Chapter 14: From Randomness to Probability
Randomness & Probability Models:

Behavior is ______________ if, while individual outcomes are uncertain, for a large number of repetitions, outcomes are regularly distributed. A situation in which we know what outcomes could happen, but we don’t know which particular outcomes will happen is called a ______________ ______________.

*Example:* If I roll a die once, I can’t predict with any certainty what number it will land on, but if I roll sixty times, I can expect it to land on 1 _______ times, 2 _______ times, 3 _______ times, etc.

The ______________ of an outcome is the proportion of times the outcome would occur for a large number of repetitions (in the long run).

*Example:* The probability of a die landing on 4 is the proportion of times a die lands on 4 for a large number of repetitions.

The set of all possible outcomes of an event is the ______________ of the event.

*Example:* For the event “roll a die and observe what number it lands on” the sample space contains all possible numbers the die could land on.

\[ S = \{ \text{_______________} \} \]

An ______________ is an outcome (or a set of outcomes) from a ______________ ______________.

*Example 1:* When flipping three coins, an event may be getting the result ______________. In this case, the event is one outcome from the sample space.

*Example 2:* When flipping three coins, an event may be getting two tails. In this case, the event is a set of outcomes (_______________) from the sample space.

An event is usually denoted by ______________________. For example, call getting two tails ______________.

The probability of event A is denoted __________.
Probability Rules:

The probability of any event is between _________. A probability of _____ indicates the event will _______ occur, and a probability of _____ indicates the event will _______ occur.

\[ \leq P(A) \leq \]

If S is the sample space, then \( P(S) = ____ \), since some outcome in the sample space is guaranteed to occur.

The probability that event A does not occur is one minus the probability that A does occur. That A will not occur is called the ___________ of A and is denoted _____.

\[ P(A^c) = \]

**Example:** When flipping two coins, the probability of getting two heads is 0.25. The probability of not getting two heads is _________.

If events A and B are ________ (they have no outcomes in common), then the probability that A or B occurs is the probability that A occurs ______ the probability that B occurs.

\[ P(A \text{ or } B) = \]

**Example:** Let event A be rolling a die and landing on an even number, and event B be rolling a die and landing on an odd number.

The outcomes for A are { ___________ } and the outcomes for B are { ___________ }. These events are disjoint because they have no outcomes in common. So the probability of A or B (landing on either an even or an odd number) equals the probability of A plus the probability of B.

\[ P(A \text{ or } B) = \]

Events A and B are _________________ if knowing that one occurs does not change the probability that the other occurs.

**Example:** Roll a yellow die and a red die. Event A is the yellow die landing on an even number, and event B is the red die landing on an odd number. These two events are _________________ because the probability of A does not change the probability of B.

If events A and B are _________________ then the probability of A and B equals the probability of A _____________ the probability of B.

\[ P(A \text{ and } B) = \]

**Example:** The probability than the yellow die lands on an even number and the red die lands on an odd number is:

\[ P(A \text{ and } B) = \]

If events A and B are independent, then their complements, \( A^c \) and \( B^c \) are also _________________ and \( A^c \) is _________________ of B.
The probability that event A occurs if we know for certain that event B will occur is called ________________ probability.

The ________________ probability of A given B is denoted: ________________

If events A and B are ________________ then knowing that event B will occur does not change the probability of A so for independent events:

\[ P(A|B) = \]

**Example:** When flipping a coin twice, what is the probability of getting heads on the second flip if the first flip was a head?

Event A: getting head on first flip  
Event B: getting head on second flip

Events A and B are independent since the outcome of the first flip does not change the probability of the second flip, so...

\[ P(A|B) = \]

The ________________ of two or more events is the event that ________________ of those events occurs.

Addition Rule for the Union of Two Events:

\[ P(A \text{ or } B) = P(A \cup B) = \]

Addition Rule for the Intersection of Two Events:

\[ P(A \text{ or } B) = P(A \cap B) = \]
**Example:** In our class of 36 students, we found that 25 students like to listen to rap music, 22 students like to listen to alternative music, and 16 students like to listen to both rap and alternative music.

Find the probabilities of the following events:

A: a student likes to listen to rap
B: a student likes to listen to alternative
A or B: a student likes to listen to rap or alternative
A and B: a student likes to listen to rap and alternative

Describe the following events:

A\(^c\):
B\(^c\):
A\(^c\) or B\(^c\):
A\(^c\) and B\(^c\):