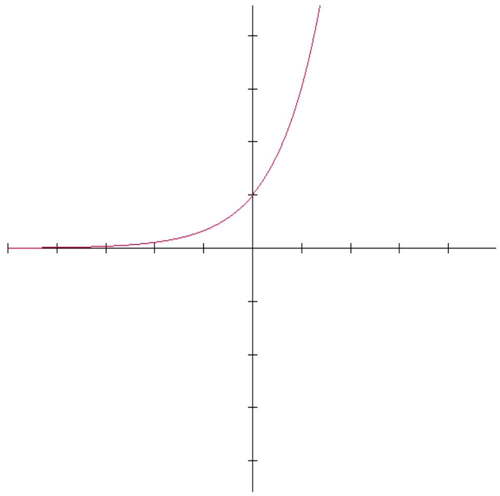
Exponential Functions

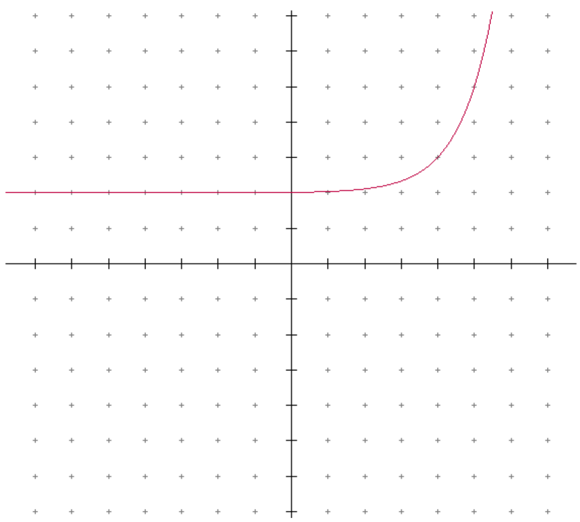
<u>Exponential Growth</u>

$$f(x) = ca^{x-h} + k$$

$$f(x) = 3^x$$

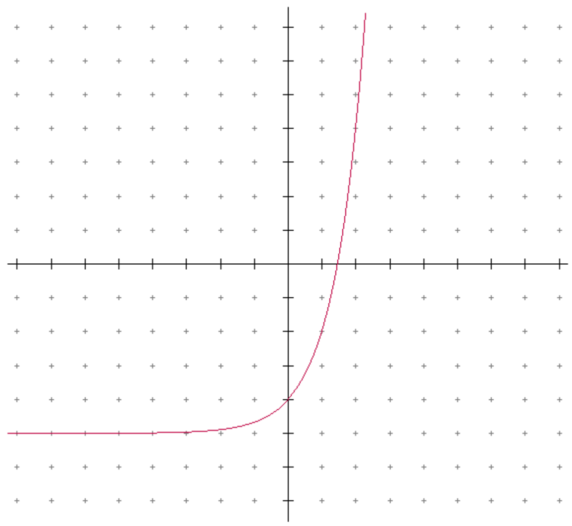


$$f(x) = 3^{x-4} + 2$$



Notice that the horizontal asymptote is at y=2. Since the graph of this function is going to be above the x axis, begin with the key point (0,1). This function shifts right 4 and up 2. The dots have been left on the graph so it would be easier to see. Adding 4 to the x value of the key point, and 2 to the y value, the new key point is at (4,3). Just remember where to begin, and do not cross the horizontal asymptote.

$$f(x) = 3^x - 5$$



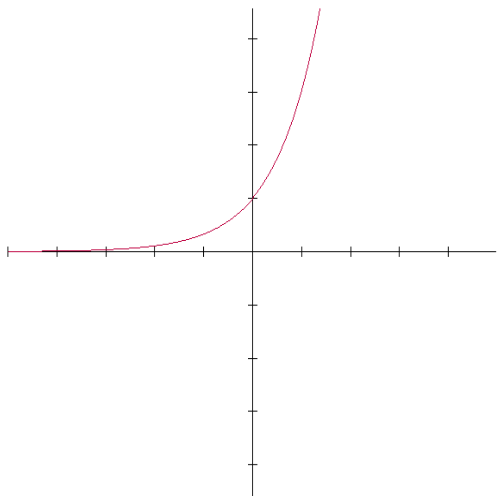
In this function, the value of k is -5. This tells you the new horizontal asymptote will be at y = -5. Since the value of the constant "c" is a positive one, begin with the key point (0,1). This function will only shift down 5 spaces. Therefore, subtract 5 from the y value of the key point which is 1. This results in: (1 – 5 = -4) therefore, the new key point is at (0,-4).

