z = 12 \\ y - 3z = -7 \\ z = 3 \end{cases}$ **Solution:** (1 , 2, 3)

$y - 3(3) = -7$

$y - 9 = -7$

$+9 \quad +9$

$y = 2$

$x + 4(2) + 3 = 12$

$x + 8 + 3 = 12$

$x + 11 = 12$

$-11 \quad -11$

$x = 1$

B) $\begin{cases} x + 2z = 30 \\ y + z = 12 \\ z = 1 \end{cases}$ **Solution:** (28 , 11 , 1)

$x + 2(1) = 30$

$x + 2 = 30$

$-2 \quad -2$

$x = 20$

$y + 1 = 12$

$-1 \quad -1$

$y = 11$

C) $\begin{cases} x+9y+z=20 \\ x+10y-2z=18 \\ 3x+27y+2z=58 \end{cases}$

Solution: (-18 , 4 , 2)

$2(x+9y+z=20)$
$x+10y-2z=18$
$2x+18y+2z=40$
$3x+28y=58$

$-2(x+9y+z=20)$
$3x+27y+2z=58$
$-2x-18y-2z=-40$
$x+9y=18$

$-3(x+9y=18)$
$-3x-27y=-54$
$3x+28y=58$
$y=4$

$x+9(4)=18$
$x+36=18$
$-36\quad -36$
$x=-18$

$x+9y+z=20$
$-18+9(4)+z=20$
$-18+36+z=20$
$18+z=20$
$-18\quad -18$
$z=2$

D) $\begin{cases} -x+y-3z=-4 \\ 3x-2y+8z=14 \\ 2x-2y+5z=7 \end{cases}$

Solution: (4 , 3 , 1)

$2(-x+y-3z=-4)$
$3x-2y+8z=14$
$-2x+2y-6z=-8$
$x+2z=6$

$2(-x+y-3z=-4)$
$2x-2y+5z=7$
$-2x+2y-6z=-8$
$-z=-1$
$z=1$

$x+2(1)=6$
$x+2=6$
$x=4$

$-x+y-3z=-4$
$-4+y-3=-4$
$y-7=-4$
$y=3$

E) $\begin{cases} 3x-6y+3z=18 \\ 2x-3y+4z=\ \ 6 \\ 2x-3y+5z=\ \ 4 \end{cases}$

Solution: (4 , -2 , -2)

$-2(2x-3y+4z=6)$
$3x-6y+3z=18$
$-4x+6y-8z=-12$
$-x \quad -5z=6$

$-2(2x-3y+5z=4)$
$3x-6y+3z=18$
$-4x+6y-10z=-8$
$-x \quad -7z=10$

$-1(-x-5z=6)$
$-x-7z=10$
$x-5z=-6$
$-2z=4$
$z=-2$

$-x-7(-2)=10$
$-x+14=10$
$-14 \quad -14$
$-x=-4$
$x=4$

$3x-6y+3z=18$
$3(4)-6y+3(-2)=18$
$12-6y-6=18$
$6-6y=18$
$-6y=12$
$y=-2$

F) $\begin{cases} x+\ \ y+\ \ z=\ \ 0 \\ 4x-2y+\ \ z=-3 \\ 4x+2y+\ \ z=\ \ 9 \end{cases}$

Solution: (2 , 3 , -5)

$4x-2y+z=-3$
$4x+2y+z=9$
$8x \quad +2z=6$

$2(x+y+z=0)$
$2x+2y+2z=0$
$4x-2y+z=-3$
$6x+3z=-3$

$3(8x+2z=6) \rightarrow 24x+6z=18$
$-2(6x+3z=-3) \rightarrow -12x-6z=6$
$12x=24$
$x=2$

$6x+3z=-3$
$6(2)+3z=-3$
$12+3z=-3$
$3z=-15$
$z=-5$

$x+y+z=0$
$2+y+-5=0$
$y-3=0$
$y=3$

Function Operations

Complete each of the following.

$(f+g)_{(x)} =$ f(x) + g(x)

$(f-g)_{(x)} =$ f(x) − g(x)

$(f \cdot g)_{(x)} =$ f(x) • g(x)

$(f \div g)_{(x)} =$ $\frac{f(x)}{g(x)}$

$(f \circ g)_{(x)} = f_{(x)} \circ g_{(x)} =$ f(g(x))

When conducting function operations, the symbol $\circ$ is used for what type of operation?

Composition Function

Perform each of the operations.

1) Given $f_{(x)} = x^2 + 6$ and $g_{(x)} = x - 6$ find each of the following.

a) $f_{(-3)}$

$f(-3) = (-3)^2 + 6 = 9 + 6 = \boxed{15}$

b) $g_{(18)}$

$g(18) = 18 - 6 = \boxed{12}$

c) $f_{(5)} + g_{(7)}$

$[(5)^2 + 6] + [7 - 6]$

$[25 + 6] + 1$

$31 + 1$

$\boxed{32}$

d) $f_{(6)} - f_{(4)}$

$[(6)^2 + 6] - [(4)^2 + 6]$

$(36 + 6) - (16 + 6)$

$42 - 22$

$\boxed{20}$

e) $f_{(a)}$

$\boxed{a^2 + 6}$

f) $g_{(y)}$

$\boxed{y - 6}$

2) Given $f_{(x)} = 3x - 6$, $g_{(x)} = x - 12$, and $h_{(x)} = -2x - 5$ find each of the following.

a) $f(a+b)$

$3(a+b) - 6$

$\boxed{3a + 3b - 6}$

b) $g(3x+2)$

$(3x+2) - 12$

$\boxed{3x - 10}$

c) $h_{(x-12)}$

$-2(x-12) - 5$

$-2x + 24 - 5$

$\boxed{-2x + 19}$

d) $f_{(g_{(x)})}$ = $f(x-12)$

$\cancel{g}\ 3(x - 12) - 6$

$3x - 36 - 6$

$\boxed{3x - 42}$

e) $f_{(2)} + g_{(2)} + h_{(2)}$

$[3(2) - 6] + [2 - 12] + [-2(2) - 5]$

$6 - 6 \quad -10 \quad -4 - 5$

$0 + -10 + -9$

$\boxed{-19}$

f) $(f+g)_{(6)}$

$f(6) + g(6)$

$[3(6) - 6] + [6 - 12]$

$18 - 6$

$12 + -6$

$\boxed{6}$

3) **Given** $f_{(x)} = x^2 + 6$, $g_{(x)} = x - 3$ **and** $h_{(x)} = 2x^2 - 5x + 7$ **find each of the following.**

a) $h_{(g_{(x)})} = h(x-3)$

$2(x-3)^2 - 5(x-3) + 7$

$2(x^2 - 6x + 9) - 5x + 15 + 7$

$2x^2 - 12x + 18 - 5x + 15 + 7$

$\boxed{2x^2 - 17x + 40}$

b) $(h \circ g)_{(4)} = h(g(4))$

$g(4) = 4 - 3 = 1$

$h(1) = 2(1)^2 - 5(1) + 7$

$2 - 5 + 7$

$\boxed{4}$

c) $f_{\left(g_{(h_{(4)})}\right)}$

$h(4) = 2(4)^2 - 5(4) + 7 = 19$

$32 - 20 + 7$

$g(19) - 19 - 3 = 16$

$f(16) = (16)^2 + 6$

$\boxed{262}$

d) $f_{(x+5)} + g_{(3x-2)}$

$[(x+5)^2 + 6] + [(3x-2) - 3]$

$[x^2 + 10x + 25 + 6] + [3x - 5]$

$\boxed{x^2 + 13x + 26}$

e) $f_{(x-3)} - f_{(x+6)}$

$[(x-3)^2 + 6] - [(x+6)^2 + 6]$

$[x^2 - 6x + 9 + 6] - [x^2 + 12x + 36 + 6]$

$x^2 - 6x + 15 - x^2 - 12x - 42$

$\boxed{-18x - 27}$

f) $h_{(f_{(x)})}$

4) **Given** $f_{(x)} = 3x^3 - 6$, $g_{(x)} = 3x^2 - 4x$, **and** $h_{(x)} = x^2 - 3x + 12$ **find each of the following.**

a) $2f_{(-3)} - 3g_{(8)}$

$2[3(-27) - 6] - 3[3(8)^2 - 4(8)]$

$2[-81 - 6] - 3[192 - 32]$

$2(-87) - 3(160)$

$-174 - 480$

$\boxed{-654}$

b) $h_{(x)} + g_{(x)} - f_{(x)}$

$[x^2 - 3x + 12] + [3x^2 - 4x] -$

$[3x^3 - 6]$

$\boxed{-3x^3 + 4x^2 - 7x + 18}$

c) $h_{(4)} - h_{(2)}$

$[16 - 3(4) + 12] - [4 - 3(2) + 12]$

$16 - [4 - 6 + 12]$

$16 - 10$

$\boxed{6}$

d) $\frac{1}{3} f_{(3)} + \frac{1}{2} g_{(-2)} - h_{(3)}$

$\frac{1}{3}[3(27) - 6] + \frac{1}{2}[3(4) - 4(-2)] - [9 - 9 + 12]$

$\frac{1}{3}(75) + \frac{1}{2}(20) - 12$

$25 + 10 - 12$

$35 - 12$

$\boxed{23}$

e) $h_{(g_{(3)})} + f_{(g_{(2)})}$

$g(3) = 27 - 12$ $\quad g(2) = 12 - 8$

$g(3) = 15$ $\quad g(2) = 4$

$h(15) + f(4)$

$225 - 45 + 12 + 186$

$\boxed{378}$

f) $f_{(x-4)}$

$3(x-4)^3 - 6$

$3(x^3 - 12x^2 + 48x - 64) - 6$

$3x^3 - 36x^2 + 144x - 192 - 6$

$\boxed{3x^3 - 36x^2 + 144x - 198}$

5) **Given** $f_{(x)} = 2x^2 - 3$, $g_{(x)} = 5x + 2$, **and** $h_{(x)} = -x^2 + 1$ **find each of the following.**

a) $2f_{(x-3)} - 3h_{(x+2)}$

$2[2(x-3)^2 - 3] - 3[-(x+2)^2 + 1]$

$2[2(x^2 - 6x + 9) - 3] - 3[-(x^2 + 4x + 4) + 1]$

$2[2x^2 - 12x + 18 - 3] - 3[-x^2 - 4x - 4 + 1]$

$4x^2 - 24x + 30 + 3x^2 + 12x + 9$

$\boxed{7x^2 - 12x + 39}$

b) $3f_{(x)} - 2h_{(x)} + g_{(x)}$

$3[2x^2 - 3] - 2[-x^2 + 1] + (5x + 2)$

$6x^2 - 9 + 4x^2 - 2 + 5x + 2$

$\boxed{8x^2 + 5x - 9}$

c) $f_{\left(g_{(h_{(2)})}\right)}$

$h(2) = -4 + 1 = -3$

$g(-3) = -15 + 2 = -13$

$f(-13) = 2(-13)^2 - 3$

$2(169) - 3$

$\boxed{335}$

Miscellaneous

Complete the following table for each of the following functions:

x	-2	-1	0	1	2	3
$f_{(x)}$						

A) $f_{(x)} = 3x - 5$

x	-2	-1	0	1	2	3
$f_{(x)}$	-11	-8	-5	-2	1	4

$f(x) = 3(-2) - 5 = -6 - 5 = -11$

$f(-1) = 3(-1) - 5 = -3 - 5 = -8$

$f(0) = 3(0) - 5 = -5$

$f(1) = 3(1) - 5 = 3 - 5 = -2$

$f(2) = 3(2) - 3 = 6 - 5 = 1$

$f(3) = 3(3) - 5 = 9 - 5 = 4$

B) $f_{(x)} = -3x + 1$

x	-2	-1	0	1	2	3
$f_{(x)}$	7	4	1	-2	-5	-8

$f(-2) = -3(-2) + 1 = 7$

$f(-1) = -3(-1) + 1 = 4$

$f(0) = -3(0) + 1 = 1$

$f(1) = -3(1) + 1 = -2$

$f(2) = -3(2) + 1 = -5$

$f(3) = -3(3) + 1 = -8$

C) $f_{(x)} = \frac{1}{2}x + 3$

x	-2	-1	0	1	2	3
$f_{(x)}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$

$f(-2) = \frac{1}{2}(-2) + 3 = 2$

$f(-1) = \frac{1}{2}(-1) + 3 = 2\frac{1}{2}$

$f(0) = \frac{1}{2}(0) + 3 = 3$

$f(1) = \frac{1}{2}(1) + 3 = 3\frac{1}{2}$

$f(2) = \frac{1}{2}(2) + 3 = 4$

$f(3) = \frac{1}{2}(3) + 3 = 4\frac{1}{2}$

D) $f_{(x)} = -\frac{2}{3}x + 6$

x	-2	-1	0	1	2	3
$f_{(x)}$	22/3	20/3	6	16/3	14/3	4

$f(-2) = -\frac{2}{3}(-2) + 6 = 22/3$

$f(-1) = -\frac{2}{3}(-1) + 6 = 20/3$

$f(0) = -\frac{2}{3}(0) + 6 = 6$

$f(1) = -\frac{2}{3}(1) + 6 = 16/3$

$f(2) = -\frac{2}{3}(2) + 6 = 14/3$

$f(3) = -\frac{2}{3}(3) + 6 = 4$

E) $f_{(x)} = \frac{1}{2}|x+3|$

x	-2	-1	0	1	2	3
$f_{(x)}$	1/2	1	3/2	2	5/2	3

$f(-2) = \frac{1}{2}|-2+3| = \frac{1}{2}$

$f(-1) = \frac{1}{2}|-1+3| = 1$

$f(0) = \frac{1}{2}|0+3| = \frac{3}{2}$

$f(1) = \frac{1}{2}|1+3| = 2$

$f(2) = \frac{1}{2}|2+3| = 5/2$

$f(3) = \frac{1}{2}|3+3| = 3$

F) $f_{(x)} = \frac{|x-2|}{x-2}$

x	-2	-1	0	1	2	3
$f_{(x)}$	-1	-1	-1	-1	unde.	1

$f(-2) = \frac{|-2-2|}{-2-2} = \frac{|-4|}{-4} = \frac{4}{-4} = -1$

$f(-1) = \frac{|-1-2|}{-1-2} = \frac{|-3|}{-3} = \frac{3}{-3} = -1$

$f(0) = \frac{|-2|}{-2} = \frac{2}{-2} = -1$

$f(1) = \frac{|1-2|}{1-2} = \frac{|-1|}{-1} = \frac{1}{-1} = -1$

$f(2) = \frac{|2-2|}{2-2} = \frac{|0|}{0}$ = undefined.

$f(3) = \frac{|3-2|}{3-2} = \frac{|1|}{1} = \frac{1}{1} = 1$

G) $f_{(x)} = \begin{cases} -\frac{1}{2}x+4, & x \le 0 \\ (x-2)^2, & x > 0 \end{cases}$

x	-2	-1	0	1	2	3
$f_{(x)}$	5	9/2	4	1	0	1

$f(-2) = -\frac{1}{2}(-2)+4 = 1+4 = 5$

$f(-1) = -\frac{1}{2}(-1)+4 = \frac{1}{2}+4 = 4\frac{1}{2}$

$f(0) = -\frac{1}{2}(0)+4 = 4$

$f(1) = (1-2)^2 = (-1)^2 = 1$

$f(2) = (2-2)^2 = (0)^2 = 0$

$f(3) = (3-2)^2 = (1)^2 = 1$

H) $f_{(x)} = \begin{cases} 9-x^2, & x < -1 \\ x-3, & x \ge -1 \end{cases}$

x	-2	-1	0	1	2	3
$f_{(x)}$	5	-4	-3	-2	-1	0

$f(-2) = 9-(-2)^2 = 9-4 = 5$

$f(-1) = -1-3 = -4$

$f(0) = 0-3 = -3$

$f(1) = 1-3 = -2$

$f(2) = 2-3 = -1$

$f(3) = 3-3 = 0$

Find the equation of the function yielding the following results.

x	–2	-1	0	1	2	3
$f_{(x)}$	7	5	3	1	-1	-3

$f_{(x)} = -2x+3$

$y = mx+b$

$y = -2x+b$ $\quad$ $(0,3)$ y int

$y = -2x+3$

$m = \frac{7-5}{-2+1} = -2$

Find the equation of the function yielding the following results.

x	–2	-1	0	1	2	3
$f_{(x)}$	-13	-8	-3	2	7	12

$f_{(x)} = 5x-3$

$m = \frac{-13+8}{-2+1} = 5$

y int $(0,-3)$

Find the equation of the function yielding the following results.

x	–2	-1	0	1	2	3
$f_{(x)}$	5	5½	6	6½	7	7½

$f_{(x)} = \frac{1}{2}x+6$

$m = \frac{5\frac{1}{2}-5}{-1} = \frac{1}{2} \quad -\frac{1}{2}$

y int $= (0,6)$

Find the equation of the function yielding the following results.

x	–2	-1	0	1	2	3
$f_{(x)}$	-5	-7	-9	-11	-13	-15

$f_{(x)} = -2x-9$

$m = \frac{-7++5}{-1++2} = \frac{-2}{1} = -2$

y int $(0,-9)$

Below are the three methods you used to graph a line. Explain, in detail, how you go about graphing a line using each of the following methods.

Graph by using a table.

Create a table ~~x|y~~ and pick at least 3 values for x, substitute those values of x into the function to find the corresponding y values. Plot the pairs and graph the line.

Graph by finding intercepts.

Set up a table with alternating zeros x|y 0, 0 substitute the value of 0 for x and find the y-intercept. substitute the value of 0 for y to find the x-intercept. Plot the points and graph the line.

Graph by using the slope intercept method.

First place the function in slope intercept form, $y=mx+b$. Go to the y-intercept and plot the point. from there, use the slope. If the slope is positive, go up and Right. If the slope is negative, go up and left. Find the 2nd point and graph the line.

This last question was asked because it will sometimes be necessary to graph a function using one particular method. If that function is graphed using a different method, the problem will me marked incorrect because the directions were not followed.

Checking Progress

You have now completed the "Linear Functions" section of the workbook. The following is a checklist so that you may check your progress. Check off each of the objectives you have accomplished.

The student should be able to…

- ☐ ***Identify whether a relation determines a function.***
- ☐ ***Verify a point exists on a line.***
- ☐ ***Find the slope of a line.***
- ☐ ***Write the equation of a line in standard form.***
- ☐ ***Find the equation of a line given the slope and y intercept.***
- ☐ ***Find the equation of a line given a point on the line and the slope of the line.***
- ☐ ***Find the equation of a line given two points on the line***
- ☐ ***Find the slope of parallel and perpendicular lines.***
- ☐ ***Find the distance between two points on a line segment.***
- ☐ ***Find the midpoint of a line segment.***
- ☐ ***Find the x and y intercepts of a line.***
- ☐ ***Graph a line using a table.***
- ☐ ***Graph a line by finding intercepts.***
- ☐ ***Graph a line using the slope intercept method.***
- ☐ ***Solve linear systems of equations in two variables by graphing.***
- ☐ ***Solve linear systems of equations in two variables using the substitution method.***
- ☐ ***Solve linear systems of equations in two variables using the linear combination method.***
- ☐ ***Determine whether or not an ordered pair is a solution to a system.***
- ☐ ***Graph linear inequalities in two variables.***
- ☐ ***Graph systems of inequalities.***
- ☐ ***Solve systems of equations in three variables.***
- ☐ ***Perform basic function operations including finding composite functions***

POLYNOMIALS

Objectives

The following is a list of objectives for this section of the workbook.

By the time the student is finished with this section of the workbook, he/she should be able to...

- ***Find the degree of a polynomial.***
- ***Add and subtract polynomial expressions.***
- ***Multiply polynomials using the foil method.***
- ***Multiply polynomials using the special product rules.***
- ***Raise a binomial to a power using Pascal's triangle.***
- ***Factor simple polynomials.***
- ***Factor by grouping.***
- ***Factor complex polynomials.***
- ***Solve polynomial equations by factoring.***
- ***Solve polynomial inequalities.***
- ***Express the solutions of polynomial inequalities using interval notation.***

Math Standards Addressed

The following state standards are addressed in this section of the workbook.

Algebra I

10.0 Students add, subtract, multiply, and divide monomials and polynomials. Students solve multi-step problems, including word problems, by using these techniques.

11.0 Students apply basic factoring techniques to second-and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

Algebra II

3.0 Students are adept at operations on polynomials, including long division.

4.0 Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.

20.0 Students know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers.

Degree of a Polynomial

How does finding the degree of a polynomial differ between a polynomial that has only one variable, versus one that has two or more?

If a polynomial has only one type of variable, the degree of the polynomial is the highest power if several varaiable are represent, you must find the sum of the exponents in each terms of the polynomial. The degree will be the largest sum.

Find the degree of each of the following polynomials.

A) $x^4 - 3x^3 + x - 7$

4th

B) $\frac{1}{2}x^2 + 5x^3 - 3x + 2$

3rd

C) $4x^2y - 3x + 2y^2$

3rd

D) $4x^3y^5 - 7x^2y + 9y^6$

8th

E) $x^6 - 3x^2y^7 + 4xy^{10}$

11th

F) $2x^3 - 3x^2 - 1$

3rd

Adding/Subtracting Polynomials

Perform the indicated operation for each of the following.

A) $2(x-1) - (3x+2) - 3(x+5)$

$2x - 2 - 3x - 2 - 3x - 15$

$-4x - 19$

B) $(x^4 - 5x^3 + 7x - 3) - (2x^4 + 4x^2 - 6x + 12)$

$x^4 - 5x^3 + 7x - 3 - 2x^4 - 4x^2 + 6x - 12$

$-x^4 - 5x^3 - 4x^2 + 13x - 15$

C) $2x - 3[x - 2[x - (x-3)]]$

$2x - 3[x - 2[x - x + 3]]$

$2x - 3[x - 2[3]]$

$2x - 3[x - 6]$

$2x - 3x + 18$

$-x + 18$

D) $2y - 3y[4 - 2(3y - 5)]$

$2y - 3y[4 - 6y + 10]$

$2y - 3y[14 - 6y]$

$2y - 42y + 18y^2$

$18y^2 - 40y$

Laws of Exponents

Complete each law of exponents.

$x^a \cdot x^b = x^{a+b}$

$x^a \div x^b = x^{a-b}$

$\left(x^a\right)^b = x^{ab}$

$\left(x^a y^b\right)^c = x^{ac} y^{bc}$

$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

$x^{-1} = \frac{1}{x}$

$x^0 = 1$

$\left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a = \frac{y^a}{x^a}$

$|a^2| = |a|^2 = a^2$

Why do you need to know the laws of exponents in order to perform operations with polynomials?

when multiplying or dividing polynomials, you need to know what to do with the exponents.

Prove anything to the zero power is one. (use an example)

$x^7 \div x^7 = x^{7-7} = x^0 = 1$

anything divided by itself is 1.

What is the difference between $2x^{-1}$ and $(2x)^{-1}$?

$2x^{-1}$ $(2x)^{-1}$

$\frac{2}{x}$ $\frac{1}{2x}$

Multiplying Polynomials

Perform the indicated operation for each of the following.

A) $(x+5)(x-6)$

x^2-x-30

B) $(x+5)(x-5)$

x^2-25

C) $(x+7)(x+2)$

$x^2+9x+14$

D) $(x+4)(x+4)$

$x^2+8x+16$

E) $3(x+5)(x-2)$

$3(x^2+3x-10)$

$3x^2+9x-30$

F) $x(2x+5)(3x-1)$

$x(6x^2-2x+15x-5)$

$x(6x^2+13x-5)$

$6x^3+13x^2-5x$

G) $(4x-3)(3x+5)$

$12x^2+20x-9x-15$

$12x^2+11x-15$

H) $(3x+7y)(2x-5y)$

$6x^2-15xy+14xy-35y^2$

$6x^2-xy-35y^2$

I) $2(3x-1)(5x+2)$

$2(15x^2+6x-5x-2)$

$2(15x^2+x-2)$

$30x^2+2x-4$

J) $3x^2(2x+1)(2x-1)$

$3x^2(4x^2-1)$

$12x^4-3x^2$

K) $(2x-3y)(x+2y)$

$2x^2+4xy-3xy-6y^2$

$2x^2+xy-6y^2$

L) $x^2(6x-1)(3x-5)$

$x^2(18x^2-30x-3x+5)$

$x^2(18x^2-33x+5)$

$18x^4-33x^3+5x^2$

M) $(x-3)(x+5)(x-1)$

$(x-3)(x^2+4x-5)$

x^3+4x^2-5x

$-3x^2-12x+15$

$x^3+x^2-17x+15$

N) $3(2x+1)(x+6)(x+2)$

$(6x+3)(x^2+8x+12)$

$6x^3+48x^2+72x$

$3x^2+24x+36$

$6x^3+51x^2+96x+36$

O) $(x-2)^3$

$(x-2)(x^2-4x+4)$

x^3-4x^2+4x

$-2x^2+8x-8$

$x^3-6x^2+12x-8$

P) $(x-2y)^2(x+2y)^2$

$(x-2y)(x+2y)(x-2y)(x+2y)$

$(x^2-4y^2)(x^2-4y^2)$

$x^4-8x^2y^2+16y^4$

Q) $(3x^n-5)(2x^n+3)$

$6x^{2n}+9x^n-10x^n-15$

$6x^{2n}-x^n-15$

R) $(x^{2n}-1)(x^n-7)$

$x^{3n}-7x^{2n}-x^n+7$

Special Product Rules

Complete each of the following special product formulas.

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a+b)(a-b) = a^2 - b^2$

Use the special product rules to perform the indicated operation

A) $(2x+5)^2$

$(2x)^2 + 2(2x)(5) + (5)^2$

$4x^2 + 20x + 25$

B) $(3x-2)^2$

$(3x)^2 - 2(3x)(2) + (2)^2$

$9x^2 - 12x + 4$

C) $(2a-3b)^2$

$(2a)^2 - 2(2a)(3b) + (3b)^2$

$4a^2 - 12ab + 9b^2$

D) $(x+5)^2(x-4)^2$

$(x^2 + 10x + 25)(x^2 - 8x + 16)$

$x^4 - 8x^3 + 16x^2$

$10x^3 - 80x^2 + 160x$

$25x^2 - 200x + 400$

$x^4 + 2x^3 - 39x^2 - 40x + 400$

E) $(5x+6)(5x-6)$

$25x^2 - 36$

F) $2(3x+2)(3x-2)$

$2(9x^2 - 4)$

$18x^2 - 8$

G) $(x+2)^3$

$(x)^3 + 3(x)^2(2) + 3(x)(2)^2 + (2)^3$

$x^3 + 6x^2 + 12x + 8$

H) $(x+2)^2(x-3)^3$

$(x^2 + 4x + 4)(x^3 - 9x^2 + 27x - 27)$

$x^5 - 9x^4 + 27x^3 - 27x^2$

$4x^4 - 36x^3 + 108x^2 - 108x$

$4x^3 - 36x^2 + 108x - 108$

$x^5 - 5x^4 - 5x^3 + 45x^2 - 108$

I) $(4x-3)^2$

$(4x)^2 - 2(4x)(3) + (3)^2$

$16x^2 - 24x + 9$

J) $[(x+y)+1][(x+y)-1]$

$(x+y)^2-1$

$x^2+2xy+y^2-1$

K) $(2x-1)(x+3)+3(x+3)$

$2x^2+5x-3+3x+9$

$2x^2+8x+6$

L) $[(x-3)+y]^2$

$(x-3)^2+2(x-3)(y)+y^2$

$x^2-6x+9+2xy-6y+y^2$

M) $(x^2+3x+2)(2x^2-x+4)$

$$\begin{array}{r} 2x^4-x^3+4x^2 \\ 6x^3-3x^2+12x \\ 4x^2-2x+8 \\ \hline 2x^4+5x^3+5x^2+10x+8 \end{array}$$

N) $[(x+y)+5][(x+y)-5]$

$(x+y)^2-25$

$x^2+2xy+y^2-25$

O) $(2x^n-3)^3$

$(2x^n)^3-3(2x^n)^2(3)+3(2x^n)(3)^2-(3)^3$

$8x^{3n}-9(4x^{2n})+27(2x^n)-27$

$8x^{3n}-36x^{2n}+54x^n-27$

Simplifying Complex Polynomials

Simplify each of the following polynomial expressions.

A) $(3x-2y)^2-(2x-3y)(2x+3y)$

$9x^2-12xy+4y^2-(4x^2-9y^2)$

$9x^2-12xy+4y^2-4x^2+9y^2$

$5x^2-12xy+13y^2$

B) $(2x-y)(x+y)-(2x-y)(x-y)$

$(2x-y)[(x+y)-(x-y)]$

$(2x-y)[x+y-x+y]$

$(2x-y)(2y)$

$4xy-2y^2$

C) $(2x+3)(2x-1)+2(2x-3)(x+5)-(3x+2)(2x+1)$

$(4x^2-2x+6x-3)+2(2x^2+10x\ 3x-15)-(6x^2+7x+2)$

$4x^2+4x-3+4x^2+14x-30-6x^2-7x-2$

$2x^2+11x-35$

D) $[(x+2)(2x^2-3)]^2$

$(x+2)^2(2x^2-3)^2$

$(x^2+4x+4)(4x^4-12x^2+9)$

$$\begin{array}{r} 4x^6-12x^4+9x^2 \\ 16x^5-48x^3+36x \\ 16x^4-48x^2+36 \\ \hline 4x^6+16x^5+4x^4-48x^3-39x^2+36x+36 \end{array}$$

E) $(x-1)[(2x-3y)(2x+3y)]^2$

$(x-1)[4x^2-9y^2]^2$

$(x-1)(16x^4-72x^2y^2+81y^4)$

$$\begin{array}{r} 16x^5-72x^3y^2+81xy^4 \\ -16x^4+72x^2y^2-81y^4 \\ \hline 16x^5-16x^4-72x^3y^2+72x^2y^2+81xy^4-81y^4 \end{array}$$

F) $(x-2)^3(x+1)^2$

$[x^3-3(x)^2(2)+3(x)(2)^2-(2)^3](x^2+2x+1)$

$(x^3-6x^2+12x-8)(x^2+2x+1)$

$$\begin{array}{r} x^5+2x^4+x^3 \\ -6x^4-12x^3-6x^2 \\ 12x^3+24x^2+12x \\ -8x^2-16x-8 \\ \hline x^5-4x^4+x^3+10x^2-4x-8 \end{array}$$

G) $3\left(5x^2-2\right)-\left(-8.1x^3-14.7x^2-17\right)$

$15x^2-6+8.1x^3+14.7x^2+17$

$8.1x^3+29.7x^2+11$

H) $\left(2x^2+1\right)-\left(x-1\right)^2$

$(2x^2+1)-(x^2-2x+1)$

$2x^2+1-x^2+2x-1$

x^2+2x

I) $2\left(3x+1\right)^2-2\left(2x-1\right)^2$

$2(9x^2+6x+1)-2(4x^2-4x+1)$

$18x^2+12x+2-8x^2+8x-2$

$10x^2+20x$

J) $\left[\left(x+y\right)+3\right]^2\left[\left(x+y\right)-3\right]^2$

K) $6\left(x-2\right)^2\left(4x+3\right)-6\left(x-2\right)\left(4x+3\right)^2$

$6(x-2)(4x+3)[(x-2)-(4x+3)]$

$6(4x^2+3x-8x-6)(x-2-4x-3)$

$6(4x^2-5x-6)(3x+5)$

$-6\left(\begin{array}{l} 12x^3+20x^2 \\ \quad -15x^2-25x \\ \quad\quad -18x-30 \end{array}\right)$

$-6(12x^3+5x^2-43x-30$

$-72x^3-30x^2+258x+180$

L) $\left(3x-2\right)\left(x-8\right)+4\left(3x-2\right)\left(x+6\right)$

$(3x-2)[(x-8)+4(x+6)]$

$(3x-2)[x-8+4x+24]$

$(3x-2)(6x+16)$

$15x^2+48x$

$-10x-32$

$15x^2+38x-32$

M) $3\left(2x^4-7x^3+x^2-x\right)-3\left(5x^2+2x-1\right)+2x-3\left[x-2\left[x-\left(2x-3\right)\right]\right]+2\left[x-5\left(2x-1\right)\right]$

$6x^4-21x^3+3x^2-3x-15x^2-6x+3+2x-3[x-2[3-x]]+2[x-10x+5]$

$6x^4-21x^3-12x^2-7x+3-3(x-6+2x)+2(5-9x)$

$6x^4-21x^3-12x^2-7x+3-3(3x-6)+10-18x$

$6x^4-21x^3-12x^2-25x+13-9x+18$

$6x^4-21x^3-12x^2-34x+31$

Using Pascal's Triangle

Use Pascal's triangle to multiply each of the following.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

A) $(x+y)^4$

$x^4+4x^3y+6x^2y^2+4xy^3+y^4$

B) $(x+y)^5$

$x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$

C) $(x-y)^4$

$x^4-4x^3y+6x^2y^2-4xy^3+y^4$

D) $(x+2)^5$

$x^5+5x^4(2)+10x^3(2)^2+10x^2(2)^3+5x(2)^4+(2)^5$

$x^5+10x^4+40x^3+80x^2+80x+32$

E) $(x-3)^6$

$x^6-6x^5(3)+15x^4(3)^2-20x^3(3)^3+15x^2(3)^4-6x(3)^5+(3)^6$

$x^6-18x^5+135x^4-540x^3+1215x^2-1458x+729$

F) $(3x-2)^3$

$(3x)^3-3(3x)^2(2)+3(3x)(2)^2-(2)^3$

$27x^3-54x^2+36x-8$

G) $(2x+1)^6$

$(2x)^6+6(2x)^5(1)+15(2x)^4(1)^2+20(2x)^3(1)^3+15(2x)^2(1)^4+6(2x)(1)^5+(1)^6$

$64x^6+6(32x^5)+15(16x^4)+20(8x^3)+15(4x^2)+12x+1$

$64x^6+192x^5+240x^4+160x^3+60x^2+12x+1$

H) $(2x-3y)^5$

$(2x)^5-5(2x)^4(3y)+10(2x)^3(3y)^2-10(2x)^2(3y)^3+5(2x)(3y)^4-(3y)^5$

$32x^5-15y(16x^4)+10(72x^3y^2)-10(108x^2y^3)+10x(81y^4)-243y^5$

$32x^5-240x^4y+720x^3y^2-1080x^2y^3+810xy^4-243y^5$

I) $\left(x-\frac{1}{2}\right)^4$

$x^4-4x^3(\frac{1}{2})+6x^2(\frac{1}{2})^2-4x(\frac{1}{2})^3+(\frac{1}{2})^4$

$x^4-2x^3+6x^2(\frac{1}{4})-4x(\frac{1}{8})+\frac{1}{16}$

$x^4-2x^3+\frac{3}{2}x^2-\frac{1}{2}x+\frac{1}{16}$

J) $\left(\frac{1}{2}x+\frac{1}{3}\right)^3$

$(\frac{1}{2}x)^3+3(\frac{1}{2}x)^2(\frac{1}{3})+3(\frac{1}{2}x)(\frac{1}{3})^2+(\frac{1}{3})^3$

$\frac{1}{8}x^3+\frac{1}{4}x^2+(\frac{3}{2}x)(\frac{1}{9})+\frac{1}{27}$

$\frac{1}{8}x^3+\frac{1}{4}x^2+\frac{1}{6}x+\frac{1}{27}$

Factoring Simple Polynomials

Factor each of the following polynomials.

