DeMOIVRE'S THEOREM AND NTH ROOTS

If $z = r(\cos\theta + i\sin\theta)$ is a complex number and n is a positive integer, then...

$$z^n = \left[r(\cos\theta + i\sin\theta)\right]^n = r^n\left[\cos n\theta + i\cos n\theta\right]$$

Example

Find $\left(-1 + i\sqrt{3}\right)^{12}$

Find $\left(2\sqrt{3}+2i\right)^4$

Nth ROOTS

Find the 6th roots of -729.

$$x^6 = -729$$

$$x^6 + 729 = 0$$

This must have 6 roots!

$$\left(x^2\right)^3 + (9)^3 = 0$$

sum of cubes $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$

$$\left(x^2 + 9\right)\left(x^4 - 9x^2 + 81\right) = 0$$

$x^2 + 9 = 0$ $x^2 = -9$ $\sqrt{x^2} = \sqrt{-9}$ $x = \pm 3i$	$x^4 - 9x^2 + 81 = 0$ Let $A = x^2$ $A^2 - 9A + 81 = 0$ $A = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $A = \dfrac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(81)}}{2(1)}$

The nth root of a Complex Number

For every positive integer *n*, the complex number $z = r\left(\cos\theta + i\sin\theta\right)$ has exactly n distinct roots given by...

$$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\theta+2\pi k}{n} + i\sin\frac{\theta+2\pi k}{n}\right)$$

where $k = 0,\ 1,\ 2,\ 3,\ 4\ldots\ n-1$

Example

Find the 6th roots of -729.

$$x^6 = -729$$

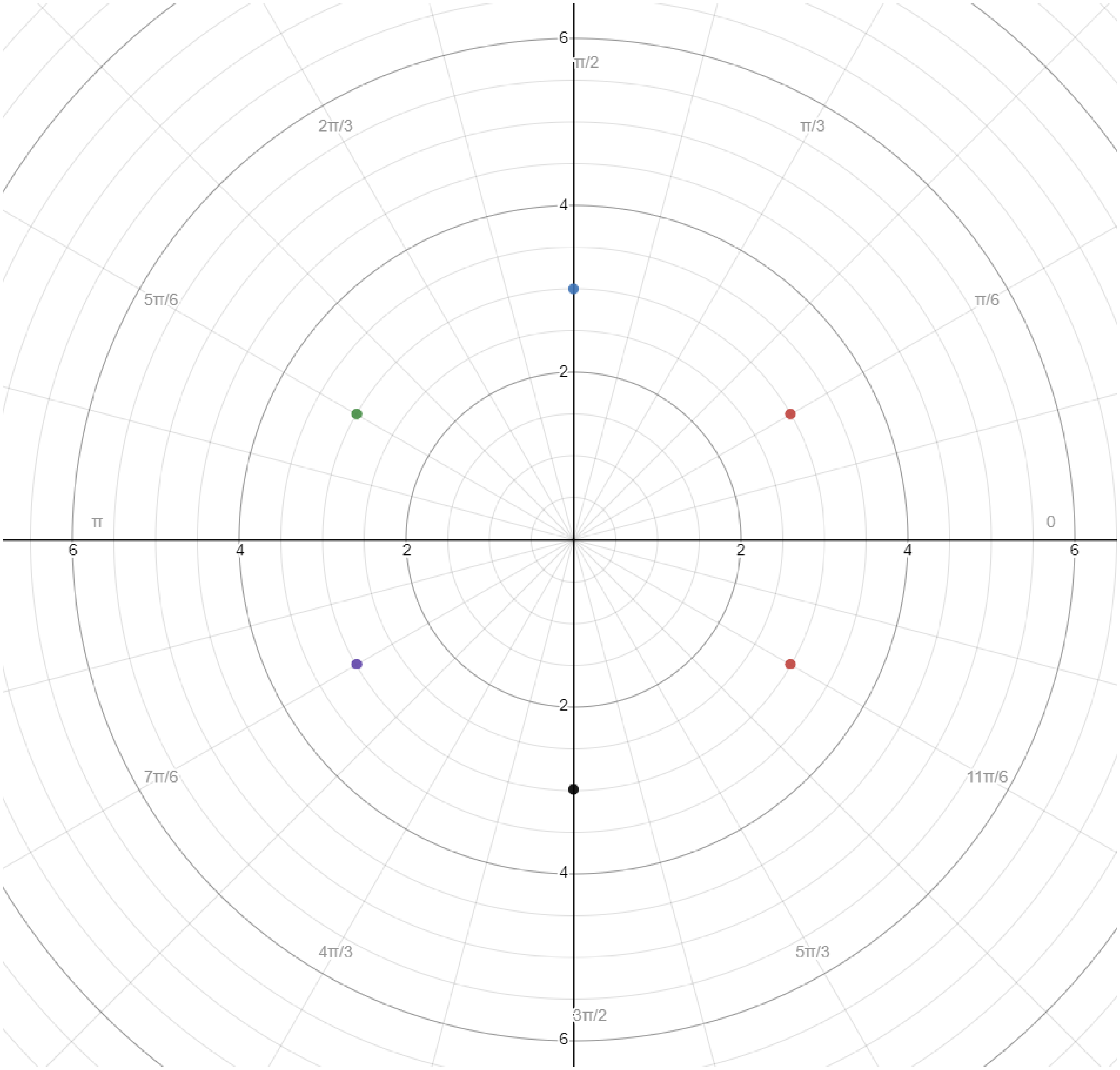
Now we need $\sqrt[6]{729\left(\cos\pi + i\sin\pi\right)}$

