Chapter 17: Probability Models

Bernoulli Trials:
A situation is called a **Bernoulli trial** if it meets the following criteria:

- There are only two possible outcomes (categorized as _______________ or _______________) for each trial
- The probability of success, denoted ______, is the same for each trial
- The trials are ________________________________

(Note that the independence assumption is violated whenever we sample without replacement, but is overridden by the __________ condition. As long as we don’t sample more than 10% of the population, the probabilities don’t change enough to matter.)

1. A new sales gimmick has 30% of the M&M’s covered with speckles. These “groovy” candies are mixed randomly with the normal candies as they are put into the bags for distribution and sale. You buy a bag and remove candies one at a time looking for the speckles.

   a. Is this situation a Bernoulli trial? Explain.

   b. What’s the probability that the first speckled candy is the fourth one we draw from the bag?

   c. What’s the probability that the first speckled candy is the tenth one?

   d. Write a general formula.

   e. What’s the probability we find the first speckled one among the first three we look at?

   f. How many do we expect to have to check, on average, to find a speckled one?

Geometric Distributions:
Suppose the random variable $X$ = the number of trials required to obtain the first success. Then $X$ is a ________________________________ if:

1. There are only two outcomes: _______________ or _______________.
2. The probability of success $p$ is _______________ for each observation.
3. The $n$ observations are _______________.
4. The variable of interest is the ________________________________.
Because \( n \) is not fixed there could be an infinite number of \( X \) values. However, the probability that \( X \) is a very large number is more and more unlikely. Therefore the probability histogram for a geometric distribution is always ____________________________.

If \( X \) is a geometric random variable, it is said to have a ____________________________, and is denoted as ______________.

The expected value (mean) of a geometric random variable is ______________.

The standard deviation of a geometric random variable is ______________.

The probability that \( X \) is equal to \( x \) is given by the following formula:

2. Refer back to the M&M’s distribution in problem 1.
   a. What’s the probability that we’ll find two speckled ones in a handful of five candies?
   b. List all possible combinations of exactly two speckled M&M’s in a handful of five candies.

**Binomial Distributions:**

Suppose the random variable \( X = \) the number of successes in \( n \) observations.

Then \( X \) is a ____________________________ __________________________ _________________ if:

1. There are only two outcomes: ______________________ or ______________________.
2. The probability of success \( p \) is ______________________ for each observation.
3. The \( n \) observations are ______________________.
4. There is a ____________________________ \( n \) of observations.

If \( X \) is a binomial random variable, it is said to have a ____________________________, and is denoted as ______________.

The expected value (mean) of a binomial random variable is ______________.

The standard deviation of a binomial random variable is ______________.
The ____________________ ____________________ ____________________ (or __________) assigns a probability to each value of \( X \).

The ____________________ ____________________ ____________________ (or __________) calculates the sum of the probabilities up to \( X \).

**Example:** Suppose each child born to Jay and Kay has probability 0.25 of having blood type O. If Jay and Kay have 5 children, what is the probability that exactly 2 of them have type O blood?

Let \( X = \) __________________________________________________________________________.

1. There are only two outcomes: success (_______________) or failure (_______________).
2. The probability of success (_______________) is _______________ for each of the ________ observations.
3. Each of the 5 observations is _________________, since one child’s blood type will not influence the next child’s blood type.
4. There is a fixed number of observations: ________.

So \( X \) is a ____________________________________________________________.

The following table shows the probability distribution function (__________) for the binomial random variable, \( X \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P( X = x ) )</td>
<td>0.2373</td>
<td>0.3955</td>
<td>0.2637</td>
<td>0.0879</td>
<td>0.0146</td>
<td>0.001</td>
</tr>
</tbody>
</table>

\( P(X = 0) = P( \ ) = ( )^5 = 0.2373 \)

\( \text{Binompdf} ( ) = 0.2373 \)
\( \text{Binompdf} ( ) = 0.3955 \)
\( \text{Binompdf} ( ) = 0.2637 \)
\( \text{Binompdf} ( ) = 0.0879 \)
\( \text{Binompdf} ( ) = 0.0146 \)
\( \text{Binompdf} ( ) = 0.0010 \)

Construct a histogram of the p.d.f. using the window \( X \) \([0, 6]\) and \( Y \) \([0, 1]\).
The following table shows the cumulative distribution function (__________) for the binomial random variable, X.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>0.2373</td>
<td>0.3955</td>
<td>0.2637</td>
<td>0.0879</td>
<td>0.0146</td>
<td>0.001</td>
</tr>
<tr>
<td>$P(X \leq x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Binomcdf ( ) = 0.2373
Binomcdf ( ) = 0.6328
Binomcdf ( ) = 0.8965
Binomcdf ( ) = 0.9844
Binomcdf ( ) = 0.999
Binomcdf ( ) = 1

Construct a histogram of the c.d.f. using the window $X_1 [0, 6]$ and $Y_{0.1} [0, 1]$.

1. Suppose I have a group of 4 students and I want to choose 1 of them as a volunteer. In how many ways can I choose 1 out of 4 students?

Call this “____________________.” There are _________ ways.

2. Suppose I have a group of 4 students and I want to choose 2 of them as volunteers. In how many ways can I choose 2 out of 4 students?

Call this “____________________.” There are _________ ways.

3. Suppose I have a group of 5 students and I want to choose 1 of them as a volunteer. In how many ways can I choose 1 out of 5 students?

Call this “____________________.” There are _________ ways.

4. Suppose I have a group of 5 students and I want to choose 3 of them as a volunteer. In how many ways can I choose 3 out of 5 students?

Call this “____________________.” There are _________ ways.
5. Suppose I have a group of 20 students and I want to choose 4 of them as a volunteer. In how many ways can I choose 4 out of 20 students?

SSSSFFFFFFFFFFFFFFFF
FSSSSFFFFFFFFFFFFF
FSFFFFSFFFFFFFFFFSF
... and so on...

Call this “________________.” There are __________ ways.

There is a mathematical way to count the total number of ways to arrange \( k \) out of \( n \) objects. This is called “_________________” or the ________________________________.

The binomial coefficient is the number of ways to arrange \( k \) successes in \( n \) observations.

It is written __________ and is called “________________.”

The value of “\( n \) choose \( k \)” is given by the formula:

Example: “5 choose 2”

So there are __________ ways to arrange 2 out of 5 objects.

Think of this as flipping a coin 5 times and getting 2 heads. In how many ways can that happen? It can happen in __________ ways.

If \( X \) is a binomial random variable with parameters \( n \) and \( p \), then

\[
P(X = x) =
\]