A) $x^2-3x-28$

$(x+4)(x-7)$

B) x^2-5x+6

$(x-2)(x-3)$

C) x^2+6x+5

$(x+1)(x+5)$

D) $x^2+8x+16$

$(x+4)(x+4)$

E) $x^2+9x-22$

$(x-2)(x+11)$

F) x^2-25

$(x+5)(x-5)$

G) x^2-9

$(x+3)(x-3)$

H) $2x^2+7x+6$

$(2x+3)(x+2)$

I) $2x^2+13x+21$

$(2x+7)(x+3)$

J) $3x^2 - 10x + 8$

$(3x-4)(x-2)$

K) $2x^2 - 3x - 35$

$(2x+7)(x-5)$

L) $5x^{2n} + 13x^n + 6$

$(5x^7+3)(x^7+2)$

M) $2x^{2n} + 2x^n - 24$

$2(x^{2n}+x^n-12)$

$2(x^n+4)(x^n-3)$

N) $20x^2 + 37x + 8$

$20x^2+5x+32x+8$

$5x(4x+1)+8(4x+1)$

$(5x+8)(4x+1)$

O) $12x^2 - 7x - 10$

$12x^2-15x+8x-10$

$3x(4x-5)+2(4x-5)$

$(3x+2)(4x-5)$

P) $3x^2 + 13x + 12$

$(3x+4)(x+3)$

Q) $3x^2 + 28x + 9$

$(3x+1)(x+9)$

R) $3x^2 - 11x - 42$

$(3x+7)(x-6)$

Factoring Formulas

Complete each of the following formulas.

$a^2 + 2ab + b^2 = (a+b)^2$

$a^2 - 2ab + b^2 = (a-b)^2$

$a^2 - b^2 = (a+b)(a-b)$

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

What is the most common mistake people make when working with the sum or difference of cube formulas? What is most likely the reason for this mistake?

The formula for sum of cubes is $(a+b)(a^2-ab+b^2)$. Difference of cubes is $(a-b)(a^2+ab+b^2)$. The most common mistake occurs at the second term of the trinomial.

Factor each of the following using the factoring formulas.

A) $x^2 - 36$

$(x+6)(x-6)$

B) $9x^2 - 36$

$9(x^2-4)$

$9(x+2)(x-2)$

C) $x^2 + 12x + 36$

$(x+6)^2$

D) $x^2 - 10x + 25$

$(x-5)^2$

E) $x^2 + x + \frac{1}{4}$

$(x+\frac{1}{2})^2$

F) $25x^2 - 10x + 1$

$(5x-1)^2$

G) $x^3 + 125$

$(x+5)(x^2-5x+25)$

H) $x^3 - 27$

$(x-3)(x^2+3x+9)$

I) $2x^3 + 54$

$2(x^3+27)$

$2(x+3)(x^2-3x+9)$

Factoring by Grouping

Factoring by grouping is commonly used when there are more than three terms in the polynomial. It is possible to group more than once in any given problem. Also, be aware that the terms do not necessarily need to be grouped evenly. This means a polynomial with 4 terms could be grouped with the first 3 terms, then the last. Pay attention to the coefficients in the polynomials. Look for patterns. There are clues telling you grouping must be used.

Factor each of the following polynomials by grouping

A) $x^3 - x^2 + 2x - 2$

$x^2(x-1)+2(x-1)$

$(x^2+2)(x-1)$

B) $x^3 + 5x^2 - 5x - 25$

$x^2(x+5)-5(x+5)$

$(x^2-5)(x+5)$

C) $x^2 - ax + cx - ac$

$x(x-a)+c(x-a)$

$(x-a)(x+c)$

D) $5x^3 - 10x^2 + 3x - 6$

$5x^2(x-2)+3(x-2)$

$(5x^2+3)(x-2)$

E) $x^3 - 4x^2 + 6x - 24$

$x^2(x-4)+6(x-4)$

$(x^2+6)(x-4)$

F) $x^2 + 2xy + y^2 - z^2$

$(x+y)^2-z^2$

$(x+y+z)(x+y-z)$

G) $10x^3 + 8x^2 + 15xy + 12y$

$2x^2(5x+4)+3y(5x+4)$

$(2x^2+3y)(5x+4)$

H) $2x^3 - 10x^2 + 4x - 20$

$2x^2(x-5)+4(x-5)$

$(2x^2+4)(x-5)$

$2(x^2+2)(x-5)$

I) $3x^2 + xy - 3xz - yz$

$x(3x+y)-z(3x+y)$

$(x-z)(3x+y)$

When factoring by grouping do you need to factor the problems in the exact order they were given to you?

No, you can change the order if needed.

Why does factoring by grouping work? (Think Algebraic Properties)

Because the foil method is really the distributive property. Factoring by grouping works because of the distributive property.

Factoring Complex Polynomials

The following questions were designed to give you a hard time ☺! You will need to use all of your knowledge on factoring for the following questions. Remember to always look at the problem to make sure there is nothing else you can do. Pay particular attention to any factor that is greater than a first degree polynomial.

Factor each of the following polynomials.

A) $x^2 - y^2 + 2yz - z^2$

$x^2 - (y^2 - 2yz + z^2)$

$x^2 - (y-z)^2$

$[x + (y-z)][x - (y-z)]$

$(x+y-z)(x-y+z)$

B) $3x^4 - 243$

$3(x^4 - 81)$

$3(x^2+9)(x^2-9)$

$3(x^2+9)(x+3)(x-3)$

C) $2x^3 - 16$

$2(x^3-8)$

$2(x-2)(x^2+2x+4)$

D) $6a^3 + 2a^2b + 21a^2 + 7ab + 15a + 5b$

$2a^2(3a+b) + 7a(3a+b) + 5(3a+b)$

$(2a^2+7a+5)(3a+b)$

$(2a+5)(a+1)(3a+b)$

E) $20a^3 + 5a^2b - 68a^2 - 17ab + 24a + 6b$

$5a^2(4a+b) - 17a(4a+b) + 6(4a+b)$

$(5a^2 - 17a + 6)(4a+b)$

$(5a-2)(a-3)(4a+b)$

F) $8x^2 + 16xy + 8y^2 + 18x + 18y + 9$

$8(x^2+2xy+y^2) + 18(x+y) + 9$

$8(x+y)^2 + 18(x+y) + 9$

let $a = x+y$ $\quad 8A^2 + 18A + 9$

$8A^2 + 12A + 6A + 9$

$\cancel{4A}4A(2A+3) + 3(2A+3)$

$(4A+3)(2A+3)$

$[4(x+y)+3][2(x+y)+3]$

$(4x+4y+3)(2x+2y+3$

G) $15x^2 + 30xy + 15y^2 - 7x - 7y - 2$

$15(x^2+2xy+y^2) - 7(x+y) - 2$

$15(x+y)^2 - 7(x+y) - 2$

let $A = x+y$ $\quad 15A^2 - 7A - 2$

$(3A-2)(5A+1)$

$[3(x+y) \; 2][5(x+y)+1]$

$(3x+3y-2)(5x+5y+1)$

H) $x^2+2xy+y^2-16z^2$

$(x+y)^2-16z^2$

$(x+y+4z)(x+y-4z)$

I) $x^2+2xy+y^2+2x+2y+1$

$(x+y)^2+2(x+y)+1$

let $A=x+y$ $\quad A^2+2A+1$

$(A+1)^2$

$[(x+y)+1]^2$

$(x+y+1)^2$

J) $2x^4-58x^2+200$

$2(x^4-29x^2+100)$

$2(x^2-4)(x^2-25)$

$2(x+2)(x-2)(x+5)(x-5)$

K) $x^5-4x^3-x^2+4$

$x^3(x^2-4)-1(x^2-4)$

$(x^3-1)(x^2-4)$

$(x-1)(x^2+x+1)(x+2)(x-2)$

L) x^6-64

$(x^3+8)(x^3-8)$

$(x+2)(x^2-2x+4)(x-2)(x^2+2x+4)$

M) $6x^2+2xy-3xz-yz$

$2x(3x+y)-z(3x+y)$

$(2x-z)(3x+y)$

N) $x^2-z^2+y^2-2xy$

$x^2-2xy+y^2-z^2$

$(x-y)^2-z^2$

$(x-y+z)(x-y-z)$

O) $16x^2-y^2-2yz-z^2$

$16x^2-(y^2+2yz+z^2)$

$16x^2-(y+z)^2$

$[4x+(y+z)][4x-(y+z)]$

$(4x+y+z)(4x-y-z)$

P) $10x^3 - 25x^2 - 26x^2y + 65xy - 12xy^2 + 30y^2$

$5x^2(2x-5) - 13xy(2x-5) - 6y^2(2x-5)$

$(5x^2 - 13xy - 6y^2)(2x-5)$

$(5x+2y)(x-3y)(2x-5)$

Q) $7(3x+2)^2(1-x)^2 + (3x+2)(1-x)^3$

$(3x+2)(1-x)^2\,[7(3x+2)+(1-x)]$

$(3x+2)(1-x)^2(21x+14+1-x)$

$(3x+2)(x-1)^2(20x+15)$

$5(3x+2)(x-1)^2(4x+3)$

R) $6x^2 + 12xy + 6y^2 - 7x - 7y - 5$

$6(x^2+2xy+y^2) - 7(x+y) - 5$

$6(x+y)^2 - 7(x+y) - 5$

Let $A = x+y$ $\quad 6A^2 - 7A - 5$

$6A^2 + 3A - 10A - 5$

$3A(2A+1) - 5(2A+1)$

$(3A-5)(2A+1)$

$[3(x+y)-5][2(x+y)+1]$

$(3x+3y-5)(2x+2y+1)$

S) $x^5 + x^2 - 4x^3 - 4$

$x^2(x^3+1) - 4(x^3+1)$

$(x^2-4)(x^3+1)$

$(x+2)(x-2)(x+1)(x^2-x+1)$

T) $16x^4 - 81y^4$

$(4x^2+9y^2)(4x^2-9y^2)$

$(4x^2+9y^2)(2x+3y)(2x-3y)$

U) $5x^3 + 40$

$5(x^3+8)$

$5(x+2)(x^2-2x+4)$

V) $x^{4n} - 29x^{2n} + 100$

$(x^{2n}-4)(x^{2n}-25)$

$(x^n+2)(x^n-2)(x^n+5)(x^n-5)$

W) $x^{6n} - 64$

$(x^{3n}+8)(x^{3n}-8)$

$(x^n+2)(x^{2n}-2x^n+4)(x^n-2)(x^{2n}+2x^n+4)$

X) $27x^6y^3 + 64z^3$

$(3x^2y)^3 + (4z)^3$

$(3x^2y+4z)(9x^4y^2 - 12x^2yz + 16z^2)$

Y) Factor the expression $x^6 - 1$ by using the difference of cubes first.

$x^6 - 1$

$(x^2)^3 - 1^3$

$(x^2-1)(x^4+x^2+1)$

$(x+1)(x-1)(x^4+x^2+1)$

Z) Factor the expression $x^6 - 1$ by using the difference of squares first.

$x^6 - 1$

$(x^3)^2 - 1$

$(x^3+1)(x^3-1)$

$(x+1)(x^2-x+1)(x-1)(x^2+x-1)$

What do you notice about the difference between your answers to Y and Z?

In answeR y, there is the polynomial (x^4+x^2+1)

in answer z, that polynomial was further factored to

$(x^2-x+1)(x^2+x+1)$

What conclusions can you make regarding any binomial where each term has a power that is some multiple of 6?

Always factor using difference of squares first

Why isn't this the case for $x^6 + 1$?

This is the sum of squares, you cannot factor, so you must use sum of rules to factor.

Solving Polynomial Equations by Factoring

Solve each of the following polynomial equations by factoring.

A) $2x(x+3)(x-5)=0$

$2x=0 \quad x+3=0 \quad x-6=0$

$x=\{-3,0,5\}$

B) $x^2-2x-48=0$

$(x+6)(x-8)=0$

$x=\{-6,8\}$

C) $x^2+x-6=6$

$-6\quad -6$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=\{-4,3\}$

D) $x^4-6x^3+9x^2=0$

$x^2(x^2-6x+9)=0$

$x^2(x-3)^2=0$

$x^2=0 \quad (x-3)^2=0$

$x=\{0,3\}$

E) $x^3+3x^2-4x-12=0$

$x^2(x+3)-4(x+3)=0$

$(x^2-4)(x+3)=0$

$(x+2)(x-2)(x+3)=0$

$-2 \quad 2 \quad -3$

$x=\{-3,\pm 2\}$

F) $3x^2-18x+27=0$

$3(x^2-6x+9)=0$

$3(x-3)^2=0$

$3\neq 0 \quad (x-3)^2=0$

$x=\{3\}$

G) $x^3+6x^2=27x$

$x^3+6x^3-27x=0$

$x(x^2+6x-27)=0$

$x(x+9)(x-3)=0$

$x=0 \quad x+9=0 \quad x-3=0$

$x=\{-9,0,3\}$

H) $x^2-12=4x$

$x^2-4x-12=0$

$(x+2)(x-6)=0$

$-2 \quad 6$

$x=\{-2,6\}$

I) $x^4-5x^2+4=0$

$(x^2-1)(x^2-4)=0$

$(x+1)(x-1)(x+2)(x-2)=0$

$-1 \quad 1 \quad -2 \quad 2$

$x=\{\pm 1,\pm 2\}$

J) $(x^2-1)(x^2+3x+2)=0$

$(x+1)(x-1)(x+1)(x+2)=0$

$-1 \quad 1 \quad -1 \quad -2$

$x=\{\pm 1,-2\}$

K) $x^6-10x^4+9x^2=0$

$x^2(x^4-10x^2+9)=0$

$x^2(x^2-9)(x^2-1)=0$

$x^2(x+3)(x-3)(x+1)(x-1)=0$

$0 \quad -3 \quad 3 \quad -1 \quad 1$

$x=\{0,\pm 1,\pm 3\}$

L) $4x^2-4x=-1$

$4x^2-4x+1=0$

$\sqrt{(2x-1)^2}=\sqrt{0}$

$2x-1=0$

$2x=1$

$x=\frac{1}{2}$

$x=\{\frac{1}{2}\}$

M) $x^4-13x^2+36=0$

$(x^2-4)(x^2-9)=0$

$(x+2)(x-2)(x+3)(x-3)=0$

$-2 \quad 2 \quad -3 \quad 3$

$x=\{\pm 2,\pm 3\}$

N) $3x(x+1)=4(x+1)$

$3x^2+3x=4x+4$

$3x^2-x-4=0$

$(3x-4)(x+1)=0$

$3x-9=0 \quad x+1=0$

$3x=4 \quad x=-1$

$x=\frac{4}{3}$

$x=\{\frac{4}{3},-1\}$

O) $3x^2-10x+3=0$

$(3x-1)(x-3)=0$

$3x-1=0 \quad x-3=0$

$\frac{3x}{3}=\frac{1}{3} \quad x=3$

$x=\{\frac{1}{3},3\}$

Solving Polynomial Inequalities

When solving polynomial inequalities, first get everything to the left of the inequality symbol. Then, factor the polynomial to find all critical points. Test each interval on the number line, and shade in the appropriate region. Express the solution using interval notation.

Example

$$x^3 - x^2 - 6x \ge 0$$

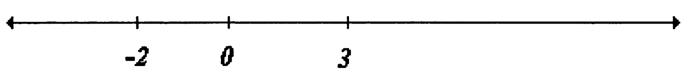
Here is the original problem.

$$x(x+2)(x-3) \ge 0$$

Completely factor the polynomial to find all critical points.

Critical Points are -2, 0 and 3

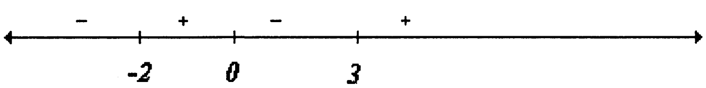
The critical points are the zeros of the polynomial.



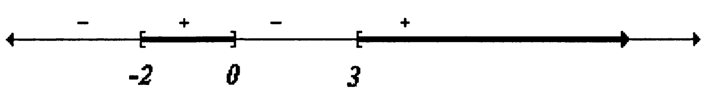
Plot each critical point on a number line, and test each interval. To test the interval, use numbers in between the critical points that were plotted.

$$-3 \Rightarrow (-3)(-3+2)(-3-3) \Rightarrow (-3)(-1)(-6) = -$$
$$-1 \Rightarrow (-1)(-1+2)(-1-3) \Rightarrow (-1)(\ 1)(-4) = +$$
$$2 \Rightarrow (\ 2)(\ 2+2)(\ 2-3) \Rightarrow (\ 2)(\ 3)(-1) = -$$
$$5 \Rightarrow (\ 5)(\ 5+2)(\ 5-3) \Rightarrow (\ 5)(\ 7)(\ 2) = +$$

Testing each interval gives us positive or negative values. These are the only thing needed. We need to determine in which interval the value of this inequality is greater than or equal to zero.



Here, the positive and negative intervals are labeled.



Shade in the appropriate solution intervals.

Solution: $[-2,0] \cup [3,\infty)$

Finally, write the solution in interval notation.

Solve each of the following polynomial inequalities. Be sure to graph each solution on the number line, and write your solution using interval notation.

A) $x^2 \le 9$

$x^2 - 9 \le 0$

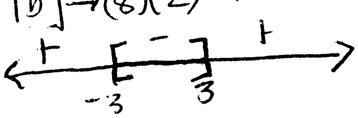
$(x+3)(x-3) \le 0$

C.P. $-3, 3$

$\boxed{-5} \to (-2)(-8) = +$

$\boxed{0} \to (3)(-3) = -$

$\boxed{5} \to (8)(2) = +$



$[-3,3]$

B) $x^2 - x - 6 > 0$

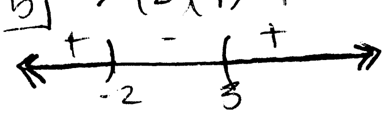
$(x-3)(x+2) > 0$

C.P. $3, -2$

$\boxed{-4} \to (-7)(-2) = +$

$\boxed{0} \to (-3)(2) = -$

$\boxed{5} \to (2)(7) = +$



$(-\infty,-2) \cup (3,\infty)$

C) $x^2 + 9 \le 6x$

$x^2 - 6x + 9 \le 0$

$(x-3)^2 \le 0$

C.P. 3

$0 \to (-3)^2 = +$

$4 \to (1)^2 = +$

$(-\infty,\infty)$

D) $(x^2+x-2)(x^2-4x+4)\le 0$

$(x+2)(x-1)(x-2)^2\le 0$

C.P. $-2, 1, 2$

$-3\to(-1)(-4)(-5)(-5)=+$

$0\to(2)(-1)(-2)(-2)=-$

$1.5\to(3.5)(.5)(-.5)(-.5)=+$

$4\to(6)(3)(2)(2)=+$

$+\ \ [-2\ \ -\ \ 1]\ \ +\ \ 2\ \ +$

$[-2,1]$

E) $4x(x+1)\ge 3$

$4x^2+4x-3\ge 0$

$(2x+3)(2x-1)\ge 0$

C.P. $-3/2\ \ +1/2$

$-5\to(-13)(-9)=+$

$0\to(-3)(1)=-$

$10\to(17)(4)=+$

$+\ \ -3/2]\ \ -\ \ [1/2\ \ +$

$(-\infty,-\frac{3}{2}]\cup[\frac{1}{2},\infty)$

F) $x^4+100\ge 29x^2$

$x^4-29x^2+100\ge 0$

$(x^2-4)(x^2-25)\ge 0$

$(x+2)(x-2)(x+5)(x-5)\ge 0$

C.P. $-2\ \ 2\ \ -5\ \ 5$

$-10\to(-)(-)(-)(-)=+\qquad 3\to(+)(+)(+)(-)$

$-3\to(-)(-)(+)(-)=-\qquad 10\to(+)(+)(+)(+)$

$0=(+)(-)(+)(-)=+$

$+\ \ -5]\ \ -\ \ [-2\ \ +\ \ 2]\ \ -\ \ [5\ \ +$

G) $(x+2)^2<25$

$x^2+4x+4<25$

$x^2+4x-21<0$

$(x+7)(x-3)<0$

C.P. $-7\ \ 3$

$-10\to(-3)(-13)=+$

$0\to(7)(-3)=-$

$5\to(12)(2)=+$

$+\ \ (-7\ \ -\ \ 3)\ \ +$

$(-7,3)$

H) $(x-3)^2\ge 1$

$x^2-6x+9\ge 1$

$x^2-6x+8\ge 0$

$(x-2)(x-4)\ge 0$

C.P. $2\ \ 4$

$0\to(-2)(-4)=+$

$3\to(1)(-1)=-$

$5\to(3)(1)=+$

$+\ \ 2]\ \ -\ \ [4\ \ +$

$(-\infty,2]\cup[4,\infty)$

I) $x^3-4x\ge 0$

$x(x^2-4)\ge 0$

$x(x+2)(x-2)\ge 0$

C.P. $0,\ -2,\ 2$

$-3\to(-3)(-2)(-5)=-\qquad 3\to(3)(5)(1)=+$

$-1\to(-1)(1)(-3)=+$

$1\to(1)(3)(-3)=-$

$-\ \ [-2\ \ +\ \ 0]\ \ -\ \ [2\ \ +$

$[-2,0]\cup[2,\infty)$

$(-\infty,-5]\cup[-2,2]\cup[5,\infty)$

J) $x^2+4x+4\le 9$

$x^2+4x-5\le 0$

$(x-1)(x+5)\le 0$

C.P. $1\ \ -5$

$-10\to(-11)(-6)=+$

$0\to(-1)(5)=-$

$3\to(2)(8)=+$

$+\ \ [-5\ \ -\ \ 1]\ \ +$

$[-5,1]$

K) $x^2-6x+9<16$

$x^2-6x-7<0$

$(x-7)(x+1)<0$

C.P. $7\ \ -1$

$-5\to(-12)(-4)=+$

$0\to(-7)(1)=-$

$10\to(3)(11)=+$

$+\ \ (-1\ \ -\ \ 7)\ \ +$

$(-1,7)$

L) $x^2(x-3)\le 0$

C.P. $0, 3$

$-2\to(-2)(-2)(-5)=-$

$2\to(2)(2)(-1)=-$

$5\to(5)(5)(2)=+$

$-\ \ 0\ \ -\ \ 3]\ \ +$

$(-\infty,3]$

M) $\dfrac{x^2+x-12}{x+1}\ge 0$

$\dfrac{(x+4)(x-3)}{x+1}\ge 0$

C.P. $-4, -1, 3$

$-6\to\dfrac{(-2)(-9)}{-5}=-$

$-3\to\dfrac{(1)(-6)}{-2}=+$

$0\to\dfrac{(4)(-3)}{1}=-$

$5\to\dfrac{(9)(2)}{6}=+$

$-\ \ [-4\ \ +\ \ -1)\ \ -\ \ [3\ \ +$

$[-4,-1)\cup[3,\infty)$

N) $\dfrac{x^2+6x+9}{x^2-4}>0$

$\dfrac{(x+3)(x+3)}{(x+2)(x-2)}>0$

C.P. $-3, -2, 2$

$-5\to\dfrac{(-2)(-2)}{(-3)(-9)}=+\qquad 0\to\dfrac{(3)(3)}{(2)(-2)}=-$

$-2.5\to\dfrac{(0)(.5)}{(-5)(-4.5)}=+\qquad 5\to\dfrac{(8)(8)}{(7)(3)}=+$

$+\ \ -3\ \ +\ \ -2)\ \ -\ \ (2\ \ +$

$(-\infty,-2)\cup(2,\infty)$

O) $\dfrac{x^3+x^2-6x}{x-5} \le 0$

$\dfrac{x(x^2+x-6)}{x-5} \le 0$

$\dfrac{x(x+3)(x-2)}{x-5} \le 0$

C.P. $-3, 0, 2, 5$

$-5 \to \dfrac{(-5)(-2)(-7)}{-10} = +$

$-1 \to \dfrac{(-1)(2)(-3)}{-6} = -$

$1 \to \dfrac{(1)(4)(-1)}{-4} = +$

$3 \to \dfrac{(3)(6)(1)}{-2} = -$

$7 \to \dfrac{(7)(10)(5)}{2} = +$

$+ \quad [\; - \;] \quad + \quad [\; - \;) \quad +$

$-3 \quad 0 \quad 2 \quad 5$

$[-3,0] \cup [2,5)$

P) $\dfrac{x+6}{x+1} - 2 < 0$

$\dfrac{x+6}{x+1} + \dfrac{-2x-2}{x+1} < 0$

$\dfrac{-x+4}{x+1} < 0$

$\dfrac{-1(x-4)}{x+1} < 0$

C.P. $-1, 4$

$-5(-1)(-9) \dfrac{}{-4} = -$

$0 \to \dfrac{(-1)(-4)}{1} = +$

$5 \to \dfrac{(-1)(1)}{6} = -$

$- \quad) \quad + \quad (\quad -$

$-1 \quad 4$

$(-\infty,-1) \cup (4,\infty)$

Q) $\dfrac{x^3+1}{x+1} \ge 0$

$\dfrac{(x+1)(x^2-x+1)}{x+1} \ge 0$

C.P. -1

$-3 \to \dfrac{(-2)(13)}{-2} = +$

$5 \to \dfrac{(6)(21)}{6} = +$

$+ \quad)(\quad +$

-1

$(-\infty,-1) \cup (-1,\infty)$

R) $\dfrac{x^3-9x}{x^2-25} < 0$

$\dfrac{x(x+3)(x-3)}{(x+5)(x-5)} < 0$

C.P. $-5, -3, 0, 3, 5$

$-6 \to \dfrac{(-6)(-3)(-9)}{(-1)(-11)} = -$

$-4 \to \dfrac{(4)(-1)(-7)}{(1)(-9)} = +$

$-1 \to \dfrac{(-1)(2)(-4)}{(4)(-6)} = -$

$1 \to \dfrac{(1)(4)(-2)}{(6)(-4)} = +$

$4 \to \dfrac{(4)(7)(1)}{(9)(-1)} = -$

$6 \to \dfrac{(6)(9)(3)}{(11)(1)} = +$

$- \quad) \quad + \quad (\quad - \quad) \quad + \quad (\quad - \quad) \quad +$

$-5 \quad -3 \quad 0 \quad 3 \quad 5$

$(-\infty,-5) \cup (-3,0) \cup (3,5)$

Word Problems

Solve each of the following word problems.

A) The sum of a number and its square is 272. Find the number.

$x^2+x=272$

$x^2+x-272=0$

$(x+17)(x-16)=0$

$x=-17 \quad x=16$

$\boxed{-17 \text{ and } 16}$

B) Find two consecutive integers whose product is 156.

$x(x+1)=156$

$x^2+x-156=0$

$(x+13)(x-12)=0$

$-13 \quad 12$

$\boxed{\begin{array}{c}-13 \text{ and } -12 \\ \text{or} \\ 12 \text{ and } 13\end{array}}$

C) The sum of the squares of two consecutive even integers is 244. Find the integers.

$x^2+(x+2)^2=244$

$x^2+x^2+4x+4=244$

$2x^2+4x-240=0$

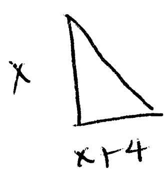
$2(x^2+2x-120)=0$

$2(x+12)(x-10)=0$

$x=-12 \quad x=10$

$\boxed{-12 \text{ and } -10 \text{ or } 10 \text{ and } 12}$

D) The base of a triangle is 4 inches longer than its height. Find the height of the triangle if it has an area of 30 in^2.



$A=\frac{1}{2}bh$

$A=\frac{1}{2}x(x+4)$

$30=\frac{1}{2}(x^2+4x)$

$60=x^2+4x$

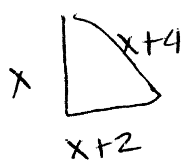
$x^2+4x-60=0$

$(x+10)(x-6)=0$

$-10 \quad 6$

height is 6 inches

E) The three sides of a right triangle are consecutive even integers. Find the integers.



$x^2+(x+2)^2=(x+4)^2$

$x^2+x^2+4x+4=x^2+8x+16$

$-x^2 \quad -8x-16 \quad -x^2-8x-16$

$x^2-4x-12=0$

$(x+2)(x-6)=0$

$-2 \quad 6$

6, 8 and 10

F) A rectangular field has a perimeter of 80 yards and an area of 396 square yards. Find the dimensions of the field.

$P=2(L+W)$ $\quad 80=2(L+W)$ $\quad A=L\cdot W$

$A=L\cdot W$ $\quad 40=L+W$ $\quad L(40-L)=396$

$W=40-L$ $\quad 40L-L^2=396$

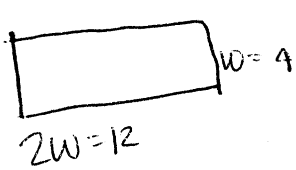
$L^2-40L+396=0$

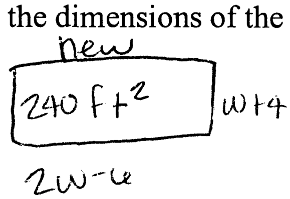
$(L-22)(L-18)=0$

$L=22 \quad L=18$

22 yds x 18 yds

G) A rectangle is twice as long as it is wide. If the width of the rectangle is increased by 4 feet, and the length decreased by 6 feet, the area of the new rectangle formed is 240 square feet. Find the dimensions of the original rectangle.





$A=L\cdot W$

$(2w-6)(w+4)=240$

$2w^2+8w-6w-24=240$

$2w^2+2w-264=0$

$2(w^2+w-132)=0$

$2(w+12)(w-11)=0$

$2\neq 0 \quad w\neq -12 \quad w=11$

11 ft by 22 ft

Checking Progress

You have now completed the "Polynomials" section of the workbook. The following is a checklist so that you may check your progress. Check off each of the objectives you have accomplished.

The student should be able to…

- ☐ ***Find the degree of a polynomial.***
- ☐ ***Add and subtract polynomial expressions.***
- ☐ ***Multiply polynomials using the foil method.***
- ☐ ***Multiply polynomials using the special product rules.***
- ☐ ***Raise a binomial to a power using Pascal's triangle.***
- ☐ ***Factor simple polynomials.***
- ☐ ***Factor by grouping.***
- ☐ ***Factor complex polynomials.***
- ☐ ***Solve polynomial equations by factoring.***
- ☐ ***Solve polynomial inequalities.***
- ☐ ***Express the solutions of polynomial inequalities using interval notation.***

RATIONAL EXPRESSIONS

Objectives

The following is a list of objectives for this section of the workbook.

By the time the student is finished with this section of the workbook, he/she should be able to…

- ***Simplify algebraic expressions using the laws of exponents.***
- ***Write any number in scientific notation.***
- ***Simplify rational expressions involving polynomials.***
- ***Completely factor a given polynomial.***
- ***Multiply and Divide rational expressions involving polynomials.***
- ***Add and subtract rational expressions involving polynomials by finding a common denominator.***
- ***Solve equations involving rational expressions.***

Math Standards Addressed

The following state standards are addressed in this section of the workbook.

Algebra I

10.0 Students add, subtract, multiply, and divide monomials and polynomials. Students solve multi-step problems, including word problems, by using these techniques.

11.0 Students apply basic factoring techniques to second-and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials.

12.0 Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.

13.0 Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques.

Algebra II

3.0 Students are adept at operations on polynomials, including long division.

4.0 Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.

7.0 Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.

Simplifying Rational Expressions

Simplify each of the following rational expressions.