$$\sqrt[6]{729(\cos\pi + i\sin\pi)}$$

$$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\frac{\theta+2\pi k}{n} + i\sin\frac{\theta+2\pi k}{n}\right)$$

where $k = 0,\ 1,\ 2,\ 3,\ 4 \ldots\ n-1$

$k=0$	$\sqrt[6]{729}\left(\cos\frac{\pi+2\pi(\ )}{6} + i\sin\frac{\pi+2\pi(\ )}{6}\right) =$		
$k=1$	$\sqrt[6]{729}\left(\cos\frac{\pi+2\pi(\ )}{6} + i\sin\frac{\pi+2\pi(\ )}{6}\right) =$		
$k=2$	$\sqrt[6]{729}\left(\cos\frac{\pi+2\pi(\ )}{6} + i\sin\frac{\pi+2\pi(\ )}{6}\right) =$		
$k=3$	$\sqrt[6]{729}\left(\cos\frac{\pi+2\pi(\ )}{6} + i\sin\frac{\pi+2\pi(\ )}{6}\right) =$		
$k=4$	$\sqrt[6]{729}\left(\cos\frac{\pi+2\pi(\ )}{6} + i\sin\frac{\pi+2\pi(\ )}{6}\right) =$		
$k=5$	$\sqrt[6]{729}\left(\cos\frac{\pi+2\pi(\ )}{6} + i\sin\frac{\pi+2\pi(\ )}{6}\right) =$		



π/2
π/3
2π/3
π/6
5π/6
π
0
7π/6
11π/6
4π/3
5π/3
3π/2
6
4
2

Example **Find all roots of:** $\sqrt[5]{6\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)}$

$k=0$	$\sqrt[5]{6}\left(\cos\frac{\frac{2\pi}{3}+2\pi(\ )}{5}+i\sin\frac{\frac{2\pi}{3}+2\pi(\ )}{5}\right)=$		
$k=1$	$\sqrt[5]{6}\left(\cos\frac{\frac{2\pi}{3}+2\pi(\ )}{5}+i\sin\frac{\frac{2\pi}{3}+2\pi(\ )}{5}\right)=$		
$k=2$	$\sqrt[5]{6}\left(\cos\frac{\frac{2\pi}{3}+2\pi(\ )}{5}+i\sin\frac{\frac{2\pi}{3}+2\pi(\ )}{5}\right)=$		
$k=3$	$\sqrt[5]{6}\left(\cos\frac{\frac{2\pi}{3}+2\pi(\ )}{5}+i\sin\frac{\frac{2\pi}{3}+2\pi(\ )}{5}\right)=$		
$k=4$	$\sqrt[5]{6}\left(\cos\frac{\frac{2\pi}{3}+2\pi(\ )}{5}+i\sin\frac{\frac{2\pi}{3}+2\pi(\ )}{5}\right)=$		