A) $5x^0-(5x)^0$

$5(1)(-1)$

$5-1$

4

B) $\left(5x^2z^6\right)^3\left(5x^2z^6\right)^{-3}$

$5^3x^6z^{18}\cdot\frac{1}{(5x^2z^6)^3}$

$\frac{5^3x^6z^{18}}{5^3x^6z^{18}}$

1

C) $\frac{\left(xy^2z^3\right)^3}{\left(x^3yz^2\right)^2}$

$\frac{x^3y^6z^9}{x^6y^2z^4}$

$\frac{y^4z^5}{x^3}$

D) $\left[x^5\cdot x^{-7}\cdot x^3\cdot\left(x^{-2}\right)^{-3}\right]^{-1}$

$[x^{-2}\cdot x^3\cdot x^6]^{-1}$

$(x\cdot x^6)^{-1}$

$(x^7)^{-1}$

$\frac{1}{x^7}$

E) $\frac{5x}{4y^2}\left(\frac{2y}{x^2}\right)^3$

$\frac{5\cancel{x}}{\cancel{4}y^2}\cdot\frac{8y^3}{x^{5}}$

$\frac{10y}{x^5}$

F) $\frac{\left(2x^1\right)^{-2}}{2\left(y^{-1}\right)^{-3}}$

$\frac{1}{(2x)^2[2y^3]}$

$\frac{1}{(4x^2)(2y^3)}$

$\frac{1}{8x^2y^3}$

G) $\left(\frac{x^3y^{-3}}{2xy^2}\right)^{-2}\div\frac{10x^{-3}y}{3x^{-5}}\cdot\left(\frac{9x^2y}{5x^5y^{-6}}\right)^{-1}$

$\frac{2y^2}{3x^3}$

H) $\left(\frac{x}{y^{-1}}\right)^{-2}\left(\frac{x^3y^2}{z^{-2}}\right)^{-3}\left(\frac{x^5y}{z}\right)^2$

$(xy)^{-2}(x^3y^2z^2)^{-3}\left(\frac{x^{10}y^2}{z^2}\right)$

$\frac{x^{10}y^2}{x^{11}y^8z^8}$

$\frac{x^{5}y^2}{x^{11}y^8z^8}$

$\frac{1}{xy^6z^8}$

I) $\frac{x^{-2}}{y^2}\cdot\left(\frac{1}{xy}\right)^{-2}$

$\frac{1}{x^2y^2}\cdot\left(\frac{xy}{1}\right)^2$

$\frac{1}{x^2y^2}\cdot\frac{x^2y^2}{1}$

1

J) $\left(-2xy^4\right)^2\left(5xy^{-3}\right)^{-2}$

$4x^2y^8\cdot\left(\frac{5x}{y^3}\right)^{-2}$

$4x^2y^8\cdot\left(\frac{y^3}{5x}\right)^2$

$\frac{4x^2y^8}{1}\cdot\frac{y^6}{25x^2}$

$\frac{4y^{14}}{25}$

K) $2^{-1}+3^{-1}-4^{-1}$

$\frac{1}{2}+\frac{1}{3}-\frac{1}{4}$

$\frac{6}{12}+\frac{4}{12}-\frac{3}{12}$

$\frac{10}{12}-\frac{3}{12}$

$\frac{7}{12}$

L) $2x\left(5x^2y\right)^0$

$2x(1)$

$2x$

M) $\left[\left(2x^{-2}y^{3}\right)\left(3x^{5}y^{-2}\right)^{2}\right]^{-2}$

$\left[\left(\frac{2y^3}{x^2}\right)\left(\frac{3x^5}{y^2}\right)^2\right]^{-2}$

$\left[\frac{2y^3}{x^2}\cdot\frac{9x^{10}}{y^4}\right]^{-2}$

$\left[\frac{18x^8}{y}\right]^{-2}$

$\frac{y^2}{32yx^{16}}$

N) $\frac{12x^{3}y^{-4}}{4x^{-2}y}$

$\frac{3x^5}{y^5}$

O) $\left(\frac{3x^{2}}{y}\right)\left(\frac{3x^{-1}y}{2x^{3}y^{-2}}\right)^{2}$

$\left(\frac{3x^2}{y}\right)\left(\frac{3y^3}{2x^4}\right)^2$

$\frac{3x^2}{y}\cdot\frac{9y^6}{4x^8}$

$\frac{27y^5}{4x^6}$

P) $\left(\frac{x^{-3}y^{-2}}{x^{-1}y^{-3}}\right)^{-2}$

$\frac{x^6y^4}{x^2y^6}$

$\frac{x^4}{y^2}$

Q) $\left[\left(3x^{-2}y^{3}\right)^{-1}\right]^{-2}$

$\left(\frac{3y^3}{x^2}\right)^2$

$\frac{9y^6}{x4}$

R) $\left(-3x^{3}y^{2}\right)^{-4}$

$\frac{1}{(-3x^3y^2)^4}$

$\frac{1}{81x^{12}y^8}$

S) $\left(3x^{2}y^{-3}z\right)^{-3}\left(2^{-3}x^{-4}y^{2}\right)^{-2}$

$\left(\frac{3x^2z}{y^3}\right)^{-3}\left(\frac{y^2}{8x4}\right)^{-2}$

$\left(\frac{y^3}{3x^2z}\right)^3\left(\frac{8x4}{y^2}\right)^2$

$\frac{y^9}{27x^6z^3}\cdot\frac{64x^8}{y^4}$

$\frac{64x^2y^5}{27\cdot z^3}$

T) $\left[\left(\frac{x^{-3}y^{4}}{6x^{5}}\right)^{-1}\left(\frac{5xy^{5}}{12x^{-3}y}\right)\right]^{2}$

$\left[\left(\frac{y^4}{6x^8}\right)^{-1}\left(\frac{5x^4y^4}{12}\right)\right]^2$

$\left(\frac{\cancel{6}x^3}{\cancel{y^4}}\cdot\frac{5x4\cancel{y^4}}{12^2}\right)^2$

$\left(\frac{5x^{12}}{2}\right)^2$

$\frac{25x^2\ 4}{4}$

U) $\frac{32x^{3}y^{-2}}{5xy^{2}}\cdot\left(\frac{2xy^{-3}}{5x^{-2}}\right)^{-3}\div\frac{50x^{-7}y^{3}}{3x^{-2}y}$

$\frac{32x^2}{5y^4}\cdot\left(\frac{5y^3}{2x^3}\right)^3\cdot\left(\frac{3x^5}{50y^2}\right)$

$\frac{32x^2}{5y^4}\cdot\frac{125y^4}{8x^9}\cdot\frac{3x^5}{50y^2}$

$\frac{32\cdot125\cdot3\cdot x^7y^{12}}{5\cdot8\cdot50\cdot x^9\cdot y^6}$

$\frac{6y^3}{x^2}$

V) $\left(\frac{6x^{3}y^{-2}}{5x^{-5}}\right)^{-2}\cdot\left(\frac{4xy^{3}}{15x^{2}}\right)\div\left(\frac{4x^{-2}y}{3x}\right)^{-1}\div\frac{5x^{-3}y^{-2}}{30x^{5}y^{-6}}$

$\left(\frac{5y^2}{6x^3}\right)^2\cdot\left(\frac{4xy^3}{15x^2}\right)\div\frac{3x^3}{4y}\cdot\frac{30x^8}{5y4}$

$\frac{25y^4}{36x^{16}}\cdot\frac{4xy^3}{15x^2}\div\frac{4y}{3x^3}\cdot\frac{30x^8}{5y4}$

$\frac{25\cdot4\cdot4\cdot30\cdot x^9\cdot y^8}{36\cdot15\cdot3\cdot5\cdot x^{21}y^4}$

$\frac{40y^4}{27x^{12}}$

Show the following statement to be true. $(x+y)^2 \neq x^2+y^2$

$(x+4)^2 = x^2+2xy+y^2$

Show the following statement to be true. $(x+y)^{-1} \neq x^{-1}+y^{-1}$

$(x+y)^{-1} = \frac{1}{x+y}$

$x^{-1}+y^{-1} = \frac{1}{x}+\frac{1}{y}$

The following types of questions are on all college entrance mathematics tests.

Simplify each of the following.

A) $\dfrac{x^{5n-7}}{x^{3n+2}}$

$x^{(5n-7)(3n+2)}$

$x^{5n-7-3n-2}$

x^{3n-9}

B) $\dfrac{4^{2x-6}}{2^{x-1}}$

$\dfrac{(2^2)^{2x-6}}{2^{x-1}}$

$2^{(4x-12)-(x-1)}$

$2^{4x-12-x+1}$

2^{3x-11}

C) $\dfrac{x^{n-1}y^{2n}}{x^{n+1}\left(y^2\right)^{n-1}}$

$\dfrac{x^{n-1}y^{2n}}{x^{n+1}y^{2n-2}}$

$x^{(n-1)-(n+1)}y^{2n-(2n-2)}$

$x^{-2}y^2 = \frac{y^2}{x^2}$

D) $\dfrac{x^{n+3}x^{n-5}}{x^n}$

$\dfrac{x^{2n-2}}{x^n}$

x^{2n-2-n}

x^{n-2}

E) $\dfrac{\left(a^{2x-3}\cdot a^{x+2}\right)^3}{\left(a^{4x+1}\right)^2}$

$\dfrac{(a^{3x-1})^3}{a^{3x+2}}$

$\dfrac{a^{9x-3}}{a^{8x+2}}$

$a^{(9x-3)-(8x+2)}$

a^{x-5}

F) $\dfrac{\left(z^{5x+3}\right)^2}{z^{6x-3}}$

$\dfrac{z^{10x+6}}{z^{6-3}}$

$z^{(10x+6)-(6x-3)}$

$z^{10x+6-6x+3}$

z^{4x+9}

G) $\dfrac{\left(x^3\right)^{2n+1}}{x^{n+2}}$

$\dfrac{x^{6x+3}}{x^{n+2}}$

$x^{(6x+3)-(n+2)}$

$x^{6x+3-n-2}$

x^{6x-n+1}

x^{5n+1}

H) $\dfrac{27^{3x-6}}{9^{2x-1}}$

$\dfrac{(3^3)^{3x-6}}{(3^2)^{2x-1}}$

$\dfrac{3^{9x-18}}{3^{4x-2}}$

$3^{(9x-18)-(4x-2)}$

3^{5x-16}

I) $\dfrac{(xy)^n}{xy^n}$

$\dfrac{x^n y^n}{xy^n}$

$x^{n-1}y^{n-n}$

x^{n-1}

Scientific Notation

Convert each of the following to scientific notation.

A) 1,250,000,000
1.25×10^9

B) 253,000,000
2.53×10^8

C) 3,802,000,000
3.802×10^9

D) 0.000256
2.56×10^{-4}

E) 0.002005
2.005×10^{-3}

F) 0.00003205
3.205×10^{-5}

G) 236.2×10^4
2.362×10^6

H) 0.32×10^8
3.2×10^7

I) 5021.3×10^{-2}
5.0213×10^1

J) 0.00035×10^{-3}
3.5×10^{-7}

K) 0.0000325×10^4
3.25×10^{-1}

L) 5800×10^{-1}
5.8×10^2

Evaluate each of the following. Write your answer in scientific notation.

A) $$\frac{(3 \times 10^3)(2.56 \times 10^5)}{2.4 \times 10^{-3}}$$

$$\frac{(3)(2.56)}{2.4} \times \frac{(10^3)(10^5)}{10^{-3}}$$

3.2×10^{11}

B) $$\frac{(7.1 \times 10^{-3})(9 \times 10^5)}{3 \times 10^8}$$

$$\frac{(7.1)(9)}{3} \times \frac{(10^{-3})(10^5)}{10^8}$$

$(7.1)(3) \times 10^{-6}$

21.3×10^{-6}

2.13×10^{-5}

C) $$\frac{(3.2 \times 10^5)(1.8 \times 10^2)}{1.2 \times 10^{-4}}$$

$$\frac{(3.2)(1.8)}{1.2} \times \frac{(10^5)(10^2)}{10^{-4}}$$

4.8×10^{11}

D) $$\frac{(1.2432 \times 10^8)}{(1.2 \times 10^2)(3.7 \times 10^{-3})}$$

$$\frac{(1.2432)}{(1.2)(3.7)} \times \frac{10^8}{(10^2)(\cancel{10^3})(10^{-3})}$$

$.28 \times 10^9$

2.8×10^8

E) $$\frac{(16{,}000)(120{,}000)}{0.000024}$$

$$\frac{(1.6 \times 10^4)(1.2 \times 10^5)}{2.4 \times 10^{-5}}$$

$$10^2 \frac{(1.6)(1.2)}{2.4} \times \frac{(10^4)(10^5)}{10^{-5}}$$

$.8 \times 10^{+14} = 8 \times 10$

F) $$\frac{(600)(0.003)(2{,}000)}{1.2 \times 10^4}$$

$$\frac{(6 \times 10^2)(3 \times 10^{-3})(2 \times 10^3)}{1.2 \times 10^4}$$

$$\frac{(6)(3)(2)}{1.2} \times \frac{(10^2)(10^{-3})(10^3)}{10^4}$$

$30 \times 10^{-2} = 3 \times 10^{-1}$

Useful Properties of Fractions

The following are useful properties when working with rational expressions.

Equivalent Fractions	$\frac{a}{b}=\frac{c}{d}$ **if and only if** $ad=bc$
Rules of Signs	$\frac{-a}{b}=\frac{a}{-b}=-\frac{a}{b}$ **and** $\frac{-a}{-b}=\frac{a}{b}$
Generating Equivalent Fractions	$\frac{a}{b}=\frac{ac}{bc}$ **where** $c\neq 0$
Add/Subtract with Like Denominators	$\frac{a}{b}\pm\frac{c}{b}=\frac{a\pm c}{b}$
Add/Subtract with Unlike Denominators	$\frac{a}{b}\pm\frac{c}{d}=\frac{ad\pm cb}{bd}$
Multiply Fractions	$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$
Divide Fractions	$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$ **where** $c\neq 0$

Simplifying Rational Expressions

A) $\frac{4x-8x^2}{10x-5}$

$\frac{4x(1-2x)}{5(2x-1)}$

$\frac{-4x(2x-1)}{5(2x-1)}$

$-\frac{4x}{5}$

B) $\frac{9x^2+9x}{2x+2}$

$\frac{9x(x+1)}{2(x+1)}$

$\frac{9x}{2}$

C) $\frac{x^2-25}{5-x}$

$\frac{(x+5)(x-5)}{-1(x-5)}$

$-(x+5)$

D) $\frac{12-4x}{x-3}$

$\frac{4(3-x)}{x-3}$

$\frac{-4(x-3)}{x-3}$

-4

E) $\frac{x^3+5x^2+6x}{x^2-4}$

$\frac{x(x^2+5x+6)}{(x+2)(x-2)}$

$\frac{x(x+2)(x+3)}{(x+2)(x-2)}$

$\frac{x(x+3)}{x-2}$

F) $\frac{x^2+8x-20}{x^2+11x+10}$

$\frac{(x+10)(x-2)}{(x+10)(x+1)}$

$\frac{x-2}{x+1}$

G) $\frac{x^2-2x-8}{x^2+7x+10}$

$\frac{(x-4)(x+2)}{(x+2)(x+5)}$

$\frac{x-4}{x+5}$

H) $\frac{x^3-x}{x^2-2x-3}$

$\frac{x(x+1)(x-1)}{(x-3)(x+1)}$

$\frac{x(x-1)}{x-3}$

I) $\frac{3x^2-12}{x^2+6x+8}$

$\frac{3(x+2)(x-2)}{(x+2)(x+4)}$

$\frac{3(x-2)}{x+4}$

J) $\dfrac{2x^2-5x-12}{3x^2-14x+8}$

$\dfrac{(2x+3)(x-4)}{(3x-2)(x-4)}$

$\dfrac{2x+3}{3x-2}$

K) $\dfrac{3x^2+x-2}{9x^2-12x+4}$

$\dfrac{(3x-2)(x+1)}{(3x-2)(3x-2)}$

$\dfrac{x+1}{3x-2}$

L) $\dfrac{x^3-2x^2-15x}{2x^2+x-15}$

$\dfrac{x(x+3)(x-5)}{(2x-5)(x+3)}$

$\dfrac{x(x-5)}{2x-5}$

M) $\dfrac{2-x+2x^2-x^3}{x-2}$

$\dfrac{-(x^3-2x^2+x-2)}{x-2}$

$\dfrac{-[x^2(x-2)+1(x-2)]}{x-2}$

$\dfrac{-(x^2+1)(x-2)}{x-2}$

$-(x^2+1)$

N) $\dfrac{x^2-9}{x^3+x^2-9x-9}$

$\dfrac{(x+3)(x-3)}{x^2(x+1)-9(x+1)}$

$\dfrac{(x+3)(x-3)}{(x^2-9)(x+1)}$

$\dfrac{(x+3)(x-3)}{(x+3)(x-3)(x+1)}$

$\dfrac{1}{x+1}$

O) $\dfrac{x^3-8}{x^2+2x+4}$

$\dfrac{(x-2)(x^2+2x+4)}{x^2+2x+4}$

$x-2$

P) $\dfrac{60a^2b^2-48a^3b}{75ab^2-60a^2b}$

$\dfrac{12a^2b(5b-4a)}{15ab(5b-4a)}$

$\dfrac{4a}{5}$

Q) $\dfrac{4x^2-9y^2}{4x^2+4xy-15y^2}$

$\dfrac{(2x+3y)(2x-3y)}{(2x-3y)(2x+5y)}$

$\dfrac{2x+3y}{2x+5y}$

R) $\dfrac{3x^2-5x-2}{3x^2-14x-5}$

$\dfrac{(3x+1)(x-2)}{(3x+1)(x-5)}$

$\dfrac{x-2}{x-5}$

S) $\dfrac{12abx^2+6abx-30ab}{36a^2bx^2-12a^2bx+108a^2b}$

$\dfrac{6ab(2x^2+x-5)}{12a^2b(3x^2-x+9)}$

$\dfrac{2x^2+x-5}{2a(3x^2-x+9)}$

T) $\dfrac{36m^2nx^2+84m^2nx-24m^2n}{18mn^2x^2-81mn^2x+27mn^2}$

$\dfrac{12m^2n(3x^2+7x-2)}{9mn^2(2x^2-9x+3)}$

$\dfrac{4m(3x^2+7x-2)}{3n(2x^2-9x+3)}$

U) $\dfrac{5rt^2x+3rt^2}{5xrt^2-3rt^2}$

$\dfrac{rt^2(5x+3)}{rt^2(5x-3)}$

$\dfrac{5x+3}{5x-3}$

Multiplication/Division of Rational Expressions

For each of the following, perform the indicated operation.

A) $\dfrac{x+13}{x^2(3-x)}\cdot\dfrac{x(x-3)}{5}$

$\dfrac{x+13}{-x^2(x-3)}\cdot\dfrac{x(x-3)}{5}$

$-\dfrac{x+13}{5x}$

B) $\dfrac{x^2-x-6}{x^2+6x+9}\cdot\dfrac{x+3}{x^2-4}$

$\dfrac{(x-3)(x+2)}{(x+3)(x+3)}\cdot\dfrac{x+3}{(x+2)(x-2)}$

$\dfrac{(x-3)\cancel{(x+2)}}{(x+3)\cancel{(x+3)}}\cdot\dfrac{\cancel{x+3}}{\cancel{(x+2)}(x-2)}$

$\dfrac{x-3}{(x+3)(x-2)}$

C) $\dfrac{6}{x-1}\cdot\dfrac{x^2-1}{36(x-2)}$

$\dfrac{6}{x-1}\cdot\dfrac{(x+1)(x-1)}{36(x-2)}$

$\dfrac{x+1}{6(x-2)}$

D) $\dfrac{x^3-4x}{x+2}\cdot\dfrac{2x^2+7x+3}{x^2+x-6}$

$\dfrac{x(x+2)(x-2)}{x+2}\cdot\dfrac{(2x+1)(x+3)}{(x+3)(x-2)}$

$\dfrac{x\cancel{(x+2)}\cancel{(x-2)}}{\cancel{(x+2)}}\cdot\dfrac{(2x+1)\cancel{(x+3)}}{\cancel{(x+3)}\cancel{(x-2)}}$

$x(2x+1)$

E) $\dfrac{2x^2+7x+3}{2x^2-13x-7}\cdot\dfrac{x^2-7x}{x^2+x-6}$

$\dfrac{(2x+1)(x+3)}{(2x+1)(x-7)}\cdot\dfrac{x(x-7)}{(x+3)(x-2)}$

$\dfrac{\cancel{(2x+1)}\cancel{(x+3)}}{\cancel{(2x+1)}\cancel{(x-7)}}\cdot\dfrac{x\cancel{(x-7)}}{\cancel{(x+3)}(x-2)}$

$\dfrac{x}{x-2}$

F) $\dfrac{x^2-9}{5x^2+14x-3}\div\dfrac{3x-x^2}{5x^2-11x+2}$

$\dfrac{(x+3)(x-3)}{(5x-1)(x+3)}\cdot\dfrac{(5x-1)(x-2)}{-x(x-3)}$

$\dfrac{\cancel{(x+3)}\cancel{(x-3)}}{\cancel{(5x-1)}\cancel{(x+3)}}\cdot\dfrac{\cancel{(5x-1)}(x-2)}{-x\cancel{(x-3)}}$

$-\dfrac{x-2}{x}$

G) $\dfrac{x^2-3x-10}{3x^2+5x-2}\cdot\dfrac{3x^2+14x-5}{x^2-25}$

$\dfrac{(x+2)(x-5)}{(3x-1)(x+2)}\cdot\dfrac{(3x-1)(x+5)}{(x+5)(x-5)}$

$\dfrac{\cancel{(x+2)}\cancel{(x-5)}}{\cancel{(3x-1)}\cancel{(x+2)}}\cdot\dfrac{\cancel{(3x-1)}\cancel{(x+5)}}{\cancel{(x+5)}\cancel{(x-5)}}$

1

H) $\dfrac{x^3+8}{x^2-4}\cdot\dfrac{x^2+9x+14}{x^2-2x+4}$

$\dfrac{(x+2)(x^2-2x+4)}{(x+2)(x-2)}\cdot\dfrac{(x+2)(x+7)}{x^2-2x+4}$

$\dfrac{\cancel{(x+2)}\cancel{(x^2-2x+4)}}{\cancel{(x+2)}(x-2)}\cdot\dfrac{(x+2)(x+7)}{\cancel{x^2-2x+4}}$

$\dfrac{(x+2)(x+7)}{x-2}$

I) $\dfrac{x^3-27}{x^2-7x+12}\div\dfrac{x^2+3x+9}{x^2-16}$

$\dfrac{(x-3)(x^2+3x+9)}{(x-3)(x-4)}\div\dfrac{x^2+3x+9}{(x+4)(x-4)}$

$\dfrac{\cancel{(x-3)}\cancel{(x^2+3x+9)}}{\cancel{(x-3)}\cancel{(x-4)}}\cdot\dfrac{(x+4)\cancel{(x-4)}}{\cancel{x^2+3x+9}}$

$x+4$

J) $\dfrac{x^4-16}{4x^2-2x-6} \div \dfrac{x^3-2x^2+4x-8}{x^2-1}$

$$\frac{(x^2+4)(x+2)(x-2)}{(2x+2)(2x-3)} \div \frac{x^4(x-2)+4(x-2)}{(x+1)(x-1)}$$

$$\frac{(x^2+4)(x+2)(x-2)}{2(x+1)(2x-3)} \cdot \frac{(x+1)(x-1)}{(x^2+4)(x-2)}$$

$$\frac{(x+2)(x-1)}{2(2x-3)}$$

K) $\dfrac{x^2+8x+12}{2x^2-4x-96} \cdot \dfrac{4x^2-52x+160}{x+3}$

$$\frac{(x+2)(x+6)}{2(x^2-2x-48)} \cdot \frac{4(x^2-13x+40)}{x+3}$$

$$\frac{(x+2)(x+6)}{2(x+6)(x-8)} \cdot \frac{4(x-8)(x-5)}{x+3}$$

$$\frac{2(x+2)(x-5)}{x+3}$$

L) $\dfrac{x^3+8}{3x^2+8x+4} \cdot \dfrac{3x^2-4x-4}{4x^3-32}$

$$\frac{(x+2)(x^2-2x+4)}{(3x+2)(x+2)} \cdot \frac{(3x+2)(x-2)}{4(x-2)(x^2+2x+4)}$$

$$\frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{(3x+2)}\cancel{(x+2)}} \cdot \frac{\cancel{(3x+2)}\cancel{(x-2)}}{4\cancel{(x-2)}(x^2+2x+4)}$$

$$\frac{x^2-2x+4}{4(x^2+2x+4)}$$

M) $\dfrac{6x^4+5x^3+x^2}{6x^2+x-1} \cdot \dfrac{9x^2-6x+1}{2x^2+x}$

$$\frac{x^2(6x^2+5x+1)}{(3x-1)(2x+1)} \cdot \frac{(3x-1)(3x-1)}{x(2x+1)}$$

$$\frac{x^2(3x+1)\cancel{(2x+1)}}{\cancel{(3x-1)}\cancel{(2x+1)}} \cdot \frac{\cancel{(3x-1)}(3x-1)}{x(2x+1)}$$

$$\frac{x(3x+1)(3x-1)}{2x+1}$$

N) $\dfrac{3r^2+7rt+2t^2}{7r^2t} \cdot \dfrac{14t^2+14tr}{3r^2+4rt+t^2}$

$$\frac{(3r+t)(r+2t)}{7r^2t} \cdot \frac{14t(t+r)}{(3r+t)(r+t)}$$

$$\frac{2(r+2t)}{r^2}$$

O) $\dfrac{10x^2+29x+10}{2x^2-9x-35} \div \dfrac{15x^2-14x-8}{6x^2+7x-20}$

$$\frac{(5x+2)(2x+5)}{2x^2-9x-35} \div \frac{(3x-4)(5x+2)}{(3x-4)(2x+5)}$$

$$\frac{(5x+2)(2x+5)}{2x^2-9x-35} \cdot \frac{(3x-4)(2x+5)}{(3x-4)(5x+2)}$$

$(2x+5)(x-7)$

$$\frac{(2x+5)^2}{(2x+5)(x-7)} = \frac{2x+5}{x-7}$$

P) $\dfrac{3x^3-9x^2-30x}{x^2-x-2} \div \dfrac{6x^4-12x^3-90x^2}{x^2+2x-3}$

$$\frac{3x(x^2-3x-10)}{(x-2)(x+1)} \div \frac{6x^2(x^2-2x-15)}{(x+3)(x-1)}$$

$$\frac{3x(x-5)(x+2)}{(x-2)(x+1)} \cdot \frac{(x+3)(x-1)}{6x^2(x-5)(x+3)}$$

$$\frac{(x+2)(x-1)}{2x(x-2)(x+1)}$$

Q) $\dfrac{3x-1}{1-2x} \div \dfrac{x+y}{2xy} \div \dfrac{4-12x}{x+2y}$

$$\frac{3x-1}{(-1)(2x-1)} \cdot \frac{2xy}{x+y} \cdot \frac{x+2y}{(-4)(3x-1)}$$

$$\frac{xy(x+2y)}{2(2x-1)(x+y)}$$

R) $\dfrac{2x^2-5xy-3y^2}{25x^2+15xy+2y^2} \div \dfrac{2x^2+7xy+3y^2}{5x^2-18xy-8y^2} \cdot \dfrac{x-4y}{3y^2-4xy+x^2}$

$$\frac{(2x+y)(x-3y)}{(5x+y)(5x+2y)} \cdot \frac{(5x+2y)(x-4y)}{(2x+y)(x+3y)} \cdot \frac{x-4y}{(x-y)(x-3y)}$$

$$\frac{\cancel{(2x+y)}\cancel{(x-3y)}}{(5x+y)\cancel{(5x+2y)}} \cdot \frac{\cancel{(5x+2y)}(x-4y)}{\cancel{(2x+y)}(x+3y)} \cdot \frac{x-4y}{(x-y)\cancel{(x-3y)}}$$

$$\frac{(x-4y)^2}{(5x+y)(x+3y)(x-y)}$$

S) $\dfrac{9x^2+6xy+y^2}{6x^2+5xy+y^2} \cdot \dfrac{x^2+10xy+25y^2}{3x^2+7xy+2y^2} \div \dfrac{3x^2+16xy+5y^2}{2x^2+5xy+2y^2}$

$$\frac{(3x+y)(3x+y)}{(3x+y)(2x+y)} \cdot \frac{(x+5y)(x+5y)}{(3x+y)(x+2y)} \cdot \frac{(2x+y)(x+2y)}{(3x+y)(x+5y)}$$

$$\frac{\cancel{(3x+y)}\cancel{(3x+y)}}{\cancel{(3x+y)}\cancel{(2x+y)}} \cdot \frac{\cancel{(x+5y)}(x+5y)}{\cancel{(3x+y)}\cancel{(x+2y)}} \cdot \frac{\cancel{(2x+y)}\cancel{(x+2y)}}{(3x+y)\cancel{(x+5y)}}$$

$$\frac{x+5y}{3x+y}$$

T) $\dfrac{x^2+xy-12y^2}{x^2+3xy+2y^2} \cdot \dfrac{x^2+4xy+3y^2}{3x^2-2xy-y^2} \div \dfrac{x^2+7xy+12y^2}{x^2+xy-2y^2}$

$$\frac{(x+4y)(x-3y)}{(x+2y)(x+y)} \cdot \frac{(x+3y)(x+y)}{(3x+y)(x-y)} \cdot \frac{(x-y)(x+2y)}{(x+3y)(x+4y)}$$

$$\frac{\cancel{(x+4y)}(x-3y)}{\cancel{(x+2y)}\cancel{(x+y)}} \cdot \frac{\cancel{(x+3y)}\cancel{(x+y)}}{(3x+y)\cancel{(x-y)}} \cdot \frac{\cancel{(x-y)}\cancel{(x+2y)}}{\cancel{(x+3y)}\cancel{(x+4y)}}$$

$$\frac{x-3y}{3x+y}$$

Addition/Subtraction of Rational Expressions

For each of the following, perform the indicated operation.

A) $\frac{3}{x-2}+\frac{5}{x+8}$

$$\frac{3(x+8)}{(x-2)(x+8)}+\frac{5(x-2)}{(x+8)(x-2)}$$

$$\frac{3x+24+5x-10}{(x-2)(x+8)}$$

$$\frac{8x+14}{(x-2)(x+8)}$$

$$\frac{2(4x+7)}{(x-2)(x+8)}$$

B) $\frac{6}{x+1}-\frac{2}{x-1}$

$$\frac{6(x-1)}{(x+1)(x-1)}-\frac{2(x+1)}{(x-1)(x+1)}$$

$$\frac{6x-6}{(x+1)(x-1)}-\frac{2x+2}{(x+1)(x-1)}$$

$$\frac{6x-6-2x-2}{(x+1)(x-1)}$$

$$\frac{4x-8}{(x+1)(x-1)}$$

$$\frac{4(x-2)}{(x+1)(x-1)}$$

C) $\frac{3}{x-1}+\frac{5x}{3x+4}$

$$\frac{3(3x+4)}{(x-1)(3x+4)}+\frac{5x(x-1)}{3x+4}$$

$$\frac{9x+12+5x^2-5x}{(x-1)(3x+4)}$$

$$\frac{5x^2+4x+12}{(x-1)(3x+4)}$$

D) $\frac{1}{x}+\frac{2}{x-3}+\frac{4}{x+3}$

$$\frac{x^2-9}{x(x+3)(x-3)}+\frac{2(x^2+3x)}{x(x+3)(x-3)}+\frac{4(x^2-3x)}{x(x+3)(x-3)}$$

$$\frac{x^2-9+2x^2+6x+4x^2-12x}{x(x+3)(x-3)}$$

$$\frac{7x^2-6x-9}{x(x+3)(x-3)}$$

E) $\frac{1}{x}+\frac{2}{x^2+1}-\frac{1}{x^3+x}$

$x(x^2+1)$

$$\frac{(x^2+1)}{x(x^2+1)}+\frac{2x}{x(x^2+1)}-\frac{1}{x(x^2+1)}$$

$$\frac{-x^2-1+2x-1}{x(x^2+1)}$$

$$\frac{-x^2+2x-2}{x(x^2+1)}$$

or

$$-\frac{x^2-2x+2}{x(x^2+1)}$$

F) $\frac{3}{x-2}+\frac{5}{2-x}$

$$\frac{3}{x-2}+\frac{5}{(-1)(x-2)}$$

$$\frac{3}{x-2}-\frac{5}{x-2}$$

$$-\frac{2}{x-2}$$

G) $\frac{2}{x+1}+\frac{3}{x-1}+\frac{15}{x^2-1}$

$(x+1)(x-1)$

$$\frac{2(x-1)}{(x+1)(x-1)}+\frac{3(x+1)}{(x+1)(x-1)}+\frac{15}{(x+1)(x-1)}$$

$$\frac{2x-2+3x+3+15}{(x+1)(x-1)}$$

$$\frac{5x+16}{(x+1)(x-1)}$$

H) $\frac{8}{x-1}-\frac{4}{x}+\frac{x+3}{x^2-1}$

$(x+1)(x-1)$

$$\frac{8(x^2+x)}{x(x+1)(x-1)}-\frac{4(x^2-1)}{x(x+1)(x-1)}+\frac{x(x+3)}{x(x+1)(x-1)}$$

$$\frac{8x^2+8x-4x^2+4+x^2+3x}{x(x+1)(x-1)}$$

$$\frac{5x^2+11x+4}{x(x+1)(x-1)}$$

I) $\dfrac{4}{x^2-x-2}-\dfrac{x}{x^2+6x+5}$

$\dfrac{4}{(x-2)(x+1)}-\dfrac{x}{(x+5)(x+1)}$

$\dfrac{4(x+5)}{(x-2)(x+5)(x+1)}-\dfrac{x(x-2)}{(x-2)(x+5)(x+1)}$

$\dfrac{4x+20-x^2+2x}{(x-2)(x+5)(x+1)}$

$\dfrac{-x^2+6x+20}{(x-2)(x+5)(x+1)}$

or $-\dfrac{x^2-6x-20}{(x-2)(x+5)(x+1)}$

J) $\dfrac{x+2}{x+3}-\dfrac{2}{x-3}+\dfrac{2x}{x^2-9}$

$(x+3)(x-3)$

$\dfrac{(x+2)(x-3)}{(x+3)(x-3)}-\dfrac{2(x+3)}{(x+3)(x-3)}+\dfrac{2x}{(x+3)(x-3)}$

$\dfrac{x^2-x-6-2x-6+2x}{(x+3)(x-3)}$

$\dfrac{x^2-x-12}{(x+3)(x-3)}$

$\dfrac{(x-4)(x+3)}{(x+3)(x-3)}=\dfrac{x-4}{x-3}$

K) $-\dfrac{3}{x}+\dfrac{6}{x+2}-\dfrac{1}{x-4}$

$\dfrac{-3(x^2-2x-8)}{x(x+2)(x-4)}+\dfrac{6(x^2-4x)}{x(x+2)(x-4)}-\dfrac{x^2+2x}{x(x+2)(x-4)}$

$\dfrac{-3x^2+6x+24+6x^2-24x-x^2-2x}{x(x+2)(x-4)}$

$\dfrac{2x^2-20x+24}{x(x+2)(x-4)}$

$\dfrac{2(x^2-10x+12)}{x(x+2)(x-4)}$

L) $\dfrac{2}{x^2-x-2}+\dfrac{12}{x^2+2x-8}$

$(x-2)(x+1)$ $(x-2)(x+4)$

$\dfrac{2(x+4)}{(x-2)(x+1)(x+4)}+\dfrac{12(x+1)}{(x-2)(x+1)(x+4)}$

$\dfrac{2x+8+12x+12}{(x-2)(x+1)(x+4)}$

$\dfrac{14x+20}{(x-2)(x+1)(x+4)}$

$\dfrac{2(7x+10)}{(x-2)(x+1)(x+4)}$

M) $\dfrac{5x-7}{10x^2+27x-28}-\dfrac{x-11}{5x^2-19x+12}$

$(5x-4)(2x+7)$ $(5x-4)(x-3)$

$\dfrac{(5x-7)(x-3)}{(5x-4)(2x+7)(x-3)}-\dfrac{(x-11)(2x+7)}{(5x-4)(2x+7)(x-3)}$

$(5x^2-15x-7x+21)-(2x^2+7x-22x-77)$

$5x^2-22x+21-2x^2+15x+77$

$\dfrac{3x^2-7x+98}{(5x-4)(2x+7)(x-3)}$

N) $\dfrac{x-1}{12x^2+5x-25}+\dfrac{4x+5}{9x^2-25}$

$(3x+5)(4x-5)$ $(3x+5)(3x-5)$

$\dfrac{(x-1)(3x-5)}{(3x+5)(3x-5)(4x-5)}+\dfrac{(4x+5)(4x-5)}{(3x+5)(3x-5)(4x-5)}$

$3x^2-5x-3x+5+16x^2-25$

$\dfrac{19x^2-8x-20}{(3x+5)(3x-5)(4x-5)}$

O) $\dfrac{5x-2}{28x^2+31x-5}-\dfrac{6x-1}{7x^2+20x-3}$

$(7x-1)(4x+5) \quad (7x-1)(x+3)$

$\dfrac{(5x-2)(x+3)}{(7x-1)(4x+5)(x+3)}-\dfrac{(6x-1)(4x+5)}{(7x-1)(4x+5)(x+3)}$

$(5x^2+15x-2x-6)-(24x^2+30x-4x-5)$

$5x^2+13x-6-24x^2-26x+5$

$\dfrac{-19x^2-13x-1}{(7x-1)(4x+5)(x+3)}$

$-\dfrac{19x^2+13x+1}{(7x-1)(4x+5)(x+3)}$

P) $\dfrac{x+4}{x^2+8x+15}-\dfrac{x+6}{x^2+12x+35}$

$(x+3)(x+5) \quad (x+5)(x+7)$

$\dfrac{(x+4)(x+7)}{(x+3)(x+5)(x+7)}-\dfrac{(x+6)(x+3)}{(x+3)(x+5)(x+7)}$

$(x^2+11x+28)-(x^2+9x+18)$

$x^2+11x+28-x^2-9x-18$

$2x+10$

$\dfrac{2(x+5)}{(x+3)(x+5)(x+7)} \quad \dfrac{2}{(x+3)(x+7)}$

Q) $a-b+a^{-1}+b^{-1}$

$a-b+\dfrac{1}{a}+\dfrac{1}{b}$

$\dfrac{a^2b}{ab}-\dfrac{ab^2}{ab}+\dfrac{b}{ab}+\dfrac{a}{ab}$

$\dfrac{a^2b+ab^2+b+a}{ab}$

R) $3x-1-\dfrac{1}{x-1}$

$\dfrac{3x(x-1)}{x-1}-\dfrac{x-1}{x-1}-\dfrac{1}{x-1}$

$\dfrac{3x^2-3x-x+1-1}{x-1}$

$\dfrac{3x^2-4x}{x-1}$

$\dfrac{x(3x-4)}{x-1}$

S) $\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}+1$

$\dfrac{(a+b)(a+b)}{(a+b)(a-b)}-\dfrac{(a-b)(a-b)}{(a+b)(a-b)}+\dfrac{(a+b)(a-b)}{(a+b)(a-b)}$

$(a^2+2ab+b^2)-(a^2-2ab+b^2)+(a^2-b^2)$

$a^2+2ab+b^2-a^2+2ab-b^2+a^2-b^2$

$\dfrac{a^2+4ab-b^2}{(a+b)(a-b)}$

Simplifying Complex Fractions

Although these problems tend to look intimidating, remember, they are simply an addition/subtraction problem, and a multiplication/division problem put together. Always rewrite the problem the long way and solve.

For example, when given the problem $\dfrac{3x-\dfrac{5x}{2x-3}}{3x+\dfrac{5x}{2x+3}}$ ***rewrite as*** $\left[3x-\dfrac{5x}{2x-3}\right]\div\left[3x+\dfrac{5x}{2x+3}\right]$

Simplify each of the following compound fractions.

A) $\dfrac{\dfrac{1}{2}-\dfrac{1}{12}}{\dfrac{1}{2}+\dfrac{1}{12}}$

$\left[\dfrac{6}{12}-\dfrac{1}{12}\right]\div\left[\dfrac{6}{12}+\dfrac{1}{12}\right]$

$\dfrac{5}{12}\div\dfrac{7}{12}$

$\dfrac{5}{12}\cdot\dfrac{12}{7}$

110 $\dfrac{5}{7}$

B) $\dfrac{\dfrac{2}{3}+\dfrac{1}{6}}{\dfrac{3}{4}+\dfrac{1}{12}}$

$\left[\dfrac{4}{6}+\dfrac{1}{6}\right]\div\left[\dfrac{9}{12}+\dfrac{1}{12}\right]$

$\dfrac{5}{6}\div\dfrac{10}{12}$

$\dfrac{5}{6}\cdot\dfrac{12}{10}$

1

C) $\dfrac{\dfrac{4x}{5x-15}}{\dfrac{8x^2}{7x-21}}$

$\dfrac{4x}{5(x-3)}\div\dfrac{8x^2}{7(x-3)}$

$\dfrac{4x}{5(x-3)}\cdot\dfrac{7(x-3)}{8x^2}$

$\dfrac{7}{10x}$

D) $\dfrac{2x+\dfrac{4x}{x-1}}{2x-\dfrac{4x}{x+1}}$

$$\left[\frac{2x(x-1)}{x-1}+\frac{4x}{x-1}\right]\div\left[\frac{2x(x+1)}{x+1}-\frac{4x}{x+1}\right]$$

$$\frac{2x^2-2x+4x}{x-1}\div\frac{2x^2+2x-4x}{x+1}$$

$$\frac{2x^2+2x}{x-1}\div\frac{2x^2-2x}{x+1}$$

$$\frac{2x(x+1)}{x-1}\cdot\frac{x+1}{2x(x-1)}$$

$$\frac{(x+1)^2}{(x-1)^2}$$

E) $\dfrac{x+\dfrac{2x}{x^2+2}}{x-\dfrac{2x+2}{x^2+2}}$

$$\left[\frac{x(x^2+2)}{x^2+2}+\frac{2x}{x^2+2}\right]\div\left[\frac{x(x^2+2)}{x^2+2}-\frac{2x+2}{x^2+2}\right]$$

$$\frac{x^3+2x+2x}{x^2+2}\div\frac{x^3+2x-2x-2}{x^2+2}$$

$$\frac{x^3+4x}{x^2+2}\div\frac{x^3-2}{x^2+2}$$

$$\frac{x(x^2+4)}{x^2+2}\cdot\frac{x^2+2}{x^3-2}$$

$$\frac{x(x^2+4)}{x^3-2}$$

F) $\dfrac{\dfrac{x}{x+1}+\dfrac{x^2}{x^2-1}}{\dfrac{3x}{x-1}-\dfrac{2x^2}{x+1}}$

$$\left[\frac{x(x-1)}{(x+1)(x-1)}+\frac{x^2}{(x+1)(x-1)}\right]\div\left[\frac{3x(x+1)}{(x+1)(x-1)}-\frac{2x^2(x-1)}{(x+1)(x-1)}\right]$$

$$\frac{x^2-x+x^2}{(x+1)(x-1)}\div\frac{3x^2+3x-2x^3+2x^2}{(x+1)(x-1)}$$

$$\frac{2x^2-x}{(x+1)(x-1)}\div\frac{-2x^3+5x^2+3x}{(x+1)(x-1)}$$

$$\frac{x(2x-1)}{(x+1)(x-1)}\div\frac{(-x)(2x+1)(x-3)}{(x+1)(x-1)}$$

$$\frac{x(2x-1)}{(x+1)(x-1)}\cdot\frac{(x+1)(x-1)}{(-x)(2x+1)(x-3)}$$

$$-\frac{2x-1}{(2x+1)(x-3)}$$

G) $\dfrac{\dfrac{2a}{2a+b}+\dfrac{2b}{2a-b}}{\dfrac{2a}{2a-b}-\dfrac{2b}{2a+b}}$

$$\left[\frac{2a(2a-b)}{(2a+b)(2a-b)}+\frac{2b(2a+b)}{(2a+b)(2a-b)}\right]\div\left[\frac{2a(2a+b)}{(2a+b)(2a-b)}-\frac{2b(2a-b)}{(2a+b)(2a-b)}\right]$$

$$\frac{4a^2-2ab+4ab+2b^2}{(2a+b)(2a-b)}\div\frac{4a^2+2ab-4ab+2b^2}{(2a+b)(2a-b)}$$

$$\frac{4a^2+2ab+2b^2}{(2a+b)(2a-b)}\div\frac{4a^2-2ab+2b^2}{(2a+b)(2a-b)}$$

$$\frac{2(2a^2+ab+b^2)}{(2a+b)(2a-b)}\cdot\frac{(2a+b)(2a-b)}{2(2a^2-ab+b^2)}$$

$$\frac{2a^2+ab+b^2}{2a^2-ab+b^2}$$

H) $\dfrac{\dfrac{x}{y}+\dfrac{x-y}{x+y}}{\dfrac{x}{x}-\dfrac{x-y}{x+y}}$

$$\left[\frac{x(x+y)}{y(x+y)}+\frac{y(x-y)}{y(x+y)}\right]\div\left[\frac{x(x+y)}{y(x+y)}-\frac{y(x-y)}{y(x+y)}\right]$$

$$\frac{x^2+xy+xy-y^2}{y(x+y)}\div\frac{x^2+xy-xy+y^2}{y(x+y)}$$

$$\frac{x^2+2xy-y^2}{y(x+y)}\div\frac{x^2+y^2}{y(x+y)}$$

$$\frac{x^2+2xy-y^2}{y(x+y)}\cdot\frac{y(x+y)}{x^2+y^2}$$

$$\frac{x^2+2xy-y^2}{x^2+y^2}$$

I) $\dfrac{x+\dfrac{x^2+5x+6}{(x+3)^2}}{\dfrac{1}{x}-\dfrac{x+2}{x^2-9}}$

$$\left[\frac{x(x^2+6x+9)}{(x+3)^2}+\frac{x^2+5x+6}{(x+3)^2}\right]\div\left[\frac{x^2-9}{x(x+3)(x-3)}-\frac{x(x+2)}{x(x+3)(x-3)}\right]$$

$$\frac{x^3+6x^2+9x+x^2+5x+6}{(x+3)^2}\div\frac{x^2-9-x^2-2x}{x(x+3)(x-3)}$$

$$\frac{x^3+7x^2+14x+6}{(x+3)^2}\cdot\frac{x(x+3)(x-3)}{-2x-9}$$

$$\frac{x^3+7x^2+14x+6}{(x+3)\cancel{(x+3)}}\cdot\frac{x\cancel{(x+3)}(x-3)}{(-1)(2x+9)}$$

$$-\frac{x(x^3+7x^2+14x+6)(x-3)}{(2x+9)(x+3)}$$

Normally when working with a base that has negative exponents, the base will need to be moved to the other side of the fraction bar. This is the case when $\frac{x}{y^{-1}}$ ***becomes*** xy***. This can only be done if you are multiplying the bases. Whenever there is an addition or subtraction symbol between two terms, they must be treated differently.***

For example, $\frac{x^{-1}+y^{-1}}{xy}$ ***will become*** $\frac{\frac{1}{x}+\frac{1}{y}}{xy}$ ***and the problem would be solved from there. Notice however in this particular example if there is a negative exponent with the*** y ***in the denominator it would jump up. The only reason it can is because there is a multiplication problem in the denominator of the fraction. Watch for possible combinations of this rule.***

J) $\dfrac{x^{-1}-\left(x^2-4\right)^{-1}}{x+(2-x)^{-1}}$

$$\left[\frac{1}{x}-\frac{1}{x^2-4}\right]\div\left[x+\frac{1}{2-x}\right]$$

$$\left[\frac{x^2-4}{x(x^2-4)}-\frac{x}{x^2-4}\right]\div\left[\frac{x(x-2)}{x-2}-\frac{1}{x-2}\right]$$

$$\frac{x^2-x-4}{x(x+2)(x-2)}\div\frac{x^2-2x-1}{x-2}$$

$$\frac{x^2-x-4}{x(x+2)(x-2)}\cdot\frac{x-2}{x^2-2x-1}$$

$$\frac{x^2-x-4}{x(x+2)(x^2-2x-1)}$$

K) $\dfrac{(x-3)^{-1}+(x+3)^{-1}}{(x+3)^{-1}}$

$$\left[\frac{1}{x-3}+\frac{1}{x+3}\right]\div\frac{1}{x+3}$$

$$\frac{x+3+x-3}{(x+3)(x-3)}\div\frac{1}{x+3}$$

$$\frac{2x}{(x+3)(x-3)}\cdot\frac{x+3}{1}$$

$$\frac{2x}{x-3}$$

L) $\dfrac{x^{-1}+y^{-1}}{xy}$

$$\left[\frac{1}{x}+\frac{1}{y}\right]\div xy$$

$$\left[\frac{y}{xy}+\frac{x}{xy}\right]\div xy$$

$$\frac{x+y}{xy}\cdot\frac{1}{xy}$$

$$\frac{x+y}{x^2y^2}$$

Solving Rational Equations

The first step to solving a rational equation is to find a common denominator. However, if there are variables or polynomials in the denominator, always find the zeros of the denominator before you solve the problem. That way, any extraneous solutions can be found at the beginning of the problem. For example, in the problem $\frac{5}{x+3}=6$, the value of x cannot be -3, because it would create a zero in the denominator.

Solve each of the following rational equations by finding a common denominator.

A) $\frac{4x}{9}+\frac{5x}{12}=\frac{2}{3}$

$\frac{4}{4}\left(\frac{4x}{9}\right)+\frac{3}{3}\left(\frac{5x}{12}\right)=\frac{12}{12}\left(\frac{2}{3}\right)$

$\frac{16x}{36}+\frac{15x}{36}=\frac{24}{36}$

$16x+15x=24$

$31x=24$

$x=\frac{24}{31}$

B) $\frac{x+1}{2}-\frac{2x+3}{7}=\frac{5x+2}{28}$

$\frac{14(x+1)}{28}-\frac{4(2x+3)}{28}=\frac{5x+2}{28}$

$14x+14-8x-12=5x+2$

$6x+2=5x+2$

$-5x\ -2\ -5x-2$

$x=0$

C) $\frac{x+2}{7}-\frac{x+3}{2}=\frac{3}{14}$

$\frac{2(x+2)}{14}-\frac{7(x+3)}{14}=\frac{3}{14}$

$2x+4-7x-21=3$

$-5x-17=3$

$+17\ +17$

$\frac{-5x}{-5}=\frac{20}{-5}$

$x=-4$

D) $\frac{x-6}{4}-\frac{3x-8}{12}=\frac{2x+1}{8}$

$\frac{6(x-6)}{24}-\frac{2(3x-8)}{24}=\frac{3(2x+1)}{24}$

$6x-36-6x+16=6x+3$

$-20=6x+3$

$-3\quad -3$

$\frac{-23}{6}=\frac{6x}{6}$

$x=-\frac{23}{6}$

E) $\frac{x-3}{12}+\frac{2x-1}{15}=\frac{3x+1}{4}$

$\frac{5(x-3)}{60}+\frac{4(2x-1)}{60}=\frac{15(3x+1)}{60}$

$5x-15+8x-4=45x+15$

$13x-19=45x+15$

$-45x+19\ -45x+19$

$\frac{-32x}{-32}=\frac{34}{-32}$

$x=-\frac{17}{16}$

F) $\frac{x+3}{3}+\frac{x-4}{2}=\frac{1}{6}$

$\frac{2(x+3)}{6}+\frac{3(x-4)}{6}=\frac{1}{6}$

$2x+6+3x-12=1$

$5x-6=1$

$+6\ +6$

$\frac{5x}{5}=\frac{7}{5}$

$x=\frac{7}{5}$

G) $\frac{1}{3}(2x-1)+\frac{1}{4}\left(\frac{1}{2}-3x\right)=-2$

$12\left[\frac{1}{3}(2x-1)\right]+12\left[\frac{1}{4}\left(\frac{1}{2}-3x\right)\right]=-2(12)$

$4(2x-1)+3\left(\frac{1}{2}-3x\right)=-24$

$8x-4+\frac{3}{2}-9x=-24$

$+4\qquad +4$

$-x+\frac{3}{2}=-20$

$-2x+3=-40$

$-3\quad -3$

$\frac{-2x}{-2}=\frac{-43}{-2}$

$x=\frac{43}{2}$

H) $\frac{1}{3}\left(\frac{1}{2}-\frac{x}{2}\right)-\frac{1}{2}\left(\frac{1}{2}+\frac{x}{3}\right)=-\frac{2}{9}$

$-18\left[-\frac{1}{3}\left(\frac{x-1}{2}\right)-\frac{1}{2}\left(\frac{2x+3}{6}\right)\right]=\left[-\frac{2}{9}\right](-18)$

$6\left(\frac{x-1}{2}\right)+9\left(\frac{2x+3}{6}\right)=4$

$3x-3+3\left(\frac{2x+3}{2}\right)=4$

$6x-6+6x+9=8$

$12x+3=8$

$-3\quad -3$

$12x=5$

$x=\frac{5}{12}$

I) $\frac{3}{5}\left(5x-\frac{1}{3}\right)-\frac{2}{3}\left(3x-\frac{3}{5}\right)=\frac{1}{2}(x+2)$

$\frac{3}{5}\left(\frac{15x-1}{3}\right)-\frac{2}{3}\left(\frac{15x-3}{5}\right)=\frac{1}{2}(x+2)$

mult. by 30 → $\frac{1}{5}(15x-1)-\frac{2}{15}(15x-3)=\frac{1}{2}(x+2)$

$6(15x-1)-4(15x-3)=15(x+2)$

$90x-6-60x+12=15x+30$

$30x+6=15x+30$

$-15x-6\quad -15x-6$

$\frac{15x}{15}=\frac{24}{15}$

$x=\frac{8}{5}$

J) $\frac{x-2}{x+1}+\frac{x-3}{x-1}=2 \quad x\neq \pm 1$

$(x-2)(x-1)+(x-3)(x+1)=2(x^2-1)$

$x^2-3x+2+x^2-2x-3=2x^2-2$

$2x^2-5x-1=2x^2-2$

$-2x^2\quad -2x^2$

$-5x-1=-2$

$+1\quad +1$

$\frac{-5x}{-5}=\frac{-1}{-5}$

$x=\frac{1}{5}$

K) $\frac{x+4}{x-3}-\frac{2x-5}{x-2}=-1 \quad x\neq 2,3$

$(x+4)(x-2)-(2x-5)(x-3)=-(x^2-5x+6)$

$x^2+2x-8-(2x^2-11x+15)=-x^2+5x-6$

$x^2+2x-8-2x^2+11x-15=-x^2+5x-6$

$-x^2+13x-23=-x^2+5x-6$

$13x-23=5x-6$

$-5x+23\quad -5x+23$

$8x=17$

$x=\frac{17}{8}$

L) $\frac{x-1}{x+3}+\frac{x+2}{x-3}=2 \quad x\neq \pm 3$

$(x-1)(x-3)+(x+2)(x+3)=2(x^2-9)$

$x^2-4x+3+x^2+5x+6=2x^2-18$

$2x^2+x+9=2x^2-18$

$x+9=-18$

$-9\quad -9$

$x=-27$

M) $\frac{x-2}{x+3}+\frac{3x+1}{x-2}=4 \quad x\neq -3,2$

$(x-2)(x-2)+(3x+1)(x+3)=4(x^2+x-6)$

$x^2-4x+4+3x^2+10x+3=4x^2+4x-24$

$4x^2+6x+7=4x^2+4x-24$

$6x+7=4x-24$

$-4x-7\quad -4x-7$

$2x=-31$

$x=-\frac{31}{2}$

N) $\frac{x+3}{x-2}+\frac{5x+1}{x-3}=6 \quad x\neq 2,3$

$(x+3)(x-3)+(5x+1)(x-2)=6(x^2-5x+6)$

$x^2-9+5x^2-9x-2=6x^2-30x+36$

$6x^2-9x-11=6x^2-30x+36$

$-6x^2\quad -6x^2$

$-9x-11=-30x+36$

$+30x+11\quad +30x+11$

$21x=47$

$x=\frac{47}{21}$

O) $\frac{3x}{3x+1}-\frac{x}{11x+1}=\frac{5x+3}{11x+1} \quad x\neq -\frac{1}{3}, -\frac{1}{11}$

$3x(11x+1)-x(3x+1)=(5x+3)(3x+1)$

$33x^2+3x-3x^2-x=15x^2+5x+9x+3$

$30x^2+2x=15x^2+14x+3$

$-15x^2-14x-3$

$15x^2-12x-3=0$

$3(5x^2-4x-1)=0$

$3(5x+1)(x-1)=0$

$-1/5\quad 1$

$\boxed{x=\{-\frac{1}{5},1\}}$

P) $\frac{3x+1}{6x}-\frac{x-1}{5x-4}=\frac{2x+1}{6x} \quad x\neq 0, 4/5$

$(3x+1)(5x-4)-6x(x-1)=(2x+1)(5x-4)$

$(15x^2-12x+5x-4)-6x^2+6x=10x^2-8x+5x-4$

$9x^2-x-4=10x^2-3x-4$

$-9x^2+x+4$

$x^2-2x=0$

$x(x-2)=0$

$\cancel{x=0}\quad x=2$

Not a solution

$x=2$

Q) $\frac{3x-1}{2x+3}-\frac{x-1}{2x-3}=\frac{x-1}{2x+3} \quad x\neq \pm\frac{3}{2}$

$(3x-1)(2x-3)-(2x+3)(x-1)=(2x-3)(x-1)$

$(6x^2-9x-2x+3)-(2x^2-2x+3x-3)=2x^2-2x-3x+3$

$6x^2-11x+3-2x^2-x+3=2x^2-5x+3$

$4x^2-12x+6=2x^2-5x+3$

$-2x^2+5x-3$

$2x^2-7x+3=0$

$(2x-1)(x-3)=0$

$x=\frac{1}{2}\quad x=3\quad x=\{\frac{1}{2},3\}$

R) $\frac{2x+1}{2x}+\frac{x-4}{2x-7}=\frac{4x-7}{2x}$ $x\neq 0,\frac{7}{2}$

$(2x+1)(2x-7)+2x(x-4)=(4x-7)(2x-7)$

$4x^2-12x-7+2x^2-8x=8x^2-28x-14x+49$

$6x^2-20x-7=8x^2-42x+49$

$-6x^2+20x-7$

$2x^2-22x+56=0$

$2(x^2-11x+28)=0$ $2(x-4)(x-7)=0$

$\boxed{x=4 \quad x=7}$

S) $\frac{x^2-5x+6}{x^2+7x+12}=\frac{x-3}{x+3}$ $x\neq -4,3$

$(x+3)(x+4)$

$x^2-5x+6=(x-3)(x+4)$

$x^2-5x+6=x^2+x-12$

$-x^2 \quad -x^2$

$-5x+6=x-12$

$-x-6 \quad -x-6$

$-6x=-18$

$\frac{-6x}{-6}=\frac{-18}{-6}$

$x=3$

T) $\frac{x+3}{x+2}+\frac{x-3}{x-2}=\frac{x^2+4}{x^2-4}$ $x\neq\pm 2$

$(x+3)(x-2)+(x-3)(x+2)=x^2+4$

$x^2+x-6+x^2-x-6=x^2+4$

$2x^2-12=x^2+4$

$-x^2 \; +12 \quad -x^2 \; +12$

$x^2=16$

$(x+4)(x-4)=0$

$x=-4 \quad x=4$

$x=\{\pm 4\}$

U) $\frac{x+1}{2x-3}-\frac{x-1}{2x^2-5x+3}=1$ $x\neq 1,\frac{3}{2}$

$(2x-3)(x-1)$

$(x+1)(x-1)-(x-1)=(2x-3)(x-1)$

$x^2-1-x+1=2x^2-5x+3$

$x^2-x=2x^2-5x+3$

$-x^2+x \quad -x^2+x$

$x^2-4x+3=0$

$(x-3)(x-1)=0$

$3 \quad 1 \leftarrow$ Not a solution

$x=3$

V) $\frac{1}{x-2}+\frac{3}{x+3}=\frac{4}{x^2+x-6}$ $x\neq -3,2$

$(x+3)+3(x-2)=4$

$x+3+3x-6=4$

$4x-3=4$

$+3 \quad +3$

$4x=7$

$x=\frac{7}{4}$

W) $\frac{4}{x-1}+\frac{6}{3x+1}=\frac{15}{3x+1}$ $x\neq -\frac{1}{3},1$

$4(3x+1)+6(x-1)=15(x-1)$

$12x+4+6x-6=15x-15$

$18x-2=15x-15$

$-15x+2 \quad -15x+2$

$3x=-13$

$x=-\frac{13}{3}$

X) $(x-3)^{-1}+(x+3)^{-1}=10(x^2-9)^{-1}$ $x\neq\pm 3$

$\frac{1}{x-3}+\frac{1}{x+3}=\frac{10}{x^2-9}$

$(x+3)+(x-3)=10$

$2x=10$

$x=5$

Y) $15x^{-1}-4=6x^{-1}+3$ $x\neq 0$

$x\left[\frac{15}{x}-4\right]=\left[\frac{6}{x}+3\right](x)$

$15-4x=6+3x$

$-6+4x \quad -6+4x$

$9=7x$

$7x=9$

$x=\frac{9}{7}$

Miscellaneous

The following problems are necessary to solve Partial Fraction Decomposition problems. Once the equations resemble the following, they are solved by grouping all similar terms. Group anything that has an x^2. Do the same for any term that has an x, and so on. You will then factor out the x^2 and the x and proceed to set up a system of equations in three variables. Then solve the system for A, B, and C.

$$x^2 - 2x + 16 = Ax^2 - 4A + Bx^2 - 2Bx + Cx^2 + 2Cx$$

Grouping similar terms will yield the following:

$$x^2 - 2x + 16 = \left(Ax^2 + Bx^2 + Cx^2\right) + \left(-2Bx + 2Cx\right) - 4A$$

Factor out the common variable from each group to find the following:

$$x^2 - 2x + 16 = \left(A + B + C\right)x^2 + \left(-2B + 2C\right)x - 4A$$

From this point on, we will create a system of equations and solve for A, B and C.

$$\begin{array}{lrl} Eq_1 & A + B + C = & 1 \\ Eq_2 & -2B + 2C = & -2 \\ Eq_3 & -4A \qquad\qquad = & 16 \end{array}$$

The left side of the equation is very important went creating the system. Since the equal sign states that the equation is in balance, it stands to reason that the coefficient of x^2 on the right must be equal to 1. Likewise, the coefficient of the x term on the right must be equal to -2.

Find the values of A, B, C and D that make the following equations true.

A) $1 = Ax - A + Bx + B$

$1 = Ax + Bx - A + B$

$1 = (A+B)x - A + B$

must be $0x + 1$ $\quad 1 = A+B)x - A + B$ $\qquad$ substitution method

$A + B = 0 \rightarrow A = -B$

$-A + B = 1$

$-A + B = 1 \qquad A = -B$

$-(-B) + B = 1 \qquad A = -\frac{1}{2}$

$B + B = 1$

$2B = 1$

$B = \frac{1}{2}$

$A = -\frac{1}{2}, B = \frac{1}{2}$

B) $3x^2 - 7x - 2 = Ax^2 - A + Bx^2 - Bx + Cx^2 + Cx$

$3x^2 - 7x - 2 = Ax^2 + Bx^2 + Cx^2 + Cx - Bx - A$

$3x^2 - 7x - 2 = (A+B+C)x^2 + (C-B)x - A$

$A+B+C=3$

$-B+C=-7$

$-A = -2$

substitution method

$-A = -2$
$\frac{-A}{-1} = \frac{-2}{-1}$
$A=2$

$-B+C=-7$
$C=B-7$
$C=4-7$
$C=-3$

$A+B+C=3$
$(2)+B+(B-7)=3$
$2B-5=3$
$2B=8$
$B=4$

$\boxed{A=2,\ B=4,\ C=-3}$

C) $x+1 = Ax + A + Bx + 3B$

$x+1 = Ax + Bx + A + 3B$

$x+1 = (A+B)x + A + 3B$

$A+B=1$

$A+3B=1$

linear combination method

$-1(A+B=1) \Rightarrow -A-B=-1$

$A+3B=1 \Rightarrow A+3B=1$

$2B=0$

$B=0$

$A+B=1$
$A+0=1$
$A=1$

$\boxed{A=1,\ B=0}$

D) $x^2+12x+12 = Ax^2-4A+Bx^2-2Bx+Cx^2+2Cx$

$x^2+12x+12 = Ax^2+Bx^2+Cx^2-2Bx+2Cx-4A$

$x^2+12x+12 = (A+B+C)x^2+(-2B+2C)x-4A$

$A+B+C=1$
$-2B+2C=12$
$-4A \quad = 12$

Substitution method

$-4A=12$
$A=-3$

$-2B+2C=12$
$2C=2B+12$
$C=B+6$
$C=(-1)+6$
$C=5$

$A+B+C=1$
$(-3)+B+(B+6)=1$
$2B+3=1$
$2B=-2$
$B=-1$

$\boxed{A=-3, B=-1, C=5}$

E) $12 = Ax^2+Ax-6A+Bx^2+3Bx+Cx^2-2Cx$

$12=Ax^2+Bx^2+Cx^2+Ax+3Bx-2Cx-6A$

$12=(A+B+C)x^2+(A+3B-2C)x-6A$

$A+B+C=0$
$A+3B-2C=0$
$-6A \quad =12$
$A=-2$

$\Rightarrow (-2)+B+C=0 \quad B+C=2$
$(-2)+3B-2C=0 \quad 3B-2C=2$

linear combination method

$2(B+C=2) \Rightarrow 2B+2C=4$
$3B-2C=2 \Rightarrow 3B-2C=2$
$5B=6$
$B=\frac{6}{5}$

$-3(B+C=2) \Rightarrow -3B-3C=-6$
$3B-2C=2 \Rightarrow 3B-2C=2$
$-5C=-4$
$C=\frac{4}{5}$

$\boxed{A=-2, B=\frac{6}{5}, C=\frac{4}{5}}$

F) $2x^2-5x+6 = Ax^2+Ax-6A+Bx^2+3Bx+Cx^2-2Cx$

$2x^2-5x+6=Ax^2+Bx^2+Cx^2+Ax+3Bx-2Cx-6A$

$2x^2-5x+6=(A+B+C)x^2+(A+3B-2C)x-6A$

$A+B+C=2 \Rightarrow (-1)+B+C=2$
$A+3B-2C=-5 \Rightarrow (-1)+3B-2C=-5$
$-6A \quad =6$
$A=-1$

$B+C=3$
$3B-2C=-4$

linear combination method

$2(B+C=3) \Rightarrow 2B+2C=6$
$3B-2C=-4 \Rightarrow 3B-2C=-4$
$5B \quad =2$
$B=\frac{2}{5}$

$-3(B+C=3) \Rightarrow -3B-3C=-9$
$3B-2C=-4 \Rightarrow 3B-2C=-4$
$-5C=-13$
$C=\frac{13}{5}$

G) $x^2-2x+16=Ax^2-4A+Bx^2-2Bx+Cx^2+2Cx$

$x^2-2x+16=Ax^2+Bx^2+Cx^2-2Bx+2Cx-4A$

$x^2-2x+16=(A+B+C)x^2+(-2B+2C)x-4A$

$A+B+C=1$
$-2B+2C=-2$
$-4A=16$
$A=-4$

Substitution method

$-4+B+C=1$
$B+C=5$
$B+(B-1)=5$
$2B-1=5$
$2B=6$
$B=3$

$-2B+2C=-2$
$2C=2B-2$
$C=B-1$
$C=3-1$
$C=2$

$\boxed{A=-4,\ B=3,\ C=2}$

H) $\dfrac{3x+6}{x^2-5x}=\dfrac{A}{x}+\dfrac{B}{x-5}$

$3x+6=A(x-5)+Bx$
$3x+6=Ax-5A+Bx$
$3x+6=Ax+Bx-5A$
$3x+6=(A+B)x-5A$

$A+B=3$
$-5A=6$

$-5A=6$
$A=-\frac{6}{5}$

$A+B=3$
$-\frac{6}{5}+B=3$
$B=3+\frac{6}{5}$
$=\frac{15}{5}+\frac{6}{5}$
$B=\frac{21}{5}$

$\boxed{A=-\frac{6}{5},\ B=\frac{21}{5}}$

I) $\dfrac{2x+12}{x^2-x-6}=\dfrac{A}{x+2}+\dfrac{B}{x-3}$

$2x+12=A(x-3)+B(x+2)$
$2x+12=Ax-3A+Bx+2B$
$2x+12=Ax+Bx-3A+2B$
$2x+12=(A+B)x-3A+2B$

$A+B=2$
$-3A+2B=12$

linear combination method

$3(A+B=2)\Rightarrow 3A+3B=6$
$-3A+2B=12\Rightarrow -3A+2B=12$
$5B=18$
$B=\frac{18}{5}$

$-2(A+B=2)\Rightarrow -2A-2B=-4$
$-3A+2B=12\Rightarrow -3A+2B=12$
$-5A=8$
$A=-\frac{8}{5}$

$\boxed{A=-\frac{8}{5},\ B=\frac{18}{5}}$

J) $\frac{4x^2+3x-8}{x^3+2x}=\frac{A}{x}+\frac{Bx+C}{x^2+2}$

$4x^2+3x-8=A(x^2+2)+(Bx+C)x$

$4x^2+3x-8=Ax^2+2A+Bx^2+Cx$

$4x^2+3x-8=Ax^2+Bx^2+Cx+2A$

$4x^2+3x-8=(A+B)x^2+Cx+2A$

$A+B=4$

$C=3$

$2A=-8$

$A=-4$

$A+B=4$

$(-4)+B=4$

$B=8$

$\boxed{A=-4,\ B=8,\ C=3}$

K) $\frac{3x+11}{(x+3)^2}=\frac{A}{x+3}+\frac{B}{(x+3)^2}$

$3x+11=A(x+3)+B$

$3x+11=Ax+3A+B$

$A=3$

$3A+B=11$

$3(3)+B=11$

$9+B=11$

$B=2$

$\boxed{A=3,\ B=2}$

L) $\dfrac{3x^3+12x^2+8x+3}{x^2(x+1)^2}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+1}+\dfrac{D}{(x+1)^2}$

$3x^3+12x^2+8x+3=A[x(x+1)^2]+B(x+1)^2+C[x^2(x+1)]+Dx^2$

$3x^3+12x^2+8x+3=Ax(x^2+2x+1)+B(x^2+2x+1)+C(x^3+x^2)+Dx^2$

$3x^3+12x^2+8x+3=Ax^3+2Ax^2+Ax+Bx^2+2Bx+B+Cx^3+Cx^2+Dx^2$

$3x^3+12x^2+8x+3=Ax^3+Cx^3+2Ax^2+Bx^2+Cx^2+Dx^2+Ax+2Bx+B$

$3x^3+12x^2+8x+3=(A+C)x^3+(2A+B+C+D)x^2+(A+2B)x+B$

$A+C=3$
$2A+B+C+D=12$
$A+2B=8$
$B=3$

$A+2B=8$
$A+2(3)=8$
$A+6=8$
$A=2$

$A+C=3$
$(2)+C=3$
$-2 \quad -2$
$C=1$

$2A+B+C+D=12$
$2(2)+(3)+(1)+D=12$
$4+3+1+D=12$
$8+D=12$
$-8 \quad -8$
$D=4$

$A=2,\ B=3,\ C=1,\ D=4$

Checking Progress

You have now completed the "Rational Expressions" section of the workbook. The following is a checklist so that you may check your progress. Check off each of the objectives you have accomplished.

The student should be able to…

- [ ] ***Simplify algebraic expressions using the laws of exponents.***
- [ ] ***Write any number in scientific notation.***
- [ ] ***Simplify rational expressions involving polynomials.***
- [ ] ***Completely factor a given polynomial.***
- [ ] ***Multiply and Divide rational expressions involving polynomials.***
- [ ] ***Add and subtract rational expressions involving polynomials by finding a common denominator.***
- [ ] ***Solve equations involving rational expressions.***

RADICALS

Objectives

The following is a list of objectives for this section of the workbook.

By the time the student is finished with this section of the workbook, he/she should be able to…

- ***Simplify radicals using the properties of radicals.***
- ***Multiply and divide radicals using the properties of radicals.***
- ***Add and subtract radicals using the properties of radicals.***
- ***Rationalize the denominator of a rational expression where a radical resides in the denominator.***
- ***Solve equations involving radicals.***
- ***Verify a solution to a radical equation is not an extraneous root.***
- ***Simplify radicals using imaginary numbers.***
- ***Simplify expressions involving imaginary numbers.***
- ***Plot complex numbers on a Complex Plane.***
- ***Multiply and divide complex numbers.***
- ***Add and subtract complex numbers.***
- ***Rationalize an expression with complex numbers in the denominator.***

Math Standards Addressed

The following state standards are addressed in this section of the workbook.

Algebra II

5.0 Students demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane.

6.0 Students add, subtract, multiply, and divide complex numbers.

7.0 Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.

15.0 Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.

Simplifying Radicals

Always remember the question you are asking yourself for radicals. When looking at the problem $\sqrt{36}$ you are asking yourself "What number times itself is 36?" This seems overly simplified now, and you already know this, but knowing what question you are being asked is very important, particularly when we get to logarithms.

Properties of Radicals

Property		Example
$\sqrt[a]{x^b} = \left(\sqrt[a]{x}\right)^b$		$\sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$
$a\sqrt{x} + b\sqrt{x} = (a+b)\sqrt{x}$	***Both index and radicand must match***	$3\sqrt{5} + 6\sqrt{5} = 9\sqrt{5}$
$\sqrt{x} \cdot \sqrt{y} = \sqrt{xy}$	***To multiply radicals with different radicands, the index must match. However, if the radicands are the same you can multiply if the indexes are different. Refer to your notes.***	$\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$
$a\sqrt{x} \cdot b\sqrt{y} = ab\sqrt{xy}$		$2\sqrt{3} \cdot 3\sqrt{5} = 6\sqrt{15}$
$\sqrt[a]{\dfrac{x}{y}} = \dfrac{\sqrt[a]{x}}{\sqrt[a]{y}}$	***Must be able to use this property backwards and forwards.***	$\sqrt{\dfrac{4}{9}} = \dfrac{\sqrt{4}}{\sqrt{9}} = \dfrac{2}{3}$
$\sqrt[a]{\sqrt[b]{x}} = \sqrt[ab]{x}$	***If you have a radical inside another radical you multiply the indexes to simplify.***	$\sqrt[4]{\sqrt[3]{17}} = \sqrt[12]{17}$
$\sqrt[a]{(xy)^a} = xy$	***If and only if a is odd.***	$\sqrt[5]{(5x)^5} = 5x$ *or* $\sqrt[3]{(-12)^3} = -12$
$\sqrt[a]{(xy)^a} = \lvert xy \rvert$	***If and only if a is even. Remember, when you take the square root of numbers, you are taking the absolute root, so absolute value symbols only need to be used with variables or an expression containing variables when writing your answer. The actual number would come out of the symbols..***	$\sqrt[4]{(3xy)^4} = 3\lvert xy \rvert$ *or* $\sqrt[4]{(x-6)^4} = \lvert x-6 \rvert$

Given $\sqrt[a]{b}$, identify the radicand and the index.

What is the index of a radical if you do not see it?

What question are you asking yourself for the problem $\sqrt[3]{12}$?

what number times itself 3 times is 12?

Simplify each of the following simple radical statements.

A) $\sqrt{64}$

8

B) $\sqrt{625}$

25

C) $\sqrt{81}$

9

D) $\sqrt{0.25}$

0.5

E) $\sqrt{\frac{4}{9}}$

$\frac{2}{3}$

F) $\left(\sqrt{\frac{25}{36}}\right)^{-1}$

$\left(\frac{5}{6}\right)^{-1}$

G) $\sqrt{12}$

$\sqrt{2\cdot 2\cdot 3}$

$2\sqrt{3}$

H) $\sqrt{18}$

$\sqrt{2\cdot 3\cdot 3}$

$3\sqrt{2}$

I) $\sqrt{250}$

$\sqrt{5\cdot 5\cdot 10}$

$5\sqrt{10}$

J) $\sqrt{294}$

$\sqrt{2\cdot 3\cdot 7\cdot 7}$

$7\sqrt{6}$

K) $\sqrt{324}$

$\sqrt{2\cdot 2\cdot 9\cdot 9}$

18

L) $\sqrt{27000}$

$\sqrt{27\cdot 1000}$

$\sqrt{3\cdot 3\cdot 3\cdot 10\cdot 10\cdot 10}$

$30\sqrt{30}$

M) $\sqrt[3]{32}$

$\sqrt[3]{4\cdot 8}$

$\sqrt[3]{2\cdot 2\cdot 2\cdot 2\cdot 2}$

$2\sqrt[3]{4}$

N) $\sqrt[3]{-8}$

-2

O) $\sqrt[5]{192}$

$\sqrt[5]{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 3}$

$2\sqrt[5]{6}$

P) $\sqrt[4]{162}$

$\sqrt[4]{2\cdot 3\cdot 3\cdot 3\cdot 3}$

$3\sqrt[4]{2}$

Q) $\sqrt[5]{1092}$

$\sqrt[5]{4\cdot 273}$

$\sqrt[5]{2\cdot 2\cdot 3\cdot 91}$

$\sqrt[5]{2\cdot 2\cdot 3\cdot 7\cdot 13}$

$\sqrt[5]{1092}$

R) $\sqrt[3]{1024}$

$\sqrt[3]{4\cdot 256}$

$\sqrt[3]{4\cdot 4\cdot 64}$

$\sqrt[3]{4\cdot 4\cdot 4\cdot 4\cdot 4}$

$\sqrt[3]{4\cdot 4\cdot 4\cdot 2\cdot 2\cdot 2\cdot 2}$

$8\sqrt[3]{2}$

S) $\sqrt[3]{-\frac{27}{8}}$

$-\frac{3}{2}$

T) $\sqrt[4]{-16}$

Not real

U) $\sqrt[3]{-216}$

-6

V) $\sqrt[3]{0}$

0

W) $\sqrt[3]{64}$

4

X) $\sqrt[4]{720}$

$\sqrt[4]{9\cdot 80}$

$\sqrt[4]{9\cdot 8\cdot 10}$

$\sqrt[4]{9\cdot 2\cdot 2\cdot 2\cdot 2\cdot 5}$

$2\sqrt[4]{45}$

Simplify each of the following. Remember to use absolute value symbols when needed.

A) $\sqrt{36x^8y^4}$

$6x^4y^2$

B) $\sqrt[3]{54x^4y^2}$

$\sqrt[3]{2\cdot 27\cdot x^4\cdot y^2}$

$\sqrt[3]{2\cdot 3\cdot 3\cdot 3\cdot x^3\cdot x\cdot y^2}$

$3x\sqrt[3]{2xy^2}$

C) $\sqrt[4]{\left(3x^2\right)^4}$

$3x^2$

D) $\sqrt[3]{\dfrac{16x^5}{27x^2}}$

$\sqrt[3]{\dfrac{2\cdot 2\cdot 2\cdot 2\cdot x^3\cdot x^2}{3\cdot 3\cdot 3\cdot x^2}}$

$\dfrac{2x}{3}\sqrt[3]{\dfrac{2x^2}{x^2}}$

$\dfrac{2x\sqrt[3]{2}}{3}$

E) $\sqrt[3]{64x^3y^6}$

$4xy^2$

F) $\sqrt{64x^8y^2}$

$8x^4|y|$

G) $\sqrt{x^4y^{-6}}$

$\sqrt{\dfrac{x^4}{y^6}}$

$\dfrac{x^2}{y^3}$

H) $\sqrt[5]{96x^{12}}$

$\sqrt[5]{3\cdot 32\cdot x^{12}}$

$\sqrt[5]{3\cdot 2^5\cdot x^5\cdot x^5\cdot x^2}$

$2x^2\sqrt[5]{3x^2}$

I) $\sqrt[3]{-32x^4y^3}$

$\sqrt[3]{(-1)^3\cdot 2\cdot 2\cdot 2^3\cdot x^3\cdot x\cdot y^3}$

$-2xy\sqrt[3]{4x}$

J) $\sqrt[3]{-162x^3y^9}$

$\sqrt[3]{(-1)^3\cdot 2\cdot 81\cdot x^3\cdot y^9}$

$\sqrt[3]{(-1)^3\cdot 2\cdot 3^3\cdot 3\cdot x^3\cdot y^3\cdot y^3\cdot y^3}$

$-3xy^3\sqrt[3]{6}$

K) $\sqrt[6]{128x^6y^{18}}$

$\sqrt[6]{2\cdot 64\cdot x^6\cdot y^{18}}$

$\sqrt[6]{2\cdot 2^6\cdot x^6\cdot y^6\cdot y^6\cdot y^6}$

$2|x|y^3\sqrt[6]{2}$

L) $\sqrt{16x^2y^2}$

$4|xy|$

M) $\sqrt{72x^3y^2z}$

$\sqrt{8\cdot 9\cdot x^3y^2z}$

$\sqrt{2\cdot 2^2\cdot 3^2\cdot x^2\cdot x\cdot y^2\cdot z}$

$6x|y|\sqrt{2xz}$

N) $\sqrt[5]{-320x^5y}$

$\sqrt[5]{(-1)^5\cdot 32\cdot 10\cdot x^5\cdot y}$

$\sqrt[5]{(-1)^5\cdot 2^5\cdot 10\cdot x^5y}$

$-2x\sqrt[5]{10y}$

O) $\sqrt[3]{56000x^2y^3}$

$\sqrt[3]{56\cdot 1000\,x^2y^3}$

$\sqrt[3]{7\cdot 8\cdot 1000\,x^2y^3}$

$\sqrt[3]{7\cdot 2^3\cdot 10^3\,x^2y^3}$

$20y\sqrt[3]{7x^2}$

P) $\sqrt{3125x^7y^4}$

$\sqrt{25\cdot 125x^7y^4}$

$\sqrt{5^2\cdot 5^2\cdot 5x^6\cdot x\cdot y^4}$

$25x^3y^2\sqrt{5x}$

Q) $\sqrt{25x^2+20x+4}$

$\sqrt{(5x+2)^2}$

$|5x+2|$

R) $\sqrt{81(3x+1)^4}$

$9(3x+1)^2$

S) $\sqrt{72(2x-5)^2}$

$\sqrt{2\cdot 36(2x-5)^2}$

$6|2x-5|\sqrt{2}$

T) $\sqrt{252x^2y^3(3x-2y)^2}$

$\sqrt{4\cdot 63x^2y^3(3x-2y)^2}$

$\sqrt{4\cdot 9\cdot 7x^2y^3(3x-2y)^2}$

$6y|x(3x-2y)|\sqrt{7y}$

U) $\sqrt{25x^2+25y^2}$

$\sqrt{25(x^2+y^2)}$

$5\sqrt{x^2+y^2}$

V) $\sqrt{405(2x+15)^2}$

$\sqrt{5\cdot 81\cdot(2x+15)^2}$

$9|2x+15|\sqrt{5}$

W) $\sqrt{25(2x+5)^2}$

$5|2x+5|$

X) $\sqrt{36x^2-36y^2}$

$\sqrt{36(x^2-y^2)}$

$6\sqrt{x^2-y^2}$

Y) $\sqrt{16x^2+64x+64}$

$\sqrt{16(x^2+4x+4)}$

$\sqrt{16(x+2)^2}$

$4|x+2|$

Multiplication with Radicals

Simplify each of the following.

A) $2\sqrt{2}\cdot 4\sqrt{5}$

$8\sqrt{10}$

B) $6\sqrt{3}\cdot 2\sqrt{2}\cdot\sqrt{6}$

$12\sqrt{36}$

72

C) $(2\sqrt{2})^6$

$(2\sqrt{2})(2\sqrt{2})(2\sqrt{2})(2\sqrt{2})(2\sqrt{2})(2\sqrt{2})$

$4\sqrt{4}\cdot 4\sqrt{4}\cdot 4\sqrt{4}$

$8\cdot 8\cdot 8$

512

D) $\sqrt{18}\cdot\sqrt{45}$

$\sqrt{9\cdot 2}\cdot\sqrt{9\cdot 5}$

$3\sqrt{2}\cdot 3\sqrt{5}$

$9\sqrt{10}$

E) $2\sqrt{27}\cdot 3\sqrt{30}$

$2\sqrt{3\cdot 3\cdot 3}\cdot 3\sqrt{30}$

$6\sqrt{3}\cdot 3\sqrt{30}$

$18\sqrt{90}$

$18\sqrt{9\cdot 10}$

$54\sqrt{10}$

F) $7\sqrt[3]{16}\cdot 4\sqrt[3]{400}$

$7\sqrt[3]{2^4}\cdot 4\sqrt[3]{8\cdot 50}$

$7\sqrt[3]{2^4}\cdot 4\sqrt[3]{2^3\cdot 50}$

$14\sqrt[3]{2}\cdot 8\sqrt[3]{50}$

$112\sqrt[3]{100}$

G) $\sqrt{15x^3}\cdot\sqrt{18x^2}\cdot\sqrt{12x^2}$

$\sqrt{15\cdot 18\cdot 12\cdot x^7}$

$\sqrt{3\cdot 5\cdot 2\cdot 3\cdot 3\cdot 2\cdot 2\cdot 3\cdot x^7}$

$\sqrt{2\cdot 2^2\cdot 3^2\cdot 3^2\cdot 5\cdot x^6\cdot x}$

$18x^3\sqrt{10x}$

H) $-6\sqrt{12}\cdot 3\sqrt{27x^4}$

$-18\sqrt{12\cdot 27\cdot x^4}$

$-18\sqrt{2^2\cdot 3^2\cdot 3^2\cdot x^4}$

$-18(18x^2)$

$-324x^2$

I) $\sqrt{32x^2y^2}\cdot 3\sqrt{18xy^3}$

$\sqrt{2\cdot 16x^2y^2}\cdot 3\sqrt{2\cdot 9xy^3}$

$4xy\sqrt{2}\cdot 9y\sqrt{2xy}$

$36xy^2\sqrt{4xy}$

$72xy^2\sqrt{xy}$

J) $\sqrt{12x^3}\cdot\sqrt{18x^2}\cdot\sqrt{27x^2}$

$2x\sqrt{3x}\cdot 3x\sqrt{2}\cdot 3x\sqrt{3}$

$18x^3\sqrt{2\cdot 3\cdot 3\cdot x}$

$54x^3\sqrt{2x}$

K) $7\sqrt{42x^4y^3}\cdot 2\sqrt{56x^5y^3}$

$14\sqrt{42\cdot 56\,x^9y^6}$

$14\sqrt{7\cdot 3\cdot 2\cdot 8\cdot 7x^9y^6}$

$14\sqrt{3\cdot 7^2\cdot 2^4\cdot x^8\cdot x\cdot y^6}$

$392x^4y^3\sqrt{3x}$

L) $3\sqrt{2}\left(4\sqrt{5}+\sqrt{6}\right)$

$12\sqrt{10}+3\sqrt{12}$

$12\sqrt{10}+6\sqrt{3}$

M) $\sqrt{48y}\left(x\sqrt{2}-y\sqrt{3}\right)$

$\sqrt{16\cdot 3y}\,(x\sqrt{2}-y\sqrt{3})$

$4x\sqrt{6y}-4y\sqrt{9y}$

$4x\sqrt{6y}-12y\sqrt{y}$

N) $\sqrt{150x^5}\left(x\sqrt{10x}+2y\sqrt{12y}\right)$

$5x^2\sqrt{6x}\,(x\sqrt{10x}+4y\sqrt{3y})$

$5x^3\sqrt{60x^2}+20x^2y\sqrt{18xy}$

$10x^4\sqrt{15}+60x^2y\sqrt{2xy}$

O) $\left(\sqrt{x}+6\right)^2$

$(\sqrt{x})^2+2(\sqrt{x})(6)+(6)^2$

$x+12\sqrt{x}+36$

P) $\left(\sqrt{2}-\sqrt{7}\right)^2$

$(\sqrt{2})^2-2(\sqrt{2})(\sqrt{7})+(\sqrt{7})^2$

$2-2\sqrt{14}+7$

$9-2\sqrt{14}$

Q) $\left(4\sqrt{3}-2\sqrt{5}\right)^2$

$(4\sqrt{3})^2-2(4\sqrt{3})(2\sqrt{5})+(2\sqrt{5})^2$

$48-16\sqrt{15}+20$

$68-16\sqrt{15}$

R) $\left(5\sqrt{2}-3\right)^2$

$(5\sqrt{2})^2-2(5\sqrt{2})(3)+(3)^2$

$50-30\sqrt{2}+9$

$59-30\sqrt{2}$

S) $\left(\sqrt{2}+\sqrt{7}\right)\left(\sqrt{11}-\sqrt{5}\right)$

$\sqrt{22}-\sqrt{10}+\sqrt{77}-\sqrt{35}$

T) $\left(\sqrt{13}+\sqrt{3}\right)\left(\sqrt{13}-\sqrt{7}\right)$

$13-\sqrt{91}+\sqrt{39}-\sqrt{21}$

U) $\left(3\sqrt{3}+\sqrt{8}\right)^2$

$(3\sqrt{3})^2+2(3\sqrt{3})(2\sqrt{2})+(\sqrt{8})^2$

$27+12\sqrt{6}+8$

$35+12\sqrt{6}$

V) $\left(\sqrt{x+1}+3\right)^2$

$(\sqrt{x+1})^2+2(\sqrt{x+1})(3)+(3)^2$

$x+1+6\sqrt{x+1}+9$

$x+10+6\sqrt{x+1}$

W) $\left(\sqrt{x+1}+\sqrt{2}\right)\left(\sqrt{x+1}-\sqrt{6}\right)$

$(x+1)-\sqrt{6x+6}+\sqrt{2x+2}-2\sqrt{3}$

$x+1-\sqrt{6x+6}+\sqrt{2x+2}-2\sqrt{3}$

X) $\left(\sqrt{x-2}-\sqrt{x}\right)^2$

$(\sqrt{x-2})^2-2(\sqrt{x-2})(\sqrt{x})+(\sqrt{x})^2$

$x-2-2\sqrt{x^2-2x}+x$

$2x-2-2\sqrt{x^2-2x}$

Rationalizing and Dividing Radicals

When working with radicals, a radical cannot be in the denominator. When left with a radical in the denominator, the expression must be rationalized. Multiply the top and bottom of the fraction by what is needed, not solely by what's in the radical. This will come into play when dividing radicals. When faced with a binomial in the denominator, the top and bottom must be multiplied by the conjugate.

Rationalize the denominator of each of the following.

A) $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\boxed{\frac{\sqrt{3}}{3}}$

B) $\frac{5}{\sqrt{10}}$

$\frac{5}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$

$\frac{5\sqrt{10}}{10}$

$\boxed{\frac{\sqrt{10}}{2}}$

C) $\frac{8}{\sqrt[5]{12}}$

$\frac{8}{\sqrt[5]{2^2 \cdot 3}} \cdot \frac{\sqrt[5]{2^3 \cdot 3^4}}{\sqrt[5]{2^3 \cdot 3^4}}$

$\frac{8\sqrt[5]{648}}{6}$

$\boxed{\frac{4\sqrt[5]{648}}{3}}$

D) $\frac{\sqrt{3}}{\sqrt{12x^3y^2}}$

$\frac{\sqrt{3}}{2xy\sqrt{3x}} \cdot \frac{\sqrt{3x}}{\sqrt{3x}}$

$\frac{3\sqrt{x}}{6x^2y}$

$\boxed{\frac{\sqrt{x}}{2x^2y}}$

E) $\frac{5}{\sqrt[3]{15x^3y^6z^2}}$

$\frac{5}{xy^2\sqrt[3]{3 \cdot 5 \cdot z^2}} \cdot \frac{\sqrt[3]{3^2 \cdot 5^2 \cdot z}}{\sqrt[3]{3^2 \cdot 5^2 \cdot z}}$

$\frac{5\sqrt[3]{225z}}{3 \cdot 5xy^2z}$

$\boxed{\frac{\sqrt[3]{225z}}{3xy^2z}}$

F) $\frac{10}{\sqrt[5]{16x^4y^8}}$

$\frac{10}{y\sqrt[5]{2^4 \cdot x^4 \cdot y^3}} \cdot \frac{\sqrt[5]{2 \cdot x \cdot y^2}}{\sqrt[5]{2xy^2}}$

$\frac{10\sqrt[5]{2xy^2}}{2xy^2}$

$\boxed{\frac{5\sqrt[5]{2xy^2}}{xy^2}}$

G) $\frac{4}{\sqrt[3]{12x^5y^7}}$

$\frac{4}{xy^2\sqrt[3]{2^2 \cdot 3 \cdot x^2 \cdot y}}$

$\frac{4\sqrt[3]{18xy^2}}{6x^2y^3}$

$\boxed{\frac{2\sqrt[3]{18xy^2}}{3x^2y^3}}$

H) $\sqrt{\frac{3}{2}}$

$\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\boxed{\frac{\sqrt{6}}{2}}$

I) $\sqrt{\frac{4}{3}}$

$\frac{\sqrt{4}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\boxed{\frac{2\sqrt{3}}{3}}$

J) $\frac{\sqrt{15x^3y^6}}{\sqrt{3x}}$

$\sqrt{\frac{15x^3y^6}{3x}}$

$\sqrt{5x^2y^6}$

$\boxed{xy^3\sqrt{5}}$

K) $\frac{3}{\sqrt[3]{36xy}}$

$\frac{3}{\sqrt[3]{6^2 \cdot xy}} \cdot \frac{\sqrt[3]{6x^2y^2}}{\sqrt[3]{6x^2y^2}}$

$\frac{3\sqrt[3]{6x^3y^2}}{6xy}$

$\boxed{\frac{\sqrt[3]{6x^2y^2}}{2xy}}$

L) $\frac{4x^5y^2}{\sqrt[4]{128x^6y^7z^2}}$

$\frac{4x^5y^2}{2xy\sqrt[4]{2^3x^2y^3z^2}}$

$\frac{2x^4y\sqrt[4]{2x^2yz^2}}{2xyz}$

$\boxed{\frac{x^3\sqrt[4]{2x^2yz^2}}{z}}$

M) $\sqrt{\frac{228x^6y^2}{x^4y^2z}}$

$\frac{\sqrt{228x^2}}{\sqrt{z}}$

$\frac{2x\sqrt{57}}{\sqrt{z}} \cdot \frac{\sqrt{z}}{\sqrt{z}}$

$\boxed{\frac{2x\sqrt{57z}}{z}}$

N) $\frac{3x^2yz}{\sqrt[5]{729x^8y^{16}z^2}}$

$\frac{3x^2yz}{3xy^3\sqrt[5]{3x^3yz^2}} \cdot \frac{\sqrt[5]{3^4x^2y^4z^3}}{\sqrt[5]{3^4x^2y^4z^3}}$

$\frac{3x^2yz\sqrt[5]{81x^2y^4z^3}}{9x^2y^4z}$

$\boxed{\frac{\sqrt[5]{81x^2y^4z^3}}{3y^3}}$

O) $\frac{14xy}{\sqrt{196x^5y^3}}$

$\frac{14xy}{14x^2y\sqrt{xy}} \cdot \frac{\sqrt{xy}}{\sqrt{xy}}$

$\frac{14xy\sqrt{xy}}{14x^3y^2}$

$\boxed{\frac{\sqrt{xy}}{x^2y}}$

P) $\frac{12xy^5}{\sqrt[4]{480x^5y^2}}$

$\frac{12xy^5}{2x\sqrt[4]{2 \cdot 3 \cdot 5 \cdot x \cdot y^2}} \cdot \frac{\sqrt[4]{2^3 \cdot 3^3 \cdot 5^3 \cdot x^3 \cdot y^2}}{\sqrt[4]{2^3 \cdot 3^3 \cdot 5^3 \cdot x^3 \cdot y^2}}$

$\frac{12xy^5\sqrt[4]{27000x^3y^2}}{2^2 \cdot 3 \cdot 5 \cdot x^2 \cdot y}$

$\boxed{\frac{\sqrt[4]{27000x^3y^2}}{5x}}$

Q) $\dfrac{\sqrt{5}+8\sqrt{15}}{\sqrt{5}}$

$\dfrac{\sqrt{5}(\sqrt{5}+8\sqrt{15})}{\sqrt{5}\cdot\sqrt{5}}$

$\dfrac{5+40\sqrt{3}}{5}$

$\dfrac{\cancel{5}(1+8\sqrt{3})}{\cancel{5}}$

$\boxed{1+8\sqrt{3}}$

R) $\dfrac{\sqrt{98}-6\sqrt{128}}{\sqrt{2}}$

$\dfrac{\sqrt{98}}{\sqrt{2}} - \dfrac{6\sqrt{128}}{\sqrt{2}}$

$\sqrt{\dfrac{98}{2}} - 6\sqrt{\dfrac{128}{2}}$

$\sqrt{49} - 6\sqrt{64}$

$7-48$

$\boxed{-41}$

S) $\dfrac{4\sqrt{275}+3\sqrt{147}}{\sqrt{3}}$

$\left(\dfrac{20\sqrt{11}+21\sqrt{3}}{\sqrt{3}}\right)\cdot\dfrac{\sqrt{3}}{\sqrt{3}}$

$\dfrac{20\sqrt{33}+21\sqrt{9}}{3}$

$\boxed{\dfrac{20\sqrt{33}+63}{3}}$

Rationalize each of the following by multiplying by the conjugate.

A) $\dfrac{2\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

$\left(\dfrac{2\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}\right)\left(\dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}\right)$

$\dfrac{14-2\sqrt{21}-\sqrt{21}+3}{7-3}$

$\boxed{\dfrac{17-3\sqrt{21}}{4}}$

B) $\dfrac{10}{2\sqrt{3}-\sqrt{7}}$

$\left(\dfrac{10}{2\sqrt{3}-\sqrt{7}}\right)\left(\dfrac{2\sqrt{3}+\sqrt{7}}{2\sqrt{3}+\sqrt{7}}\right)$

$\dfrac{10(2\sqrt{3}+\sqrt{7})}{12-7}$

$\dfrac{^{2}\cancel{10}(2\sqrt{3}+\sqrt{7})}{\cancel{5}}$

$\boxed{4\sqrt{3}+2\sqrt{7}}$

C) $\dfrac{2}{5-\sqrt{3}}$

$\left(\dfrac{2}{5-\sqrt{3}}\right)\left(\dfrac{5+\sqrt{3}}{5+\sqrt{3}}\right)$

$\dfrac{2(5+\sqrt{3})}{25-3}$

$\dfrac{\cancel{2}(5+\sqrt{3})}{\cancel{22}\ 11}$

$\boxed{\dfrac{5+\sqrt{3}}{11}}$

D) $\dfrac{6}{\sqrt{6}+\sqrt{5}}$

$\left(\dfrac{6}{\sqrt{6}+\sqrt{5}}\right)\left(\dfrac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}\right)$

$\dfrac{6(\sqrt{6}-\sqrt{5})}{6-5}$

$\boxed{6\sqrt{6}-6\sqrt{5}}$

E) $\dfrac{5}{2\sqrt{10}+5}$

$\left(\dfrac{5}{2\sqrt{10}+5}\right)\left(\dfrac{2\sqrt{10}-5}{2\sqrt{10}-5}\right)$

$\dfrac{5(2\sqrt{10}-5)}{40-25}$

$\dfrac{\cancel{5}(2\sqrt{10}-5)}{\cancel{15}\ 3}$

$\boxed{\dfrac{2\sqrt{10}-5}{3}}$

F) $\dfrac{10}{2\sqrt{3}-\sqrt{2}}$

$\left(\dfrac{10}{2\sqrt{3}-\sqrt{2}}\right)\left(\dfrac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}+\sqrt{2}}\right)$

$\dfrac{10(2\sqrt{3}+\sqrt{2})}{12-2}$

$\dfrac{10(2\sqrt{3}+\sqrt{2})}{10}$

$\boxed{2\sqrt{3}+\sqrt{2}}$

G) $\dfrac{30}{7+\sqrt{2}}$

$\left(\dfrac{30}{7+\sqrt{2}}\right)\left(\dfrac{7-\sqrt{2}}{7-\sqrt{2}}\right)$

$\dfrac{30(7-\sqrt{2})}{49-2}$

$\dfrac{30(7-\sqrt{2})}{47}$

$\boxed{\dfrac{210-30\sqrt{2}}{47}}$

H) $\dfrac{12}{\sqrt{5}-\sqrt{6}}$

$\left(\dfrac{12}{\sqrt{5}-\sqrt{6}}\right)\left(\dfrac{\sqrt{5}+\sqrt{6}}{\sqrt{5}+\sqrt{6}}\right)$

$\dfrac{12(\sqrt{5}+\sqrt{6})}{5-6}$

$\dfrac{12(\sqrt{5}+\sqrt{6})}{-1}$

$\boxed{-12\sqrt{5}-12\sqrt{6}}$

I) $\dfrac{20}{3\sqrt{2}-4}$

$\left(\dfrac{20}{3\sqrt{2}-4}\right)\left(\dfrac{3\sqrt{2}+4}{3\sqrt{2}+4}\right)$

$\dfrac{20(3\sqrt{2}+4)}{18-16}$

$\dfrac{20(3\sqrt{2}+4)}{2}$

$\boxed{30\sqrt{2}+40}$

J) $\frac{27}{12-\sqrt{15}}$

$\left(\frac{27}{12-\sqrt{15}}\right)\left(\frac{12+\sqrt{15}}{12+\sqrt{15}}\right)$

$\frac{27(12+\sqrt{15})}{144-15}$

$\frac{\overset{9}{\cancel{27}}(12+\sqrt{15})}{\underset{43}{\cancel{129}}}$

$\boxed{\frac{108+9\sqrt{15}}{43}}$

K) $\frac{18}{3\sqrt{2}+2\sqrt{5}}$

$\left(\frac{18}{3\sqrt{2}+2\sqrt{5}}\right)\left(\frac{3\sqrt{2}-2\sqrt{5}}{3\sqrt{2}-2\sqrt{5}}\right)$

$\frac{18(3\sqrt{2}-2\sqrt{5})}{18-20}$

$\frac{18(3\sqrt{2}-2\sqrt{5})}{-2}$

$\boxed{18\sqrt{5}-27\sqrt{2}}$

L) $\frac{3\sqrt{512}}{\sqrt{18}+\sqrt{50}}$

$\frac{48\sqrt{2}}{3\sqrt{2}+5\sqrt{2}}$

$\frac{48\sqrt{2}}{8\sqrt{2}}$

$\boxed{6}$

M) $\frac{4-x}{\sqrt{x}+2}$

$\left(\frac{4-x}{\sqrt{x}+2}\right)\left(\frac{\sqrt{x}-2}{\sqrt{x}-2}\right)$

$\frac{(-1)(x-4)(\sqrt{x}-2)}{x-4}$

$(-1)(\sqrt{x}-2)$

$\boxed{2-\sqrt{x}}$

N) $\frac{\sqrt{x}-\sqrt{3}}{\sqrt{x}+\sqrt{3}}$

$\left(\frac{\sqrt{x}-\sqrt{3}}{\sqrt{x}+\sqrt{3}}\right)\left(\frac{\sqrt{x}-\sqrt{3}}{\sqrt{x}-\sqrt{3}}\right)$

$\frac{(\sqrt{x}-\sqrt{3})^2}{x-3}$

$\boxed{\frac{x-2\sqrt{3x}+3}{x-3}}$

O) $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

$\left(\frac{\sqrt{2}-\sqrt{3}}{\sqrt{6}-\sqrt{2}}\right)\left(\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)$

$\frac{\sqrt{12}+2-3\sqrt{2}-\sqrt{6}}{6-2}$

$\boxed{\frac{2\sqrt{3}+2-3\sqrt{2}-\sqrt{6}}{4}}$

P) $\frac{\sqrt{21}-1}{\sqrt{3}-\sqrt{7}}$

$\left(\frac{\sqrt{21}-1}{\sqrt{3}-\sqrt{7}}\right)\left(\frac{\sqrt{3}+\sqrt{7}}{\sqrt{3}+\sqrt{7}}\right)$

$\frac{3\sqrt{7}+7\sqrt{3}-\sqrt{3}-\sqrt{7}}{3-7}$

$\frac{2\sqrt{7}+6\sqrt{3}}{-4}$

$\boxed{\frac{-3\sqrt{3}-\sqrt{7}}{2}}$

Q) $\frac{3\sqrt{2}+4}{4-\sqrt{8}}$

$\left(\frac{3\sqrt{2}+4}{4-2\sqrt{2}}\right)\left(\frac{4+2\sqrt{2}}{4+2\sqrt{2}}\right)$

$\frac{12\sqrt{2}+12+16+8\sqrt{2}}{16-8}$

$\frac{20\sqrt{2}+28}{8}$

$\boxed{\frac{5\sqrt{2}+7}{2}}$

R) $\frac{3x-4}{\sqrt{3x}+2}$

$\left(\frac{3x-4}{\sqrt{3x}+2}\right)\left(\frac{\sqrt{3x}-2}{\sqrt{3x}-2}\right)$

$\frac{(3x-4)(\sqrt{3x}-2)}{3x-4}$

$\boxed{\sqrt{3x}-2}$

S) $\frac{\sqrt{15}-1}{\sqrt{3}+\sqrt{5}}$

$\left(\frac{\sqrt{15}-1}{\sqrt{3}+\sqrt{5}}\right)\left(\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}\right)$

$\frac{3\sqrt{5}-5\sqrt{3}-\sqrt{3}+\sqrt{5}}{3-5}$

$\frac{4\sqrt{5}-6\sqrt{3}}{-2}$

$\boxed{3\sqrt{3}-2\sqrt{5}}$

T) $\frac{\sqrt{x}+4}{\sqrt{x}-3}$

$\left(\frac{\sqrt{x}+4}{\sqrt{x}-3}\right)\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right)$

$\frac{x+3\sqrt{x}+4\sqrt{x}+12}{x-9}$

$\boxed{\frac{x+7\sqrt{x}+12}{x-9}}$

U) $\frac{\sqrt{12}-\sqrt{27}}{\sqrt{18}+\sqrt{12}}$

$\left(\frac{2\sqrt{3}-3\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}\right)\left(\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}\right)$

$\frac{6\sqrt{6}-12-9\sqrt{6}+18}{18-12}$

$\frac{6-3\sqrt{6}}{6} = \boxed{\frac{2-\sqrt{6}}{2}}$

V) $\frac{\sqrt{2x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

$\left(\frac{\sqrt{2x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}\right)\left(\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$

$\boxed{\frac{x\sqrt{2}+\sqrt{2xy}+\sqrt{xy}+y}{x-y}}$

W) $\frac{\sqrt{x+2}-\sqrt{x}}{\sqrt{x+2}+\sqrt{x}}$

$\left(\frac{\sqrt{x+2}-\sqrt{x}}{\sqrt{x+2}+\sqrt{x}}\right)\left(\frac{\sqrt{x+2}-\sqrt{x}}{\sqrt{x+2}-\sqrt{x}}\right)$

$\frac{x+2-2\sqrt{x^2+2x}+x}{x+2-x}$

$\frac{2x+2-2\sqrt{x^2+2x}}{2}$

$\boxed{x+1-\sqrt{x^2+2x}}$

X) $\frac{3\sqrt{y}-3\sqrt{x}}{2\sqrt{x}-2\sqrt{y}}$

$\left(\frac{3\sqrt{y}-3\sqrt{x}}{2\sqrt{x}-2\sqrt{y}}\right)\left(\frac{2\sqrt{x}+2\sqrt{y}}{2\sqrt{x}+2\sqrt{y}}\right)$

$\frac{6\sqrt{xy}+6y-6x-6\sqrt{xy}}{4x-4y}$

$\frac{-6(x-y)}{4(x-y)}$

$\boxed{-\frac{3}{2}}$

Addition/Subtraction with Radicals

Simplify each of the following.

A) $6\sqrt{3}+7\sqrt{3}$

$13\sqrt{3}$

B) $4\sqrt{2}-9\sqrt{2}+6\sqrt{2}$

$\sqrt{2}$

C) $\sqrt{12}+\sqrt{27}$

$2\sqrt{3}+3\sqrt{3}$

$5\sqrt{3}$

D) $4\sqrt{12}-6\sqrt{18}+\sqrt{8}$

$8\sqrt{3}-18\sqrt{2}+2\sqrt{2}$

$8\sqrt{3}-16\sqrt{2}$

E) $3\sqrt[3]{16}+\sqrt[3]{54}-\sqrt{50}$

$6\sqrt[3]{2}+3\sqrt[3]{2}-5\sqrt{2}$

$9\sqrt[3]{2}-5\sqrt{2}$

F) $\sqrt[3]{9}-\sqrt[3]{81}$

$\sqrt[3]{9}-3\sqrt[3]{3}$

G) $2\sqrt{112}+3\sqrt{63}-8\sqrt{175}$

$8\sqrt{7}+9\sqrt{7}-40\sqrt{7}$

$-23\sqrt{7}$

H) $5\sqrt{27}-4\sqrt{3}+2\sqrt{12}$

$15\sqrt{3}-4\sqrt{3}+2\sqrt{3}$

$15\sqrt{3}$

I) $9a^2\sqrt{3}-36a^2\sqrt{3}$

$-27a^2\sqrt{3}$

J) $9\sqrt{48}-3\sqrt{27}+12\sqrt{147}$

$36\sqrt{3}-9\sqrt{3}+84\sqrt{3}$

$111\sqrt{3}$

K) $13\sqrt{90}-4\sqrt{40}+7\sqrt{250}$

$39\sqrt{10}-8\sqrt{10}+35\sqrt{10}$

$66\sqrt{10}$

L) $12\sqrt[3]{24}-6\sqrt[3]{81}$

$24\sqrt[3]{3}-18\sqrt[3]{3}$

$6\sqrt[3]{3}$

M) $3a\sqrt{612b^2}+6b\sqrt{425a^2}$

$18ab\sqrt{17}+30ab\sqrt{17}$

$48ab\sqrt{17}$

N) $15\sqrt{740x^5y^5}-7xy\sqrt{1215x^3y^3}$

74 · 10   81 15

37 2 2 5   9 9

$30x^2y^2\sqrt{185xy}-63x^2y^2\sqrt{15xy}$

O) $25ab\sqrt{396a^3b^2} + 14\sqrt{891a^5b^4}$

$25ab\sqrt{36\cdot 11\cdot a^2\cdot a\cdot b^2} + 14\sqrt{81\cdot 11\cdot a^4\cdot a\cdot b^4}$

$150a^2b^2\sqrt{11a} + 126a^2b^2\sqrt{11a}$

$276a^2b^2\sqrt{11a}$

P) $14x^2y^2\sqrt{1100xy} - 27xy\sqrt{704x^3y^3}$

$14x^2y^2\sqrt{11\cdot 100xy} - 27xy\sqrt{64\cdot 11x^3y^3}$

$140x^2y^2\sqrt{11xy} - 216x^2y^2\sqrt{11xy}$

$-76x^2y^2\sqrt{11xy}$

Q) $2\sqrt[5]{729x^6y^2} + 3x\sqrt[5]{106xy^2}$

$2\sqrt[5]{27\cdot 27\cdot x^6\, y^2} + 3x\sqrt[5]{2\cdot 53\cdot xy^2}$

$6x\sqrt[5]{3xy^2} + 3x\sqrt[5]{106xy^2}$

R) $\sqrt{\frac{80}{3}} + \sqrt{\frac{15}{16}} + \sqrt{\frac{27}{5}}$

$\frac{\sqrt{16\cdot 5}}{\sqrt{3}} + \frac{\sqrt{15}}{\sqrt{16}} + \frac{\sqrt{27}}{\sqrt{5}}$

$\frac{4\sqrt{5}}{\sqrt{3}} + \frac{\sqrt{15}}{4} + \frac{3\sqrt{3}}{\sqrt{5}}$

$\frac{4\sqrt{15}}{3} + \frac{\sqrt{15}}{4} + \frac{3\sqrt{15}}{5}$

$\frac{80\sqrt{15}}{60} + \frac{15\sqrt{15}}{60} + \frac{36\sqrt{15}}{60}$

$\boxed{\frac{131\sqrt{15}}{60}}$

S) $\sqrt{\frac{8}{5}} - \sqrt{\frac{40}{49}} - \sqrt{\frac{45}{2}}$

$\frac{2\sqrt{2}}{\sqrt{5}} - \frac{2\sqrt{10}}{7} - \frac{3\sqrt{5}}{\sqrt{2}}$

$\frac{2\sqrt{10}}{5} - \frac{2\sqrt{10}}{7} - \frac{3\sqrt{10}}{2}$

$\frac{28\sqrt{10}}{70} - \frac{20\sqrt{10}}{70} - \frac{105\sqrt{10}}{70}$

$-\frac{97\sqrt{10}}{70}$

T) $\sqrt{\frac{4}{15}} + \sqrt{\frac{48}{10}} - \sqrt{\frac{245}{6}}$

$\frac{2}{\sqrt{15}} + \frac{4\sqrt{3}}{\sqrt{10}} - \frac{7\sqrt{5}}{\sqrt{6}}$

$\frac{2\sqrt{15}}{15} + \frac{4\sqrt{30}}{10} - \frac{7\sqrt{30}}{6}$

$\frac{4\sqrt{15}}{30} + \frac{12\sqrt{30}}{30} - \frac{35\sqrt{30}}{30}$

$\boxed{\frac{4\sqrt{15} - 23\sqrt{30}}{30}}$

U) $\sqrt{\frac{180b^2}{7a}} + \sqrt{\frac{45b^2}{7a}}$

$\frac{\sqrt{36\cdot 5b^2}}{\sqrt{7a}} + \frac{3b\sqrt{5}}{\sqrt{7a}}$

$\frac{6b\sqrt{5} + 3b\sqrt{5}}{\sqrt{7a}}$

$\frac{9b\sqrt{5}}{\sqrt{7a}}$

$\boxed{\frac{9b\sqrt{35a}}{7a}}$

V) $\sqrt{\frac{112}{3}} - \sqrt{\frac{75}{7}} - \sqrt{\frac{84}{25}}$

$\frac{\sqrt{112}}{\sqrt{3}} - \frac{\sqrt{75}}{\sqrt{7}} - \frac{\sqrt{84}}{\sqrt{25}}$

$\frac{4\sqrt{7}}{\sqrt{3}} - \frac{5\sqrt{3}}{\sqrt{7}} - \frac{2\sqrt{21}}{5}$

$\frac{4\sqrt{21}}{3} - \frac{5\sqrt{21}}{7} - \frac{2\sqrt{21}}{5}$

$\frac{140\sqrt{21}}{105} - \frac{75\sqrt{21}}{105} - \frac{42\sqrt{21}}{105}$

$\boxed{\frac{23\sqrt{21}}{105}}$

W) $\sqrt{\frac{3}{5}}\left(\sqrt{\frac{20}{3}}-\frac{\sqrt{15}}{6}\right)$

$\frac{\sqrt{3}}{\sqrt{5}}\left(\frac{2\sqrt{5}}{\sqrt{3}}-\frac{\sqrt{15}}{6}\right)$

$2-\frac{3\sqrt{5}}{6\sqrt{5}}$

$2-\frac{1}{2}$

$\frac{4}{2}-\frac{1}{2}$

$\boxed{\frac{3}{2}}$

X) $\frac{\sqrt{x-\sqrt{x}}\cdot\sqrt{x+\sqrt{x}}}{\sqrt{x-1}}$

$\frac{\sqrt{(x-\sqrt{x})(x+\sqrt{x})}}{\sqrt{x-1}}$

$\frac{\sqrt{x^2-x}}{\sqrt{x-1}}\cdot\frac{\sqrt{x-1}}{\sqrt{x-1}}$

$\frac{\sqrt{x(x-1)}\cdot\sqrt{x-1}}{x-1}$

$\frac{\sqrt{x(x-1)^2}}{x-1}$

$\frac{(x-1)\sqrt{x}}{x-1}$

$\boxed{\sqrt{x}}$

Y) $\frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}+\frac{\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

$\frac{\sqrt{x}(\sqrt{x}-\sqrt{y})}{x-y}+\frac{\sqrt{y}(\sqrt{x}+\sqrt{y})}{x-y}$

$\frac{x-\sqrt{xy}+\sqrt{xy}+y}{x-y}$

$\boxed{\frac{x+y}{x-y}}$

Z) $\frac{\sqrt{x}}{\sqrt{x}+1}+\frac{\sqrt{x}}{\sqrt{x}-1}$

$\frac{\sqrt{x}(\sqrt{x}-1)}{x-1}+\frac{\sqrt{x}(\sqrt{x}+1)}{x-1}$

$\frac{x-\sqrt{x}+x+\sqrt{x}}{x-1}$

$\boxed{\frac{2x}{x-1}}$

When solving an equation you always use inverse operations. For equations involving a square root, you must square both sides of the equation to get your solution. Fill in the missing numbers to show how squaring the $\sqrt{x}$ yields a result of x.

$$\left(\sqrt{x}\right)^2=\left(x^{1/2}\right)^2=x^{\frac{2}{2}}=x^1$$

It was stated earlier that you cannot multiply radicals that have different indexes if the radicand is different. However, you can multiply them if the radicands are the same. Refer to your class notes to show that $\sqrt{x}\cdot\sqrt[3]{x}$ can actually be done.

$x^{1/2}\cdot x^{1/3}$

$x^{1/2+1/3}$

$x^{3/6+2/6}$

$x^{5/6}$

$\sqrt[6]{x^5}$

Equations with Radicals

When solving an equation with radicals, always check the solutions for any extraneous roots. Any solution that does not make the original equation true is an extraneous root.

Find all real solutions for each of the following equations. ***Be sure to check for extraneous roots.***

A) $2\sqrt{x}-3=1$

$2\sqrt{x}=4$

$(\sqrt{x})^2=(2)^2$

$\boxed{x=4}$

Check

$2\sqrt{4}-3=1$

$4-3=1$

$1=1$ ✓

B) $\sqrt{2x}=4$

$\frac{2x}{2}=\frac{16}{2}$

$\boxed{x=8}$

Check

$\sqrt{2(8)}=4$

$\sqrt{16}=4$

$4=4$ ✓

C) $\sqrt{x}-7=0$

$\sqrt{x}=7$

$\boxed{x=49}$

Check

$\sqrt{49}-7=0$

$7-7=0$

$0=0$ ✓

D) $\sqrt{x}+6=0$

$(\sqrt{x})^2=(-6)^2$

$x=36$

Check

$\sqrt{36}+6=0$

$6+6=0$

$12\neq 0$

No solution

E) $\sqrt[3]{x}+5=3$

$(\sqrt[3]{x})^3=(-2)^3$

$\boxed{x=-8}$

Check

$\sqrt[3]{-8}+5=3$

$-2+5=3$

$3=3$ ✓

F) $\sqrt[3]{2x}+5=3$

$(\sqrt[3]{2x})^3=(-2)^3$

$2x=-8$

$\boxed{x=-4}$

Check

$\sqrt[3]{2(-4)}+5=3$

$\sqrt[3]{-8}+5=3$

$-2+5=3$

$3=3$ ✓

G) $\sqrt{x}+\sqrt{x+2}=0$

$(\sqrt{x})^2=(-\sqrt{x+2})^2$

$x=x+2$

$-x \quad -x$

$0\neq 2$

No solution

H) $\dfrac{\sqrt[4]{x^2}}{2}=\sqrt[4]{x-4}$

$(\sqrt[4]{x^2})^4=(2\sqrt[4]{x-4})^4$

$x^2=16(x-4)$

$x^2-16x+64=0$

$(x-8)^2=0$

$\boxed{x=8}$

Check

$\frac{\sqrt[4]{8^2}}{2}=\sqrt[4]{8-4}$

$\frac{\sqrt[4]{64}}{2}=\sqrt[4]{4}$ $\quad \frac{2\sqrt[4]{4}}{2}=\sqrt[4]{4}$ ✓

I) $(\sqrt{2x+15})^2=(x)^2$

$2x+15=x^2$

$x^2-2x-15=0$

$(x-5)(x+3)=0$

$x=5$ $\quad x=-3$

Not a solution

Check

$\sqrt{2(5)+15}=5$

$\sqrt{25}=5$

$5=5$

$\boxed{x=5}$

J) $\sqrt{2x+5}-3=x$

$(\sqrt{2x+5})^2=(x+3)^2$

$2x+5=x^2+6x+9$

$-2x-5$

$x^2+4x+4=0$

$\sqrt{(x+2)^2}=\sqrt{0}$

$x+2=0$

$\boxed{x=-2}$

Check

$\sqrt{2(-2)+5}-3=-2$

$\sqrt{-4+5}-3=-2$

$\sqrt{1}-3=-2$

$1-3=-2$

$-2=-2$ ✓

K) $7+\sqrt{21-3x}=x$

$(\sqrt{21-3x})^2=(x-7)^2$

$21-3x=x^2-14x+49$

$x^2-11x+28=0$

$(x-4)(x-7)=0$

$\cancel{x=4}$ $\quad x=7$

Check

$x=7$

$7+\sqrt{21-3(7)}=7$

$7+\sqrt{21-21}=7$

$7+0=7$ ✓

L) $\sqrt{x+33}-\sqrt{x}=3$

$(\sqrt{x+33})^2=(\sqrt{x}+3)^2$

$x+33=x+6\sqrt{x}+9$

$\frac{24}{6}=\frac{6\sqrt{x}}{6}$

$\sqrt{x}=4$

$\boxed{x=16}$

Check

$\sqrt{16+33}-\sqrt{16}=3$

$\sqrt{49}-4=3$

$7-4=3$ ✓

M) $\sqrt{3x+1}-\sqrt{2x-1}=1$

$(\sqrt{3x+1})^2=(\sqrt{2x-1}+1)^2$

$3x+1=2x-1+2\sqrt{2x-1}+1$

$(x+1)=(2\sqrt{2x-1})^2$

$x^2+2x+1=4(2x-1)$

$x^2+2x+1=8x-4$

$-8x-4=$

$x^2-6x+5=0$

$(x-1)(x-5)=0$

$x=1 \quad x=5$

Check

$x=1 \quad \sqrt{4}-\sqrt{1}=1$

$2-1=1$

$1=1$ ✓

$x=5 \quad \sqrt{16}-\sqrt{9}=1$

$4-3=1$

$1=1$ ✓

N) $(\sqrt{x+3}+1)^2=(\sqrt{x+4})^2$

$x+3+2\sqrt{x+3}+1=x+4$

$x+4+2\sqrt{x+3}=x+4$

$2\sqrt{x+3}=0$

$(\sqrt{x+3})^2=(0)^2$

$x+3=0$

$\boxed{x=-3}$

O) $\sqrt{x+6}-\sqrt{x}=\sqrt{2}$

$(\sqrt{x+6})^2=(\sqrt{x}+\sqrt{2})^2$

$x+6=x+2\sqrt{2x}+2$

$\frac{4}{2}=\frac{2\sqrt{2x}}{2}$

$(2)^2=(\sqrt{2x})^2$

$2x=4$

$\boxed{x=2}$

Check

$\sqrt{8}-\sqrt{2}=\sqrt{2}$

$2\sqrt{2}-\sqrt{2}=\sqrt{2}$

$\sqrt{2}=\sqrt{2}$ ✓

P) $(\sqrt{2x+26})^2=(3\sqrt{2}+4\sqrt{3})^2$

$2x+26=18+24\sqrt{6}+48$

$2x+26=66+24\sqrt{6}$

$\frac{2x}{2}=\frac{40+24\sqrt{6}}{2}$

$\boxed{x=20+12\sqrt{6}}$

Q) $(\sqrt{5x+1})^2=(\sqrt{6}-\sqrt{5})^2$

$5x+1=6-2\sqrt{30}+5$

$5x+1=11-2\sqrt{30}$

$\frac{5x}{5}=\frac{10-2\sqrt{30}}{5}$

$\boxed{x=\frac{10-2\sqrt{30}}{5}}$

R) $(\sqrt{5x+3})^2=(4\sqrt{3})^2$

$5x+3=48$

$-3 \quad -3$

$5x=45$

$\boxed{x=9}$

S) $(\sqrt{3x+12})^2=(\sqrt{x+2}-2)^2$

$3x+12=x+2-4\sqrt{x+2}+4$

$3x+12=x+6-4\sqrt{x+2}$

$-x-6 \quad -x-6$

$\frac{2x+6}{2}=\frac{-4\sqrt{x+2}}{2}$

$(x+3)^2=(-2\sqrt{x+2})^2$

$x^2+6x+9=4x+8$

$-4x-8 \quad -4x-8$

$x^2+2x+1=0$

$(x+1)=0$

$\boxed{\text{No solution}}$

T) $\sqrt{\frac{1}{2}x+4}=\frac{1}{4}x+1$

$(4\sqrt{\tfrac{1}{2}x+4})^2=(x+4)^2$

$16(\tfrac{1}{2}x+4)=x^2+8x+16$

$8x+64=x^2+8x+16$

$48=x^2$

$\sqrt{x^2}=\sqrt{48}$

$\boxed{x=\pm 4\sqrt{3}}$

U) $(\sqrt[3]{x^2+47})^3=(6)^3$

$x^2+47=216$

$-47 \quad -47$

$x^2=169$

$\boxed{x=\pm 13}$

V) $\sqrt[4]{(3x+1)^3}=27$

$((3x+1)^{3/4})^{4/3}=(27)^{4/3}$

$3x+1=81$

$-1 \quad -1$

$3x=80$

$\boxed{x=\frac{80}{3}}$

W) $x-11\sqrt{x}+28=0$

$(x+28)^2=(11\sqrt{x})^2$

$x^2+56x+781=121x$

$-121x \quad -121x$

$x^2-65x+784=0$

$(x-16)(x-49)=0$

$\boxed{x=16 \quad x=49}$

X) $(\sqrt{x^2+7})^2=(4\sqrt{2})^2$

$x^2+7=32$

$-7 \quad -7$

$\sqrt{x^2}=\sqrt{25}$

$\boxed{x=\pm 5}$

Y) $\frac{10}{x}-\frac{3}{\sqrt{x}}=1$

$\frac{10}{x}-\frac{3\sqrt{x}}{x}=\frac{x}{x}$

$10-3\sqrt{x}=x$

$x+3\sqrt{x}-10=0$

$(\sqrt{x}+5)(\sqrt{x}-2)=0$

$\sqrt{x}+5=0 \quad x-2=0$

$\sqrt{x}=-5 \quad \sqrt{x}=2$

$x=25 \quad \boxed{x=4}$

Z) $\frac{21}{x}-\frac{10}{\sqrt{x}}+1=0$

$\frac{21}{x}-\frac{10\sqrt{x}}{x}+\frac{x}{x}=0$

$x-10\sqrt{x}+21=0$

$(\sqrt{x}-3)(\sqrt{x}-7)=0$

$\sqrt{x}-3=0 \quad \sqrt{x}-7=0$

$\sqrt{x}=3 \quad \sqrt{x}=7$

$\boxed{x=9 \quad x=49}$

Imaginary and Complex Numbers

Since the square root of a negative number is not real, a different type of number was invented to represent them. Imaginary numbers come from taking the square root of a negative number. The following is useful when dealing with imaginary numbers.

$i = \sqrt{-1} = i$

$i^5 = i \cdot i^4 = i \cdot (1) = i$

$i^2 = \left(\sqrt{-1}\right)^2 = -1$

$i^6 = i^2 \cdot i^4 = (-1)(1) = -1$

$i^3 = i \cdot i^2 = i \cdot (-1) = -i$

$i^7 = i^3 \cdot i^4 = (-i)(1) = -i$

$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$

$i^8 = i^4 \cdot i^4 = (1) \cdot (1) = 1$

This pattern will always repeat and continue. When asked to simplify something like i^{22}, just divide 22 by 4. The remainder is 2. The reason this is done is that for every power that is a multiple of 4, the result is one (see above examples). The next step is to evaluate i^2. Here is what is actually happening.

$i^{22} = i^{20} \cdot i^2 = \left(i^4\right)^5 \cdot i^2 = (1)^5 \cdot i^2 = i^2 = -1$

Using the power of a power rule, the term i^{20} can be rewritten as $\left(i^4\right)^5$. The term i^4 is used inside the parenthesis because $i^4 = 1$.

Evaluate and simplify each of the following.

A) $\sqrt{-9}$

$3i$

B) $\sqrt{-12}$

$2\sqrt{3}i$

C) $\sqrt[3]{-27}$

-3

D) $\sqrt{-49}$

$7i$

E) $\sqrt{-121}$

$11i$

F) $\sqrt[3]{-8}$

-2

G) $4\sqrt{-16}$

$16i$

H) $-\sqrt{-54}$

$-3\sqrt{6}i$

I) $\sqrt{-75}$

$5\sqrt{3}i$

J) $3i^2$

$3(-1)$

-3

K) $-12i^{17}$

$-12(i^4)^4 \cdot i$

$-12(1)^4 \cdot i$

$-12i$

L) $6i^{254}$

$6(i^4)^{63} \cdot i^2$

$6(1)^{63}(-1)$

-6

M) $16i^{218}$

$16(i^4)^{54} \cdot i^2$

$16(1)^{54} \cdot (-1)$

-16

N) i^{417}

$(i^4)^{104} \cdot i$

i

O) $17i^{2436}$

$17(i^4)^{609}$

$17(1)^{609}$

17

P) $-i^{513}$

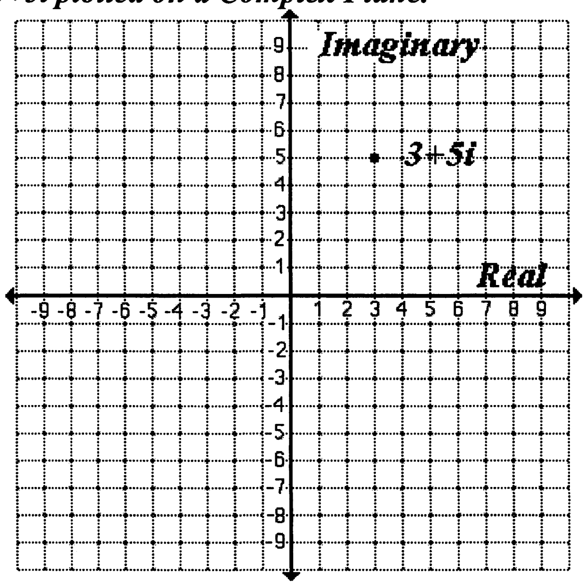
$-(i^4)^{128} \cdot i$

$-i$

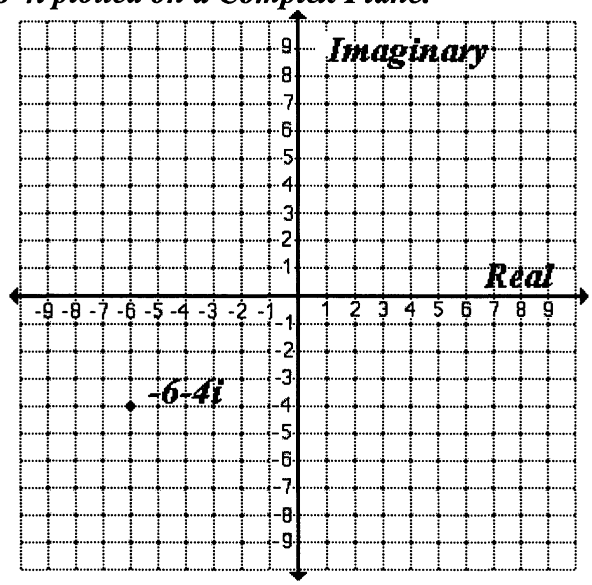
Plotting Complex Numbers

A complex number can be plotted on a plane. Normally, to plot an ordered pair on a plane, we need an x and y axis. In the case of complex numbers, a real number axis and an imaginary axis will be used. The horizontal axis will be the real number axis, while the vertical is the imaginary. This plane can be called the complex plane when it is used to plot complex or imaginary numbers.

Here is an example of the complex number 3+5i plotted on a Complex Plane.



Here is an example of the complex number -6-4i plotted on a Complex Plane.



Are all real numbers complex? Why or why not?

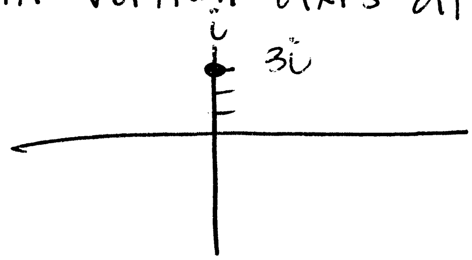
yes, a complex number is made of a real and an imaginary number. In the case of a real number, the imaginary number is 0.

complex # → a + bi

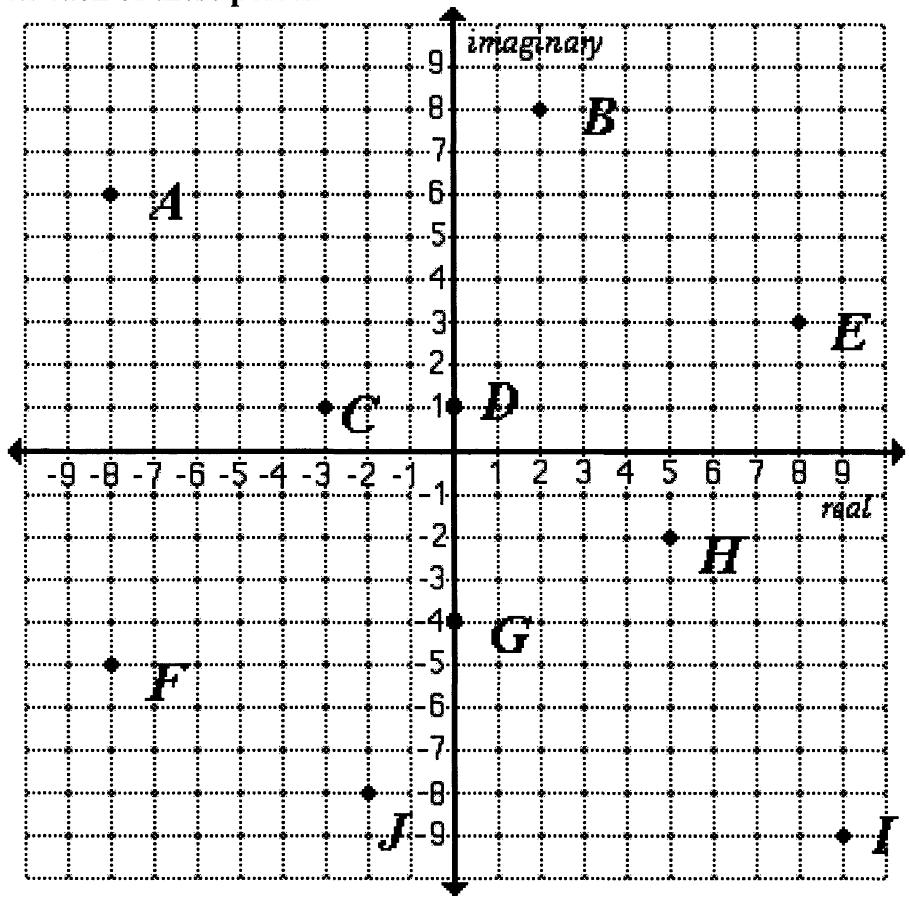
real ↗ ↖ imaginary

How do you think the graph of a simple imaginary number like 3*i* would look?

on the vertical axis at 3



The following is a complex plane with various points labeled. Identify the complex number that represents each of these points.

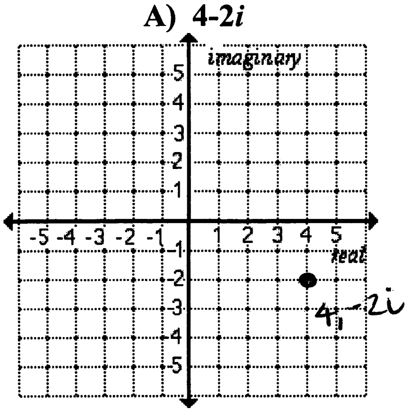


Letter	Complex Number
A	-8 +6i
B	2+8i
C	-3+i
D	i
E	8+3i

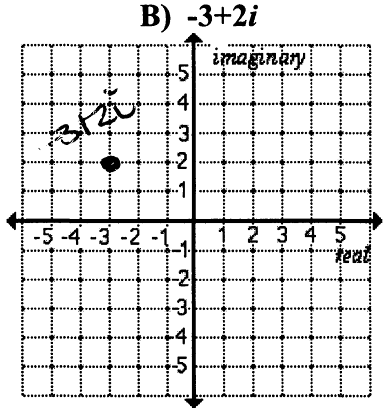
Letter	Complex Number
F	-8-5i
G	-4i
H	5-2i
I	9-9i
J	-2-8i

Plot the following complex numbers on a complex plane.

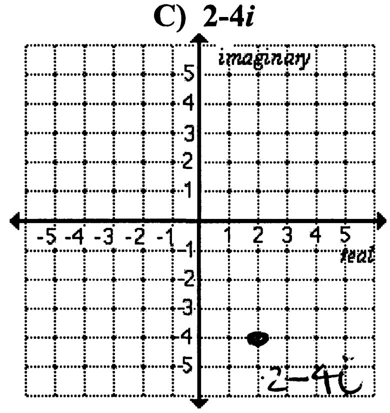
A) 4-2*i*



B) -3+2*i*



C) 2-4*i*



Operations with Complex Numbers

A complex number is a real number and an imaginary number put together. Complex numbers always come in the standard form $a+bi$. When solving a problem or evaluating an expression where the solution is a complex number, the solution must be written in standard form.

Evaluate each of the following expressions.

A) $2(3-4i)-6(8+2i)$

$6-8i-48-12i$

$-42-20i$

B) $6-(-4+3i)-(-8-12i)$

$6+4-3i+8+12i$

$18+9i$

C) $15i-(12-10i)$

$15i-12+10i$

$-12+25i$

D) $\sqrt{-144}-\sqrt{-81}$

$12i-9i$

$3i$

E) $2\sqrt{-81}+\sqrt{-121}-\sqrt{-9}$

$2(9i)+11i-3i$

$18i+11i-3i$

$26i$

F) $3\sqrt{-18}+4\sqrt{-32}$

$3(3\sqrt{2}i)+4(4\sqrt{2}i)$

$9\sqrt{2}i+16\sqrt{2}i$

$25\sqrt{2}i$

G) $\dfrac{-8+\sqrt{-32}}{12}$

$\dfrac{-8}{12}+\dfrac{4\sqrt{2}}{12}i$

$-\dfrac{2}{3}+\dfrac{\sqrt{2}}{3}i$

H) $\dfrac{15-\sqrt{-27}}{18}$

$\dfrac{15}{18}-\dfrac{3\sqrt{3}}{18}i$

$\dfrac{5}{6}-\dfrac{\sqrt{3}}{6}i$

I) $\dfrac{-12-\sqrt{-28}}{32}$

$-\dfrac{12}{32}-\dfrac{2\sqrt{7}}{32}i$

$-\dfrac{3}{8}-\dfrac{\sqrt{7}}{16}i$

J) $(5-3i)(2-6i)$

$10-30i-6i+18i^2$

$10-36i-18$

$-8-36i$

K) $4(5-3i)-(2-i)^2$

$(20-12i)-(4-4i+i^2)$

$(20-12i)-(4-4i-1)$

$(20-12i)-(3-4i)$

$20-12i-3+4i$

$17-8i$

L) $(5-12i)^2$

$25-120i+144i^2$

$25-120i-144$

$-119-120i$

M) $(4-3i)^2-(3+2i)^2$

$(16-24i+9i^2)-(9+12i+4i^2)$

$(16-24i-9)-(9+12i-9)$

$(7-24i)-(5+12i)$

$7-24i-5-12i$

$2-36i$

N) $5i(6-3i)+12i(8-i)$

$30i-15i^2+96i-12i^2$

$30i+15+96i+12$

$27+126i$

O) $2(3-2i)(4+5i)$

$2(12+15i-8i-10i^2)$

$2(12+7i+10)$

$2(22+7i)$

$44+14i$

P) $(3+4i)(3-4i)$

$9-16i^2$

$9+16$

25

Q) $(6-5i)(6+5i)$

$36-25i^2$

$36+25$

61

R) $\left(\sqrt{-12}+\sqrt{3}\right)^2$

$(2\sqrt{3}i+\sqrt{3})^2$

$12i^2+12i+3$

$-12+12i+3$

$-9+12i$

S) $\left(4\sqrt{5}+3\sqrt{-20}\right)\left(2\sqrt{45}+\sqrt{-5}\right)$

$(4\sqrt{5}+6\sqrt{5}i)(6\sqrt{5}+\sqrt{5}i)$

$24(5)+4(5i)+36(5i)+6(5i^2)$

$120+20i+180i+30i^2$

$120+200i-30$

$90+200i$

T) $\left(3\sqrt{2}+4\sqrt{3}i\right)\left(5\sqrt{8}+3\sqrt{-27}\right)$

$(3\sqrt{2}+4\sqrt{3}i)(10\sqrt{2}+9\sqrt{3}i)$

$30(2)+27\sqrt{6}i+40\sqrt{6}i+36(3i^2)$

$60+67\sqrt{6}i-108$

$-48+67\sqrt{6}i$

U) $\left(4\sqrt{3}-2\sqrt{7}i\right)^2$

$(4\sqrt{3})^2-2(4\sqrt{3})(2\sqrt{7}i)+(2\sqrt{7}i)^2$

$16(3)-16\sqrt{21}i+4(7)i^2$

$48-16\sqrt{21}i-28$

$20-16\sqrt{21}i$

V) $\sqrt{-25x^3}-\sqrt{-225x^3}+20x\sqrt{-x}$

$5x\sqrt{x}i-15x\sqrt{x}i+20x\sqrt{x}i$

$10x\sqrt{x}i$

W) $\sqrt{-3x^2}+\sqrt{-27x^2}-\sqrt{-45x^2}$

$x\sqrt{3}i+3x\sqrt{3}i-3x\sqrt{5}i$

$4x\sqrt{3}i-3x\sqrt{5}i$

X) $\sqrt{-\frac{x}{5}}\cdot\sqrt{-\frac{20}{x}}$

$\frac{\cancel{\sqrt{x}}}{\cancel{\sqrt{5}}}i\cdot\frac{2\cancel{\sqrt{5}}}{\cancel{\sqrt{x}}}i$

$2i^2$

-2

Y) $\sqrt{-\frac{x^5}{2}}\cdot\sqrt{-\frac{2}{x^3}}$

$\frac{x^2\sqrt{x}}{\cancel{\sqrt{2}}}i\cdot\frac{\cancel{\sqrt{2}}}{x\sqrt{x}}i$

xi^2

$-x$

Z) $\sqrt{-\frac{6x^3}{5}}\cdot\sqrt{-\frac{45}{12x}}$

$\frac{x\sqrt{6x}}{\sqrt{5}}i\cdot\frac{3\sqrt{5}}{2\sqrt{3x}}i$

$\frac{x\sqrt{2}\cdot\sqrt{3x}}{\cancel{\sqrt{5}}}i\cdot\frac{3\cancel{\sqrt{5}}}{2\sqrt{3x}}i$

$\frac{3x\sqrt{2}}{2}i^2$

$-\frac{3x\sqrt{2}}{2}$

Division with Complex Numbers

Division with complex numbers is much like rationalizing a denominator. You cannot have a complex number in the denominator, so multiply top and bottom by the conjugate. Remember, your answer must be written in standard form.

Divide each of the following.

A) $\frac{2}{3-i} \cdot \left(\frac{3+i}{3+i}\right)$

$\frac{2(3+i)}{9-i^2}$

$\frac{2(3+i)}{10}$

$\boxed{\frac{3}{5}+\frac{1}{5}i}$

B) $\frac{8i}{4-3i}\left(\frac{4+3i}{4+3i}\right)$

$\frac{8i(4+3i)}{16-9i^2}$

$\frac{32i+24i^2}{25}$

$\frac{-24+32i}{25}$

$\boxed{\frac{-24}{25}+\frac{32}{25}i}$

C) $\frac{15}{2-i} \cdot \left(\frac{2+i}{2+i}\right)$

$\frac{15(2+i)}{4-i^2}$

$\frac{15(2+i)}{5}$

$\boxed{6+3i}$

D) $\frac{10}{1+7i} \cdot \left(\frac{1-7i}{1-7i}\right)$

$\frac{10(1-7i)}{1-49i^2}$

$\frac{10(1-7i)}{50}$

$\boxed{\frac{1}{5}-\frac{7}{5}i}$

E) $\frac{3+i}{3-i}\left(\frac{3+i}{3+i}\right)$

$\frac{9+6i+i^2}{9-i^2}$

$\frac{9+6i-1}{9+1}$

$\frac{8+6i}{10}$ $\boxed{\frac{4}{5}+\frac{3}{5}i}$

F) $\frac{3-4i}{4+3i}\left(\frac{4-3i}{4-3i}\right)$

$\frac{12-9i-16i+12i^2}{16-9i^2}$

$\frac{12-25i-12}{25}$

$\boxed{-\frac{25}{25}i = -i}$

G) $\frac{8-5i}{4-3i}\left(\frac{4+3i}{4+3i}\right)$

$\frac{32+24i-20i-15i^2}{16-9i^2}$

$\frac{47+4i}{25}$

$\boxed{\frac{47}{25}+\frac{4}{25}i}$

H) $\frac{5-\sqrt{3}i}{2+\sqrt{3}i}\left(\frac{2-\sqrt{3}i}{2-\sqrt{3}i}\right)$

$\frac{10-5\sqrt{3}i-2\sqrt{3}i+3i^2}{4-3i^2}$

$\frac{7-7\sqrt{3}i}{7}$

$\boxed{1-\sqrt{3}i}$

I) $\frac{3-\sqrt{5}i}{3+\sqrt{5}i}\left(\frac{3-\sqrt{5}i}{3-\sqrt{5}i}\right)$

$\frac{9-6\sqrt{5}i+5i^2}{9-5i^2}$

$\frac{4-6\sqrt{5}i}{14}$

$\boxed{\frac{2}{7}-\frac{3\sqrt{5}}{7}i}$

J) $\frac{-2+\sqrt{3}i}{-2-\sqrt{3}i}\left(\frac{-2+\sqrt{3}i}{-2+\sqrt{3}i}\right)$

$\frac{4-2\sqrt{3}i-2\sqrt{3}i+3i^2}{4-3i^2}$

$\frac{1-4\sqrt{3}i}{7}$

$\boxed{\frac{1}{7}-\frac{4\sqrt{3}}{7}i}$

K) $\frac{6i}{\sqrt{2}-\sqrt{3}i}\left(\frac{\sqrt{2}+\sqrt{3}i}{\sqrt{2}+\sqrt{3}i}\right)$

$\frac{6\sqrt{2}i+6\sqrt{3}i^2}{2-3i^2}$

$\frac{-6\sqrt{3}+6\sqrt{2}i}{5}$

$\boxed{-\frac{6\sqrt{3}}{5}+\frac{6\sqrt{2}}{5}i}$

L) $\frac{3-\sqrt{2}i}{4+\sqrt{3}i}\left(\frac{4-\sqrt{3}i}{4-\sqrt{3}i}\right)$

$\frac{12-3\sqrt{3}i-4\sqrt{2}i+\sqrt{6}i^2}{16-3i^2}$

$\frac{12-\sqrt{6}-3\sqrt{3}i-4\sqrt{2}i}{19}$

$\boxed{\frac{12-\sqrt{6}}{19}-\frac{(3\sqrt{3}+4\sqrt{2})}{19}i}$

More Equations

Solve each of the following equations.

A) $x^2+400=0$

$\sqrt{x^2}=\sqrt{-400}$

$x=\pm 20i$

B) $4x^2+39=7$

$4x^2=-32$

$\sqrt{x^2}=\sqrt{-8}$

$x=\pm 2\sqrt{2}i$

C) $5x^2=-20$

$\sqrt{x^2}=\sqrt{-4}$

$x=\pm 2i$

D) $2x^2+90=0$

$2x^2=-90$

$\sqrt{x^2}=\sqrt{-45}$

$x=\pm 3\sqrt{5}i$

E) $3x^2+14=8$

$3x^2=-6$

$\sqrt{x^2}=\sqrt{-2}$

$x=\pm\sqrt{2}i$

F) $3x^2+48=0$

$3x^2=-48$

$\sqrt{x^2}=\sqrt{-16}$

$x=\pm 4i$

Checking Progress

You have now completed the "Radicals" section of the workbook. The following is a checklist so that you may check your progress. Check off each of the objectives you have accomplished.

The student should be able to…

- [ ] ***Simplify radicals using the properties of radicals.***
- [ ] ***Multiply and divide radicals using the properties of radicals.***
- [ ] ***Add and subtract radicals using the properties of radicals.***
- [ ] ***Rationalize the denominator of a rational expression where a radical resides in the denominator.***
- [ ] ***Solve equations involving radicals.***
- [ ] ***Verify a solution to a radical equation is not an extraneous root.***
- [ ] ***Simplify radicals using imaginary numbers.***
- [ ] ***Simplify expressions involving imaginary numbers.***
- [ ] ***Plot complex numbers on a Complex Plane.***
- [ ] ***Multiply and divide complex numbers.***
- [ ] ***Add and subtract complex numbers.***
- [ ] ***Rationalize an expression with complex numbers in the denominator.***

RATIONAL EXPONENTS

Objectives

The following is a list of objectives for this section of the workbook.

By the time the student is finished with this section of the workbook, he/she should be able to…

- ***Express a radical using a rational exponent.***
- ***Evaluate expressions containing rational exponents.***
- ***Solve equations involving rational exponents.***
- ***Identify an exponential function as being either growth or decay.***
- ***Graph an exponential function using a table.***

Math Standards Addressed

The following state standards are addressed in this section of the workbook.

Algebra II

12.0 Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.

15.0 Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.

Review Properties of Exponents

Property		Example
$x^a \cdot x^b = x^{a+b}$	***When multiplying variable expressions with the same base, add exponents.***	$x^5 \cdot x^6 = x^{5+6} = x^{11}$
$x^a \div x^b = x^{a-b}$	***When dividing variable expressions with the same base, subtract exponents.***	$x^{12} \div x^8 = x^{12-8} = x^4$
$\left(x^a\right)^b = x^{ab}$	***The power of a power rule states that if the exponents are separated by parenthesis, multiply them.***	$\left(x^2\right)^4 = x^{2\cdot 4} = x^8$
$\left(x^a y^b\right)^c = x^{ac} y^{bc}$	***This is an extension of the power of a power rule telling you to distribute the power. Be careful with coefficients.***	$\left(2x^3y^2\right)^3 = 2^{1\cdot 3}x^{3\cdot 3}y^{2\cdot 3} = 8x^9y^6$
$\left(\dfrac{x}{y}\right)^a = \dfrac{x^a}{y^a}$	***This is an extension of the power of a power rule telling you to distribute the power. Be careful with coefficients.***	$\left(\dfrac{x^3}{y^2}\right)^2 = \dfrac{x^{3\cdot 2}}{y^{2\cdot 2}} = \dfrac{x^6}{y^4}$
$x^{-1} = \dfrac{1}{x}$	***If you have a number to a negative power, just think of it as being on the wrong side of the fraction bar. Move it to the correct side and then work on the problem. Watch for the example.***	$(3x)^{-1} = \dfrac{1}{3x}$ *or* $3x^{-1} = \dfrac{3}{x}$
$x^0 = 1$	***Anything to the zero power is 1. This can be proven by showing a division problem when powers of divisor and dividend are the same.***	$\left(12x^5\right)^0 = 1$
$\left(\dfrac{x}{y}\right)^{-a} = \dfrac{x^{-a}}{y^{-a}} = \dfrac{y^a}{x^a}$	***For a fraction to a negative power; first flip the fraction, then work with the power. Observe how this works with the example.***	$\left(\dfrac{2}{3}\right)^{-1} = \dfrac{1}{\left(\dfrac{2}{3}\right)} = 1 \div \dfrac{2}{3} = \dfrac{3}{2}$
$\lvert x^2 \rvert = \lvert x \rvert^2$	***Regardless of whether a number is squared and the subsequent absolute value is found, or the absolute value of the actual number is squared, the result will be the same.***	$\lvert 3^2 \rvert = \lvert 9 \rvert = 9$ *same as* $\lvert 3^2 \rvert = \lvert 3 \rvert^2 = 3^2 = 9$
$a^{1/n} = \sqrt[n]{a}$	***All radicals are rational exponents. The denominator of the power is the index of the radical.***	$x^{1/4} = \sqrt[4]{x}$
$a^{m/n} = \left(a^{1/n}\right)^m = \left(\sqrt[n]{a}\right)^m$	***See description below for specifics on this rule.***	$9^{3/2} = \left(9^{1/2}\right)^3 = \left(\sqrt{9}\right)^3 = 3^3 = 27$

The third example from above is really the key to evaluating simple expressions involving rational exponents. We will be using a slight extension of this rule. The commutative property of multiplication is used to solve these problems. If we refer to the following, $\left(x^a\right)^b = \left(x^b\right)^a$ the power of a power rule says the two exponents should be multiplied. The commutative property of multiplication states $a \cdot b = b \cdot a$. Therefore, it would follow that $x^{a/b} = \left(x^a\right)^{1/b}$ or $x^{a/b} = \left(x^{1/b}\right)^a$.

Using numerical examples makes this easier to see: $\left(x^5\right)^3 = \left(x^3\right)^5 = x^{15}$ ***therefore, the following must be true.***

$$16^{3/2} = \left(16^3\right)^{1/2} = \left(16^{1/2}\right)^3 = (4)^3 = 64$$

You can plainly see that this problem will be easier to simplify if we find the square root first then, cube the result. This is the way you want to approach all of the following problems.

Practice Using Rational Exponents

Rewrite each of the following expressions using rational exponents.

A) $\sqrt[3]{5x^2}$

$(5x^2)^{1/3}$

$5^{1/3}x^{2/3}$

B) $\sqrt[5]{(x+6)^3}$

$((x+6)^3)^{1/5}$

$(x+6)^{3/5}$

C) $\sqrt[3]{x^4}$

$(x^4)^{1/3}$

$x^{4/3}$

D) $\sqrt[4]{(3x+5)^2}$

$((3x+5)^2)^{1/4}$

$(3x+5)^{2/4}$

$(3x+5)^{1/2}$

E) $\sqrt{x^2+y^2}$

$(x^2+y^2)^{1/2}$

F) $\sqrt{x+\sqrt{y}}$

$(x+\sqrt{y})^{1/2}$

$(x+y^{1/2})^{1/2}$

G) $\sqrt[3]{3x^5y^4}$

$(3x^5y^4)^{1/3}$

$3^{1/3}x^{5/3}y^{4/3}$

H) $\sqrt[5]{(6x^2)^3}$

$((6x^2)^3)^{1/5}$

$(6x^2)^{3/5}$

$6^{3/5}x^{6/5}$

Rewrite each of the following expressions using a radical.

I) $\left(4x^3\right)^{2/3}$

$((4x^3)^2)^{1/3}$

$\sqrt[3]{(4x^3)^2}$

J) $15^{1/3}$

$\sqrt[3]{15}$

K) $(2x+3)^{1/4}$

$\sqrt[4]{2x+3}$

L) $(5x-1)^{3/5}$

$[(5x-1)^3]^{1/5}$

$\sqrt[5]{(5x-1)^3}$

M) $\left(2x+y^{1/2}\right)^{1/3}$

$\sqrt[3]{2x+\sqrt{y}}$

N) $\left(x^2-y^2\right)^{1/2}$

$\sqrt{x^2-y^2}$

O) $3^{1/4}x^{3/4}y^{5/4}$

$(3x^3y^5)^{1/4}$

$\sqrt[4]{3x^3y^5}$

P) $8-x^{1/3}$

$8-\sqrt[3]{x}$

Evaluating Simple Rational Exponents

Keep the following in mind when dealing with rational exponents.

$$a^{1/n} = \sqrt[n]{a}$$

$$a^{m/n} = \left(a^{1/n}\right)^m = \left(\sqrt[n]{a}\right)^m$$

Evaluate each of the following without the aid of a calculator.

A) $27^{4/3}$

$(27^{1/3})^4$

3^4

81

B) $4^{1/2}$

2

C) $36^{-3/2}$

$\left(\frac{1}{36}\right)^{3/2}$

$\left[\left(\frac{1}{36}\right)^{1/2}\right]^3$

$\left(\frac{1}{6}\right)^3$ $\quad \frac{1}{216}$

D) $\left(25^3\right)^{1/2}$

$(25^{1/2})^3$

(5^3)

125

E) $32^{-2/5}$

$\left(\frac{1}{32}\right)^{2/5}$

$\left[\left(\frac{1}{32}\right)^{1/5}\right]^2$

$\left(\frac{1}{2}\right)^2$

$\frac{1}{4}$

F) $(-8)^{5/3}$

$\left[(-8)^{1/3}\right]^5$

$[-2]^5$

-32

G) $\left(\frac{4}{9}\right)^{3/2}$

$\left[\left(\frac{4}{9}\right)^{1/2}\right]^3$

$\left(\frac{2}{3}\right)^3$

$\frac{8}{27}$

H) $\left(\frac{27}{8}\right)^{-1/3}$

$\left(\frac{8}{27}\right)^{1/3}$

$\frac{2}{3}$

I) $-27^{2/3}$

$-(27^{1/3})^2$

-3^2

9

J) $\left(\frac{1}{8}\right)^{-2/3}$

$8^{2/3}$

$(8^{1/3})^2$

2^2

4

K) $\left(\frac{27}{64}\right)^{2/3}$

$\left[\left(\frac{27}{64}\right)^{1/3}\right]^2$

$\left(\frac{3}{4}\right)^2$

$\frac{9}{16}$

L) $\left(64x^{12}y^{15}\right)^{1/3}$

$64^{1/3}\, x^{12/3}\, y^{15/3}$

$4x^4y^5$

M) $\left(64^{1/2}\right)^{-1/3}$

$8^{-1/3}$

$\left(\frac{1}{8}\right)^{1/3}$

$\frac{1}{2}$

N) $\left[\left(\frac{16}{49}\right)^3\right]^{1/2}$

$\left[\left(\frac{16}{49}\right)^{1/2}\right]^3$

$\left(\frac{4}{7}\right)^3$

$\frac{64}{343}$

O) $\sqrt[3]{4^6}$

$(4^6)^{1/3}$

$4^{6/3}$

4^2

16

P) $\left(9x^2y^6\right)^{-3/2}$

$\left(\frac{1}{9x^2y^6}\right)^{3/2}$

$\left[\left(\frac{1}{9x^2y^6}\right)^{1/2}\right]^3$

$\left(\frac{1}{3xy^3}\right)^3$ $\quad \frac{1}{27x^3y^9}$

Q) $-36^{-3/2}$

$\left(-\frac{1}{36}\right)^{3/2}$

$-\left[\left(\frac{1}{36}\right)^{1/2}\right]^3$

$-\left(\frac{1}{6}\right)^3$ $\quad -\frac{1}{216}$

R) $\left(\frac{25}{36}\right)^{-0.5}$

$\left(\frac{25}{36}\right)^{-1/2}$

$\left(\frac{36}{25}\right)^{1/2}$

$\frac{6}{5}$

S) $(-64)^{-2/3}$

$\left[\left(-\frac{1}{64}\right)^{\frac{1}{3}}\right]^2$

$\left(-\frac{1}{4}\right)^2$

$\frac{1}{16}$

T) $\left(-8x^6y^{-3}\right)^{1/3}$

$-8^{1/3}x^{6/3}y^{-3/3}$

$-2x^2y^{-1}$

$\frac{-2x^2}{y}$

U) $\left(125x^9y^{12}z^6\right)^{2/3}$

$\left[\left(125x^9y^{12}z^6\right)^{1/3}\right]^2$

$\left(125^{1/3}x^{9/3}y^{12/3}z^{6/3}\right)^2$

$\left(5x^3y^4z^2\right)^2$

$25x^6y^8z^4$

Multiplication/Division with Rational Exponents

Evaluate each of the following. Write your solutions in <u>both</u> exponential and simple radical form.

A) $3^{1/2}\cdot 3^{2/3}$

$3^{1/2+2/3}$

$3^{3/6+4/6}$

$3^{7/6}$

$3^{7/6}$ or $\sqrt[6]{3^7}$

$3^{7/6}$ or $3\sqrt[6]{3}$

B) $4^{3/2}\cdot 2^{1/5}$

$(2^2)^{3/2}\cdot 2^{1/5}$

$2^3\cdot 2^{1/5}$

$2^{15/5}\cdot 2^{1/5}$

$2^{16/5}$

$2^{16/5}$ or $8\sqrt[5]{2}$

C) $\sqrt{5}\cdot\sqrt[3]{5}$

$5^{1/2}\cdot 5^{1/3}$

$5^{1/2+1/3}$

$5^{3/6+2/6}$

$5^{5/6}$ or $\sqrt[6]{5^5}$

D) $\sqrt{2}\cdot\sqrt[3]{4}$

$2^{1/2}\cdot 4^{1/3}$

$2^{1/2}\cdot 2^{2/3}$

$2^{1/2+2/3}$

$2^{3/6+4/6}$

$2^{7/6}$ or $2\sqrt[6]{3}$

E) $9^{2/3}\cdot 27^{1/2}$

$9^{2/3}\cdot 27^{1/2}$

$3^{4/3}\cdot 3^{3/2}$

$3^{8/6}\cdot 3^{9/6}$

$3^{17/6}$

$3^{17/6}$ or $\sqrt[6]{3^{17}}$

F) $16^{1/4}\cdot 8^{1/2}$

$(2^4)^{1/4}\cdot(2^3)^{1/2}$

$2\cdot 2^{3/2}$

$2^{2/2}\cdot 2^{3/2}$

$2^{2/2+3/2}$

$2^{5/2}$ or $\sqrt{32}$

G) $\sqrt{2}\cdot\sqrt[3]{4}\cdot\sqrt{3}$

$2^{1/2}\cdot 4^{1/3}\cdot 3^{1/2}$

$2^{1/2}\cdot 2^{2/3}\cdot 3^{1/2}$

$2^{3/6+4/6}\cdot 3^{1/2}$

$2^{7/6}\cdot 3^{1/2}$ or

$2\sqrt[6]{2}\cdot\sqrt{3}$

H) $81^{3/4}\cdot 9^{-3/2}$

$(9^2)^{3/4}\cdot(9)^{-3/2}$

$9^{3/2}\cdot 9^{-3/2}$

$9^{3/2-3/2}$

9^0

1

I) $16^{-3/4}\cdot 32^{4/5}$

$(2^4)^{-3/4}\cdot(2^5)^{4/5}$

$(2^{-3})\cdot(2^4)$

2^{-3+4}

2

J) $9^{1/3} \div 3^{1/2}$

$(3^2)^{1/3} \div 3^{1/2}$

$3^{2/3} \div 3^{1/2}$

$3^{2/3 - 1/2}$

$3^{4/6 - 3/6}$

$3^{1/6}$ or $\sqrt[6]{3}$

K) $16^{3/4} \div 8^{-2/3}$

$(2^4)^{3/4} \div (2^3)^{-2/3}$

$2^3 \div 2^{-2}$

$2^{3 - -2}$

2^5 or 32

L) $4 \cdot 4^{2/3}$

$4^{1 + 2/3}$

$4^{3/3 + 2/3}$

$4^{5/3}$ or $\sqrt[3]{45}$

M) $25^{1/4} \div 5^{1/4}$

$(5^2)^{1/4} \div 5^{1/4}$

$5^{2/4 - 1/4}$

$5^{1/4}$

$5^{1/4}$ or $\sqrt[4]{5}$

N) $\left(x^{1/2} + 3\right)^2$

$(x^{1/2})^2 + 2(3)(x^{1/2}) + (3)^2$

$x + 6x^{1/2} + 9$

or

$x + 6\sqrt{x} + 9$

O) $\left(2x^{1/3} - 5\right)^2$

$(2x^{1/3})^2 - 2(5)(2x^{1/3}) + (5)^2$

$4x^{2/3} - 20x^{1/3} + 25$

or

$4\sqrt[3]{x^2} - 20\sqrt[3]{x} + 25$

P) $\sqrt[3]{a^2} \cdot \sqrt[3]{a^4}$

$a^{2/3} \cdot a^{4/3}$

$a^{6/3}$

a^2

Q) $27^{3/5} \div 9^{2/5}$

$(3^3)^{3/5} \div (3^2)^{2/5}$

$3^{9/5} \div 3^{4/5}$

$3^{9/5 - 4/5}$

$3^{5/5}$

3

R) $\left(x^{5/2} - 2x^{3/2}\right) \div x^{1/2}$

$x^{5/2 - 1/2} - 2x^{3/2 - 1/2}$

$x^{4/2} - 2x^{2/2}$

$x^2 - 2x$

S) $\left(25^{1/2} + 27^{1/3}\right)^2$

$(5 + 3)^2$

$(8)^2$

64

T) $\sqrt[4]{\dfrac{81^2 x^{-3}}{y^{16}}}$

$\left(\dfrac{81^2}{x^3 y^{16}}\right)^{1/4}$

$\dfrac{81^{2/4}}{x^{3/4} y^{16/4}}$

$\dfrac{81^{1/2}}{x^{3/4} y^4}$

$\dfrac{9}{x^{3/4} y^4}$

U) $\left[\left(x^{1/2}\right)^{-2/3}\right]^{-3/4}$

$(x^{1/2})^{-2/3 \cdot -3/4} = $

$(x^{1/2})^{1/2}$

$x^{1/4}$ or $\sqrt[4]{x}$

Solving Equations Involving Rational Exponents

Solve each of the following equations. Don't forget to check for extraneous roots.

A) $(2x-3)^{2/3}=16$

$[(2x-3)^{2/3}]^{3/2} = 16^{3/2}$

$2x-3 = 64$

$+3 \quad +3$

$2x = 67$

$x = 67/2$

B) $x^{-1/2}=6$

$(x^{-1/2})^{-2} = 6^{-2}$

$x = 6^{-2}$

$x = 1/36$

C) $4x^{1/2}-3=9$

$+3 \quad +3$

$\frac{4x^{1/2}}{4} = \frac{12}{4}$

$(x^{1/2})^2 = 3)^2$

$x = 9$

D) $(4x-1)^{3/4}=-8$

$[(4x-1)^{3/4}]^{4/3} = -8^{4/3}$

$4x-1 = 16$

$+1 \quad +1$

$4x = 17$

$x = 17/4$

E) $(3x-1)^{-2/3}=\frac{1}{4}$

$[(3x-1)^{-2/3}]^{-3/2} = (1/4)^{-3/2}$

$3x-1 = 4^{3/2}$

$3x-1 = 8$

$+1 \quad +1$

$3x = 9$

$x = 3$

F) $(5x+6)^{2/5}=4$

$[5x+6)^{2/5}]^{5/2} = 4^{5/2}$

$5x+6 = 32$

$-6 \quad -6$

$5x = 26$

$x = 26/5$

G) $(9x)^{-2/3}=4$

$[(9x)^{-2/3}]^{-3/2} = 4^{-3/2}$

$9x = (1/4)^{3/2}$

$9x = 1/8$

$x = \frac{1}{72}$

H) $\sqrt[3]{(x+1)^2}=9$

$[(x+1)^{2/3}]^{3/2} = 9^{3/2}$

$x+1 = 27$

$-1 \quad -1$

$x = 26$

I) $9x^{-2/3}=4$

$x^{-2/3} = 4/9$

$(x^{-2/3})^{-3/2} = (4/9)^{-3/2}$

$x = (9/4)^{3/2}$

$x = 27/8$

J) $\sqrt[3]{x^2-1}=3$

$(\sqrt[3]{x^2-1})^3 = 3^3$

$x^2-1 = 27$

$\sqrt{x^2} = \sqrt{28}$

$x = 2\sqrt{7}$

K) $x^{2/3}-2x^{1/3}-15=0$

$(x^{1/3}-5)(x^{1/3}+3) = 0$

$x^{1/3} = 5$

$(x^{1/3})^3 = (5)^3$

$x = 125$

$(x^{1/3})^3 = (-3)^3$

$x = -27$

L) $\sqrt{(3x-1)^3}=27$

$(3x-1)^{3/2} = 27$

$[(3x-1)^{3/2}]^{2/3} = (27)^{2/3}$

$3x-1 = 9$

$+1 \quad +1$

$3x = 10$

$x = 10/3$

M) $8^{2x-4}=16^{x+5}$

$(2^3)^{2x-4}=(2^4)^{x+5}$

$2^{6x-12}=2^{4x+20}$

$6x-12=4x+20$

$2x=32$

$x=16$

N) $9^{3x-2}=27^2$

$(3^2)^{3x-2}=(3^3)^2$

$3^{6x-4}=3^6$

$6x-4=6$

$6x=10 \quad x=5/3$

O) $16^{5x+1}=32^{2x+8}$

$(2^4)^{5x+1}=(2^5)^{2x+8}$

$2^{20x+4}=2^{10x+40}$

$20x+4=10x+40$

$10x=36$

$x=18/5$

Miscellaneous Practice

Simplify each of the following expressions.

A) $3^{\sqrt{2}}\cdot 3^{2\sqrt{2}}$

$3^{2(2)}$

3^4

81

B) $\left(4^{\sqrt{3}}\right)^{\sqrt{3}}$

4^3

64

C) $9^{\pi/2}$

$(9^{1/2})^{\pi}$

3^{π}

D) $\dfrac{5^{\sqrt{3}+6}}{125}$

$\dfrac{5^{\sqrt{3}+6}}{5^3}$

$5^{\sqrt{3}+6-3}$

$5^{\sqrt{3}+3}$

E) $\left(\sqrt{2}\right)^{\sqrt{3}}\left(\sqrt{2}\right)^{-\sqrt{3}}$

$\sqrt{2}^{\sqrt{3}+-\sqrt{3}}$

2^0

1

F) $\dfrac{3^{\sqrt{3}}\cdot 3^{\sqrt{27}}}{3^{2\sqrt{3}}}$

$\dfrac{3^{\sqrt{3}}\cdot 3^{3\sqrt{3}}}{3^{2\sqrt{3}}}$

$\dfrac{3^{4\sqrt{3}}}{3^{2\sqrt{3}}}$

$3^{4\sqrt{3}-2\sqrt{3}}$

$3^{2\sqrt{3}}$

$(3^2)^{\sqrt{3}}$

$9^{\sqrt{3}}$

G) $\dfrac{32^{\sqrt{7}}}{4^{\sqrt{7}}}$

$\dfrac{2^{5\sqrt{7}}}{2^{2\sqrt{7}}}$

$2^{5\sqrt{7}-2\sqrt{7}}$

$2^{3\sqrt{7}}$

$8^{\sqrt{7}}$

H) $8^{1.2}\cdot 2^{-3.1}$

$(2^3)^{1.2}\cdot 2^{-3.1}$

$2^{3.6}\cdot 2^{-3.1}$

$2^{3.6-3.1}$

$2^{.5}$

$2^{1/2}$

$\sqrt{2}$

I) $\sqrt{\dfrac{2^{\sqrt{3}+3}}{8}}$

$\left(\dfrac{2^{\sqrt{3}+3}}{2^3}\right)^{1/2}$

$2^{\sqrt{3}+3-3}$

$(2^{\sqrt{3}})^{1/2} \quad 2^{\sqrt{3}/2}$

J) $\dfrac{\left(\sqrt{5}-1\right)^{2+\pi}}{\left(\sqrt{5}-1\right)^{\pi}}$

$(\sqrt{5}-1)^{2+\pi-\pi}$

$(\sqrt{5}-1)^2$

$5-2\sqrt{5}+1$

$6-2\sqrt{5}$

K) $16^{2.3}\div 8$

$(2^4)^{2.3}\div 2^3$

$2^{9.2}\div 2^3$

$2^{9.2-3}$

$2^{6.2}$

L) $x^{4/3}\cdot x^{3/2}\cdot x^{1/6}$

$x^{4/3+3/2+1/6}$

$x^{8/6+9/6+1/6}$

$x^{18/6}$

x^3

M) $\dfrac{243^{2\sqrt{3}+1}}{27^{3\sqrt{3}-5}}$

$\dfrac{(3^5)^{2\sqrt{3}+1}}{(3^3)^{3\sqrt{3}-5}}$

$\dfrac{3^{10\sqrt{3}+5}}{3^{9\sqrt{3}-15}}$

$3^{(10\sqrt{3}+5)-(9\sqrt{3}-15)}$

$3^{\sqrt{3}+20}$

N) $\left(16^{3\sqrt{2}/8}\right)^{\sqrt{2}}$

$(16^{3\sqrt{2}/8})^{\sqrt{2}}$

$16^{6/8}$

$16^{3/4}$

$(2^4)^{3/4}$

2^3

8

O) $64^{7x-4}=2\cdot 8^{x-1}$

$(2^6)^{7x-4}=2(2^3)^{x-1}$

$2^{42x-24}=2\cdot 2^{3x-3}$

$2^{42x-24}=2^{3x-2}$

$42x-24=3x-2$

$39x=22 \quad x=22/39$

P) $4^{3x-1}\cdot 8^{x+2}\cdot 16^x$

$(2^2)^{3x-1}\cdot(2^3)^{x+2}\cdot(2^4)^x$

$2^{6x-2}\cdot 2^{3x+6}\cdot 2^{4x}$

$2^{6x-2+3x+6+4x}$

2^{13x+4}

Exponential Functions

We will now take a brief look at exponential functions.

Standard exponential function

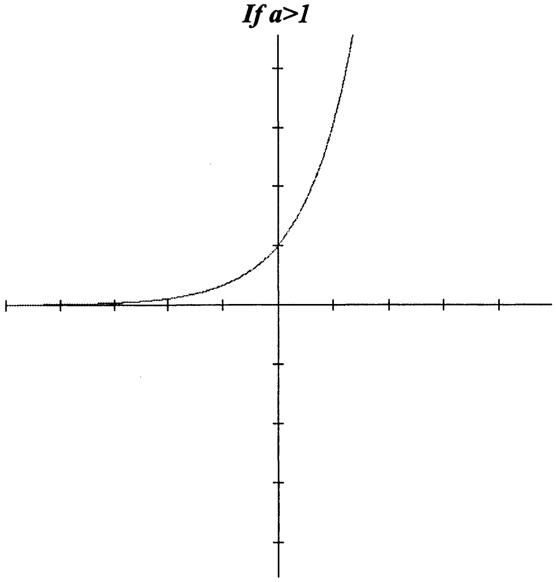
$f_{(x)} = ca^{x-h} + k$

The c term is a constant that can make to graph reflect about a horizontal axis or change the scale of the graph of the function. If c is a positive value, then you will have a standard looking growth or decay curve. If c is negative, the growth or decay curve will flip upside down. We will get into the effects different values of h and k have on this function in the future. What we will concentrate on here is identifying an exponential function as being growth or decay, and finding the range, domain and key point of the function.

Exponential Growth

$f_{(x)} = ca^{x-h} + k$

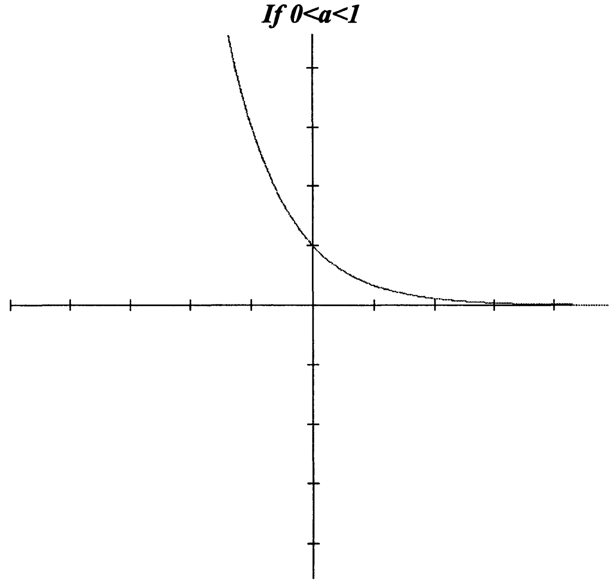
If a>1



Exponential Decay

$f_{(x)} = ca^{x-h} + k$

If 0<a<1



Obviously if $a = 1$ ***then we are raising 1 to various powers, and we wind up getting a horizontal line because no matter what you do, raising one to any power still yields a result of one. Pay special attention to the exponential decay function. The statement*** $0 < a < 1$ ***is saying that the value of*** a ***is a fraction whose value is between zero and one. Do not make the mistake of just looking for a fraction to determine whether or not the function is decay. Make sure the value of the fraction is between zero and one.***

The values for variables $h,\ k,\ and\ \ c$ ***act to make the graph shift left/right, up/down, or will reflect the function about a horizontal axis and change the scale of the graph of the function.***

You will notice the key point for each of these functions is the point $(0,1)$. This information is vital. This key point will shift depending on the values of h, k and c. To find the x value of the key point, evaluate $x-h=0$. In other words, find the value of x that would create a problem like 3 to the zero power. This number is the x value of the key point. To find the y value, substitute the x value back in. Refer to the following example. The value of c will affect the scale of the graph. If for example c is 4, the key point is (0,4). Think of it as a scalar multiplier.

$$f_{(x)}=2^{x-3}+5$$
$$to \;\; find \;\; the \;\; key \;\; point \;\; evaluate \;\; x-3=0$$
$$x-3=0$$
$$x=3 \quad this \;\; is \;\; the \;\; x \;\; value \;\; of \;\; the \;\; key \;\; point$$

$$now \;\; substitute \;\; 3 \;\; back \;\; into \;\; the \;\; problem \;\; for \;\; x$$
$$f_{(3)}=2^{3-3}+5$$
$$f_{(3)}=2^{0}+5$$
$$f_{(3)}=1+5$$
$$f_{(3)}=6$$
$$so \;\; the \;\; key \;\; point \;\; is \;\; (3,6)$$

The domain of any exponential function is all real numbers, because we can raise these numbers to any power we want.

Notice that the graphs do not cross the x axis. This means the range of the parent function is $(0,\infty)$, and that the x axis is a horizontal asymptote. Just think of asymptotes as imaginary lines that cannot be crossed.

In an exponential function, we are always raising the a value to a power. For the function $f_{(x)}=3^{x}$, why is the range of the function $(0,\infty)$?

If you raise x to a negative power, you get a fraction.
That is the worst thing you can do, it will never
equal zero or a negative number

Consider the problem $f_{(x)} = -3^x$ as it relates to $f_{(x)} = 3^x$. If I substitute various values for x, I will get the following results.

$f_{(x)} = 3^x$

x	$f_{(x)}$
-2	$1/9$
-1	$1/3$
0	1
1	3
2	9

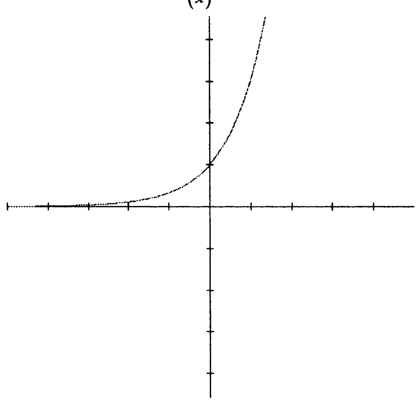
Notice that the y values of the second function are opposite the y values of the first. This tells you that the second function will be flipped upside down.

$f_{(x)} = -3^x$

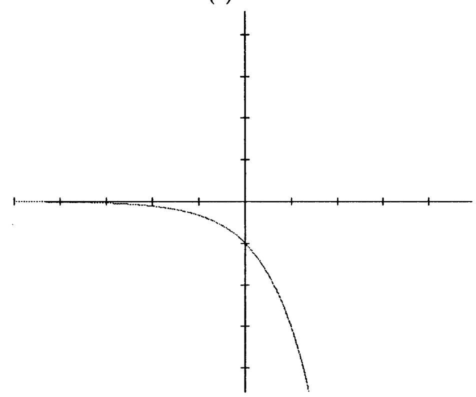
x	$f_{(x)}$
-2	$-1/9$
-1	$-1/3$
0	-1
1	-3
2	-9

When graphed, this is how each would look.

$f_{(x)} = 3^x$



$f_{(x)} = -3^x$



As you can see from the previous example, if the value of c is negative, you will be graphing a reflection of the original function. This particular reflection takes place about the x axis. The x axis, in the case of these two functions is the horizontal asymptote. This will not always be the case. Sometimes, these curves can shift vertically. If there is a vertical shift, then the function would reflect about that particular horizontal axis. Finding the key point will help you determine exactly where that horizontal asymptote is.

For many of our applications in this class we will use the number e. This number is called a natural base and has an approximate value of 2.718281828... We will use this as a constant in our future problems.

Identify each of the following exponential functions as being growth or decay.

A) $f_{(x)} = 3^{x-4} + 1$

Growth

B) $f_{(x)} = \left(\frac{2}{3}\right)^{x} + 1$

Decay

C) $f_{(x)} = 2^{x+2} - 5$

Growth

D) $f_{(x)} = \left(\frac{3}{7}\right)^{x} + 16$

Decay

E) $f_{(x)} = e^{2x-1} + 2$

Growth

F) $f_{(x)} = \left(\frac{1}{e}\right)^{x-3}$

Decay

G) $f_{(x)} = \left(\frac{4}{7}\right)^{5-x} + 3$

Growth

H) $f_{(x)} = \left(\frac{4}{5}\right)^{x-1} + 2$

Decay

I) $f_{(x)} = -2^{x+1}$

Growth

Explain why the graphs for $f_{(x)} = \left(\frac{1}{2}\right)^{x}$ **and** $f_{(x)} = 2^{-x}$ **are identical.** **(Hint, properties of exponents.)**

$y = 2^{-x}$

$y = 2^{-1 \cdot x}$

$y = (2^{-1})^{x}$

$y = (\frac{1}{2})^{x}$

Find the key point for each of the following functions. Refer to page 155 for an example.

A) $f_{(x)} = 3^{x+2} - 4$

$x+2=0$ $\quad f(-2) = 3^0 - 4$

$x=-2$ $\quad 1-4$

-3

$\boxed{(-2,-3)}$

B) $f_{(x)} = \left(\frac{1}{2}\right)^{x-5} + 1$

$x-5=0$ $\quad f(5) = \left(\frac{1}{2}\right)^0 + 1$

$x=5$ $\quad 1+1$

2

$\boxed{(5,2)}$

C) $f_{(x)} = -2^x + 3$

$x=0$ $\quad f(0) = -2^0 + 3$

$-1+3$

2

$\boxed{(0,2)}$

D) $f_{(x)} = \left(\frac{5}{3}\right)^{x+2} - 3$

$x+2=0$ $\quad f(-2) = \left(\frac{5}{3}\right)^0 - 3$

$x=-2$ $\quad 1-3$

-2

$\boxed{(-2,-2)}$

E) $f_{(x)} = -\left(\frac{1}{2}\right)^{x+7} + 1$

$x+7=0$ $\quad f(-7) = -\left(\frac{1}{2}\right)^0 + 1$

$x=-7$ $\quad -1+1$

0

$\boxed{(-7,0)}$

F) $f_{(x)} = 3^{2x-3} + 2$ $\quad = (3/2)^{-3} + 2$

$2x-3=0$ $\quad f(3/2) = 3$

$2x=3$ $\quad 3^0 + 2$

$x = 3/2$ $\quad 1+2$

3

$\boxed{(3/2, 3)}$

G) $f_{(x)} = \left(\frac{1}{5}\right)^{3-x} + 6$

$3-x=0$ $\quad f(3) = \left(\frac{1}{5}\right)^0 + 6$

$x=3$ $\quad 1+6$

7

$\boxed{(3,7)}$

H) $f_{(x)} = -3^{2x-1} - 4$

$2x-1=0$ $\quad f\left(\frac{1}{2}\right) = -3^{2\left(\frac{1}{2}\right)-1} - 4$

$2x=1$ $\quad -3^0 - 4$

$x=\frac{1}{2}$ $\quad -1-4$

-5

$\boxed{\left(\frac{1}{2}, -5\right)}$

I) $f_{(x)} = 5^{2-3x} - 4$

$2-3x=0$ $\quad f(2/3) = 5^{2-3\left(\frac{2}{3}\right)} - 4$

$3x=2$ $\quad = 5^0 - 4$

$x = 2/3$ $\quad 1-4$

-3

$\boxed{\left(\frac{2}{3}, -3\right)}$

J) $f_{(x)} = 2(3)^{x+1} - 4$

$x+1=0$ $\quad f(-1) = 2(3)^0 - 4$

$x=-1$ $\quad 2(1)-4$

$2-4$

-2

$\boxed{(-1,-2)}$

K) $f_{(x)} = -2(5)^{x+6} - 7$

$x+6=0$ $\quad f(-6) = -2(5)^0 - 7$

$x=-6$ $\quad -2(1)-7$

$-2-7$

-9

$\boxed{(-6,-9)}$

L) $f_{(x)} = e^{x+3} - 4$

$x+3=0$ $\quad f(-3) = e^0 - 4$

$x=-3$ $\quad 1-4$

-3

$\boxed{(-3,-3)}$

M) $f_{(x)} = -e^{x+2} + 2$

$x+2=0$ $\quad f(-2) = -e^0 + 2$

$x=-2$ $\quad -1+2$

1

$\boxed{(-2,1)}$

N) $f_{(x)} = 2e^{x+3} + 5$

$x+3=0$ $\quad f(-3) = 2e^0 + 5$

$x=-3$ $\quad 2+5$

7

$\boxed{(-3,7)}$

O) $f_{(x)} = -\left(\frac{1}{3}\right)^{2x+1} + 3$

$2x+1=0$ $\quad f\left(-\frac{1}{2}\right) = -\left(\frac{1}{3}\right)^{2\left(-\frac{1}{2}\right)+1} + 3$

$2x=-1$ $\quad -\left(\frac{1}{3}\right)^0 + 3$

$x=-\frac{1}{2}$ $\quad -1+3$

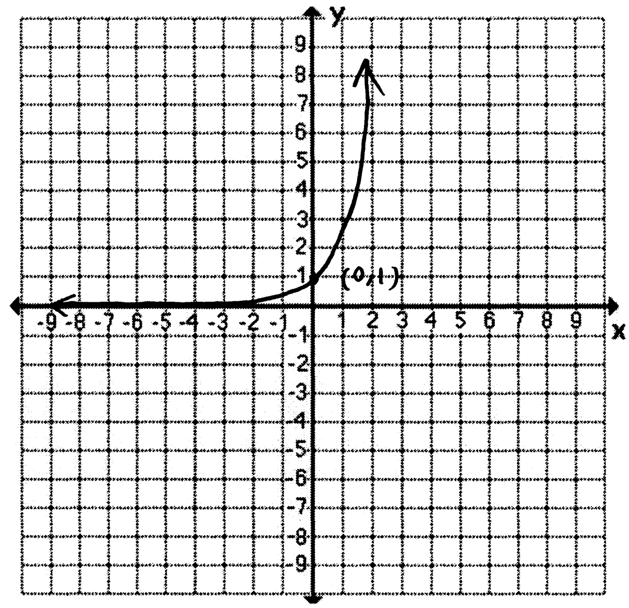
2

$\boxed{\left(-\frac{1}{2}, 2\right)}$

Graph each of the following functions by setting up a table for x and y values. Label the key point for each of the functions. ***Do not worry about the intercepts right now.***

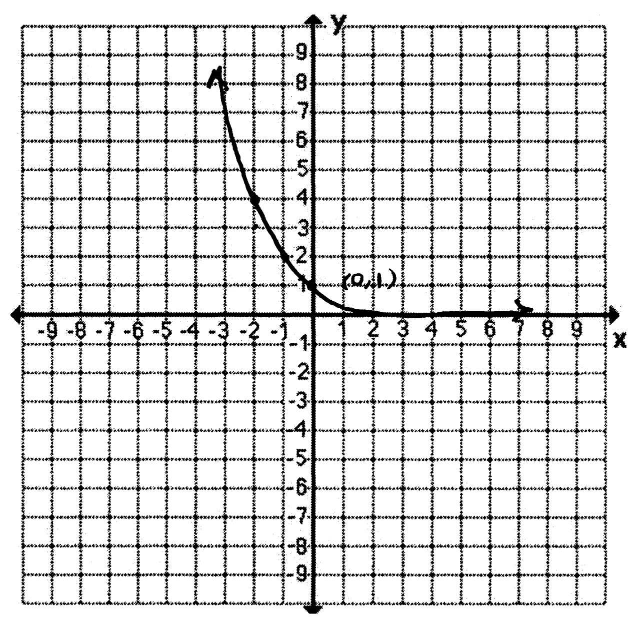
A) $f_{(x)} = 3^x$

x	$f_{(x)}$
-2	1/9
-1	1/3
0	1
1	3
2	9



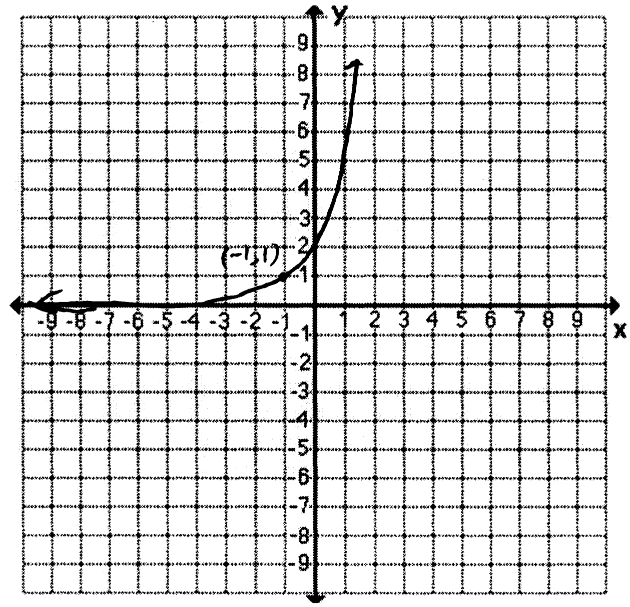
B) $f_{(x)} = \left(\frac{1}{2}\right)^x$

x	$f_{(x)}$
-2	4
-1	2
0	1
1	1/2
2	1/4



C) $f_{(x)} = 2^{x+1}$

x	$f_{(x)}$
-2	1/2
-1	1
0	2
1	4
2	8



D) $f_{(x)} = \left(\frac{1}{3}\right)^{x-3}$

$x-3=0$

$x=3$

$\left(\frac{1}{3}\right)^{1-3} = \left(\frac{1}{3}\right)^{-2} = 9$

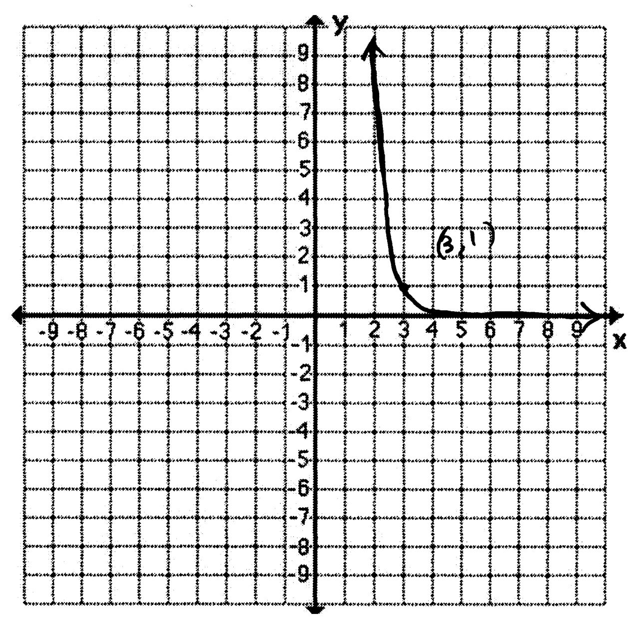
$\left(\frac{1}{3}\right)^{2-3} = \left(\frac{1}{3}\right)^{-1} = 3$

$\left(\frac{1}{3}\right)^{3-3} = \left(\frac{1}{3}\right)^{0} = 1$

$\left(\frac{1}{3}\right)^{4-3} = \left(\frac{1}{3}\right)^{1} = \frac{1}{3}$

$\left(\frac{1}{3}\right)^{5-3} = \left(\frac{1}{3}\right)^{2} = \frac{1}{9}$

x	$f_{(x)}$
1	9
2	3
3	1
4	1/3
5	1/9



E) $f_{(x)} = -2^x$

$-2^{-2} = -\frac{1}{4}$

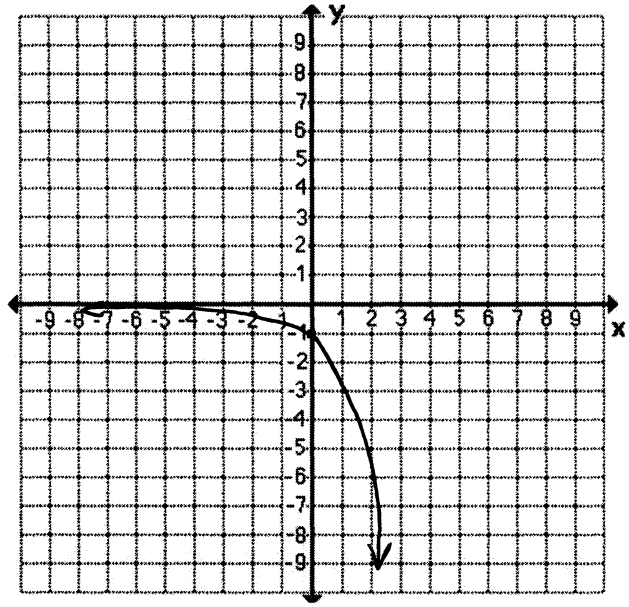
$-2^{-1} = -\frac{1}{2}$

$-2^{0} = -1$

$-2^{1} = -2$

$-2^{2} = -4$

x	$f_{(x)}$
-2	$-\frac{1}{4}$
-1	$-\frac{1}{2}$
0	-1
1	-2
2	-4



F) $f_{(x)} = 3^{x+2} + 3$

$x + 2 = 0$

$x = -2$

$3^{-4+2} + 3 = 3^{-2} + 3 = \frac{1}{9} + 3$

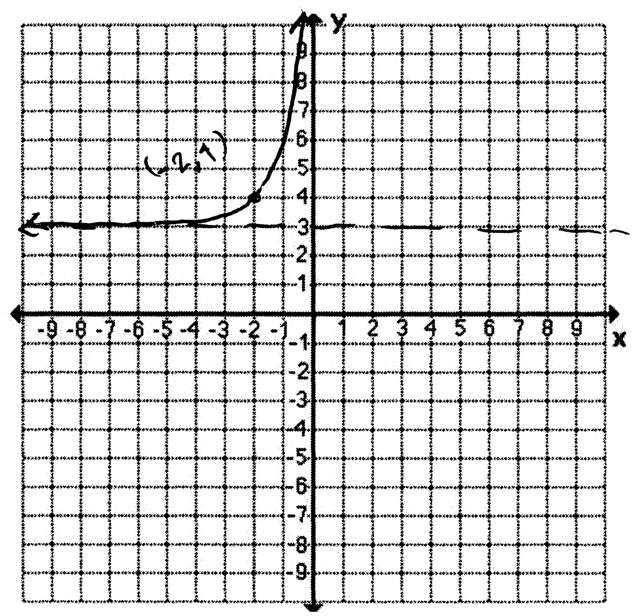
$3^{-3+2} = 3^{-1} + 3 = \frac{1}{3} + 3$

$3^{-2+2} + 3 = 3^{0} + 3 = 1 + 3 = 4$

$3^{-1+2} + 3 = 3^{1} + 3 = 6$

$3^{-2+2} + 3 = 3^{0+2} + 3 = 9 + 3 = 12$

x	$f_{(x)}$
-4	$3\frac{1}{9}$
-3	$3\frac{1}{3}$
-2	4
-1	6
0	12



In the "Functions" section of this workbook, we will concentrate on graphing functions by translation. We will be shifting the graph left, right, up or down depending on the values of h and k. What we will actually be shifting is this key point.

Checking Progress

You have now completed the "Rational Exponents" section of the workbook. The following is a checklist so that you may check your progress. Check off each of the objectives you have accomplished.

The student should be able to…

- ☐ ***Express a radical using a rational exponent.***
- ☐ ***Evaluate expressions containing rational exponents.***
- ☐ ***Solve equations involving rational exponents.***
- ☐ ***Identify an exponential function as being either growth or decay.***
- ☐ ***Graph an exponential function using a table.***

Glossary

Contained in this section are some of the terms you may hear in class. An attempt has been made to make their meaning as easy to understand as possible.

Abscissa:
The horizontal, or x coordinate in an ordered pair.

Absolute Value:
The absolute value of a number is the distance between that number and the origin.

Acute Angle:
An angle that is less than 90°.

Acute Triangle:
A triangle where all interior angles are less than 90°.

Adjacent:
Something that is next to.

Algebra:
A branch mathematics that involves working with variables.

Algorithm:
A step by step procedure designed to solve a problem. Mathematical instructions used to solve a problem or obtain a desired result.

Angle:
Two rays or lines that share a common endpoint.

Arc of a Circle:
A segment of a circle, the connected section of the outer edge of a circle.

Arithmetic Mean:
The average. If you have a set of n numbers, the average can be found by finding the sum of all digits, and dividing that sum by n.

Arithmetic Sequence:
A series of numbers that has a constant difference between them, such as 4, 7, 10, 13, 16….

Asymptote:
A straight line that the graph of a relation approaches very closely as the graph is followed. Is the limiting value of the curve of the function.

Average:
Usually refers the arithmetic mean, however, can be a single value representing the center of a set of values.

Axis:
a. A straight line about which a body or a geometric figure rotates or may be supposed to rotate.
b. One of the reference lines of a coordinate system.

Axis of Symmetry:
A line of symmetry for a graph. The two sides of the graph on either side of the axis of symmetry look like mirror reflections of one another.

Binomial:
A polynomial that has two terms.

Cartesian Coordinates:
Also known as rectangular coordinates. Come as an ordered pair (x,y) or in the form (x,y,z).

Cartesian Plane:
Also called the coordinate plane. A plane that is formed by a vertical and horizontal axis usually labeled as the x and y axes.

Central Angle:
An angle whose vertex is located at the center of a circle.

Coefficient:
The constant factor of a term that is being multiplied to a variable. For example, 12 is the coefficient of the term $12x^5y^2$.

Collinear:
Lying on the same straight line. Usually referring to points.

Complementary Angles:
Two acute angles whose sum is 90°.

Complex Numbers:
The sum or difference of real number and an imaginary number in the form $a + bi$, where $b \neq 0$.

Composite Number:
A number that has factors other than 1 and itself.

Composite Function:
A function that is made by combining one or more functions together. The process involves substituting a function's formula for the variable of the second function's formula.

Compound Fraction:
A fraction that has one or more fractions in its denominator or numerator. Also called complex fractions.

Compound Inequality:
Two inequalities put together.

Compound Interest:
A method of computing interest where interest is earned not only from the principal, but interest as well.

Conjugates:
The result of writing the sum of two terms as a difference, or vice versa. For example, x+7 and x-7 are conjugates.

Conjugate Pair Theorem:
If a polynomial has real coefficients, then any complex zeros always come in conjugate pairs. i.e.
$2 + 3i \quad and \quad 2 - 3i$

Consistent System of Equations:
A system of equations that has at least one solution.

Constant:
A term with no variables. For example, in the equation $3x^2 - 2x + 4 = 0$, 4 is the constant.

Constant Function:
The equation of a horizontal line. In other words, y = #.

Coordinates:
A set of numbers referring to the location of a specific point in a two or three dimensional space. An ordered pair, or ordered triple.

Coterminal Angles:
Angles when drawn in standard position who share the same terminal side.

Cubic Polynomial:
A third degree polynomial.

Degree of a Polynomial:
The greatest degree of any term in the polynomial itself.

Degree of a Term:
If the term has only one variable, the degree of the term is the exponent of that variable. If the term has more than one variable, the degree is the sum of the exponents of those variables.

Dependent Variable:
A variable whose value is dependent on one or more variables in a function. He who attempts to stand alone is dependent upon….

Descartes' Rule of Signs:
A method for determining the possible number of positive or negative zeros for a polynomial.

Domain:
The set of values of the independent variable of a function for which the function is defined. Usually, all possible x values that have a corresponding y value.

Even Function:
A function that is symmetrical to the y axis. A function is even if and only if $f_{(-x)} = f_{(x)}$.

Factor of a Polynomial:
Any polynomial that divides evenly into another polynomial.

Finite:
Having definite or definable limits.

Function:
A function is a rule that produces a correspondence between two sets of elements such that for each element in the domain, there corresponds exactly one element in the range.

Function Operations:
The process of adding, subtracting, multiplying, dividing and composing functions together.

Fundamental Theorem of Algebra:
Any polynomial to the nth degree has exactly n zeros.

Fundamental Theorem of Arithmetic:
All numbers have their own unique prime factorization.

Geometric Mean:
This is a sort of average. To find the geometric mean of a set of n numbers, multiply the given set of numbers, then take the nth root of their product.

Geometric Sequence:
A series of numbers in which there exists a common ratio between each term.

Greatest Common Factor:
The largest number that divides evenly into a given set of numbers.

Half-Life:
A term used to describe the decaying of a substance at an exponential rate. The length of time it takes for an amount of a particular substance to diminish by half.

Horizontal Line Test:
A test used to determine if a function is one-to-one. If a horizontal line crosses the function more than once, the function is not one-to-one.

Horizontal Reflection:
A reflection in which a two dimensional figure flips horizontally. The reflection takes place about a vertical axis.

Horizontal Translation:
A shift in which a two dimensional figure, or graph, moves horizontally to the left or right.

Imaginary Number:
A complex number with no real part. In other words, a complex number in the form $a + bi$, where $a = 0$. This leaves you with the pure imaginary number in the form bi.

Inconsistent System of Equations:
A system of equations in which there are no solutions.

Infinite:
To continue on forever, subject to no limitation, ∞.

Independent Variable:
The variable in an equation in which you may freely choose any value to substitute without considering other variables.

Initial Side of an Angle:
The ray where the measurement of an angle begins. In trigonometry, an angle is in standard position if its initial side is on the positive side of the x axis.

Interval Notation:
Using a pair of numbers rather than using an inequality to represent a specific interval. You will use parenthesis and brackets in interval notation. The smallest value is on the left of the interval.

Leading Coefficient:
The coefficient of the polynomial's first term when in the form $ax^n + bx^{n-1} + cx^{n-2}...$

Least Common Denominator:
The smallest number that can be used as a denominator for two or more fractions. This is the LCM of the denominators.

Least Common Multiple:
The smallest number that a given set of numbers can multiply to be.

Like Terms:
Terms that have the same variables, and corresponding powers. When dealing with radicals, terms that have the same index and radicand.

Limit:
The value that a function approaches as the domain variable approaches a specified value. In other words, if you are given a function, a limit says something to the affect of "What is the value of the function when x is 12?"

Mean:
Another word for average.

Midpoint:
A point on a line segment that is half way between two given points.

Mode:
The number that occurs most often in a set of data.

Monomial:
A polynomial that has only one term.

Oblique:
To be tilted at an angle. Something is oblique if it is neither vertical nor horizontal.

Odd Function:
A function that is symmetrical to the origin. A function is odd if and only if $f_{(-x)} = -f_{(x)}$.

One-Sided Limit:
Either a limit from the left, or a limit from the right.

One-to-One Function:
For every element in the range, there exists only one corresponding value in the domain. For each y value, there exists only one corresponding x value. Test using the horizontal line test.

Ordered Pair:
A set of numbers on the Cartesian plane that corresponds to the location of a particular point. In a 3-dimensional space, it is an ordered triple.

Ordinate:
The ordinate is the y coordinate in a point. In the point (2,5), 5 is the ordinate.

Origin:
Where the vertical and horizontal axis intersect on the Cartesian plane. The point (0,0).

Parent Functions:
The basic function that is used to build a more complicated one.

Period of a Periodic Function:
In trigonometry, a period is the horizontal length required for the graph of a periodic function to complete one full cycle.

Periodic Function:
A function whose graph continually repeats itself. Sine and cosine functions are examples of periodic functions.

Piecewise Function:
Pieces of different functions put together to form one graph. You only graph each of the different function in a specified domain.

Polynomial:
One or more algebraic terms joined by operation symbols.

Prime Factorization:
Completely factoring an integer to its primes. For example, $18 = 2 \cdot 3^2$.

Prime Number:
A positive number that can be divided only by 1 or itself. Remember, 1 is not a prime number.

Quadratic Equation:
An equation that contains only a second degree polynomial. A quadratic equation may be expressed in the general form $ax^2 + bx + c = 0$

Radian:
A unit used to measure angles. Found by multiplying the degree of an angle by $\frac{\pi}{180^{\circ}}$. If you were to take the length of the radius, and lay it on the arc of the circle, the number of radians in a circle, is the number of radii it takes to measure the arc of the circle.

Radicand:
The number under the $\sqrt{\ }$ symbol. In the case of $5\sqrt{3}$, 3 is the radicand.

Range:
The set of all values of the dependent variable of a function. Usually, all possible y values of a function.

Scalar Multiplication:
Multiplying one number (scalar) by another. Used in the multiplication of matricies and in vector operations.

Scalene Triangle:
A triangle where all three sides have different lengths.

Sinusoid:
A wave shaped graph as in $y = \sin x$.

Solution Set:
All values of the variable that satisfiy an equation, inequality, system of equations, etc.

Standard Position of an Angle:
An angle drawn on the x and y plane where the initial side of the angle is on the positive side of the x axis, and turns counterclockwise.

Subtend:
To cut into. To determine the measure of by marking off endpoints. For example, if you extend the legs of a central angle, the angle will subtend an arc on the circle.

Supplementary Angles:
Two angles whose sum is 180°.

System of Equations:
Two or more equations that contain common variables.

Terminal Side of an Angle:
The ray where the measurement of an angle stops. In trigonometry, angles are measured counterclockwise.

Translation:
The process of shifting a graph without changing its scale or the direction. The horizontal and vertical shifts of the graph of a function.

Vertical Line Test:
A test used to determine if a relation is a function. If a vertical line crosses the graph more than once, it is not a function.

Vertical Reflection:
A reflection in which a two dimensional figure flips over. Figures that are vertical reflections have a horizontal axis of reflection.

Vertical Translation:
A shift in which a two dimensional figure, or graph, moves vertically up or down.

***x*-intercept:**
The point at which a graph intersects the x axis.

***y*-intercept**
The point at which a graph intersects the y axis.

Zero of a Function:
The value of x for which $f_{(x)} = 0$. These are the x intercepts of the graph of the function.