Graphing exponential functions by translation is relatively simple. The most difficult part will be finding the x and y intercepts as the x-intercept will involve the use of logarithms.

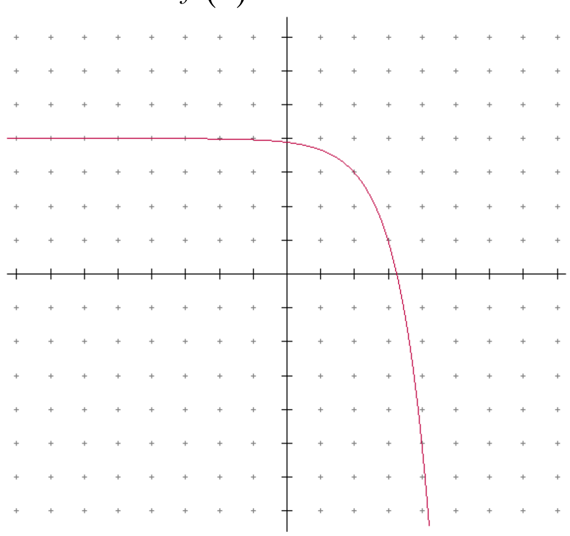
<u>***Exponential Growth***</u>

$$f\left(x\right)=ca^{x-h}+k$$

$$f\left(x\right)=3^{x}$$

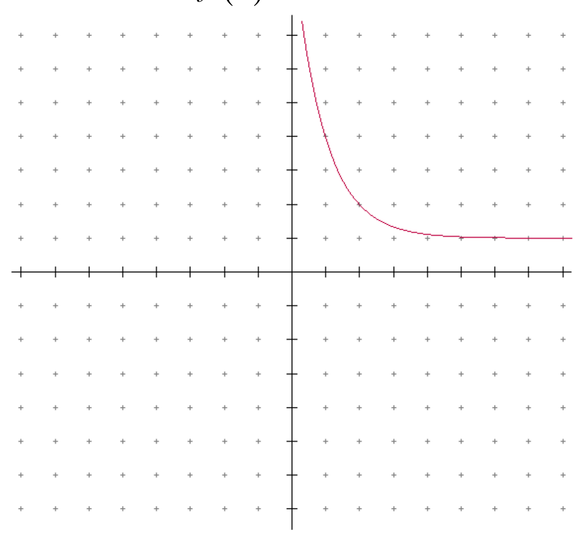


$$f\left(x\right)=-3^{x-2}+4$$



Since the value of "c" in the equation of this function is -1, we must begin with the key point of (0,-1). This is the key point, because that value of "c" caused the graph to reflect about the horizontal asymptote. The entire function will shift up 4, so the new horizontal asymptote is y = 4. The curve is going to shift right 2 and up 4. By adding 2 to the x value of the key point, and 4 to the y value, the new point can be found at (2,3). Notice the graph runs right though that point.

$$f\left(x\right)=3^{2-x}+1$$

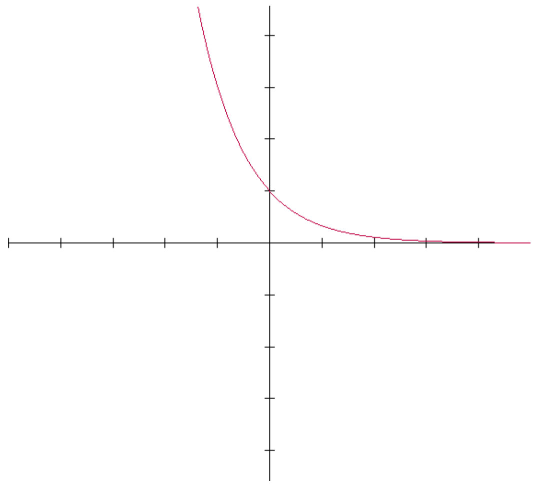


Here we have something that looks like decay. The value of "a" in this function is greater that one, so it should be growth. What really happened here, is the laws of exponents went to work. 3^{2-x} ***Is the same thing as*** $3^{-(x-2)}$***. The power of a power rule says this can be seen as*** $\left(3^{-1}\right)^{x-2}$***. This simplifies to*** $\left(\frac{1}{3}\right)^{x-2}$ ***, a decay curve. OK, so we begin with a decay curve that has a key point of (0,1). Add 2 to the x value, and 1 to the y value of the key point, and the new key point is (2,2), with a horizontal asymptote of y = 1.***

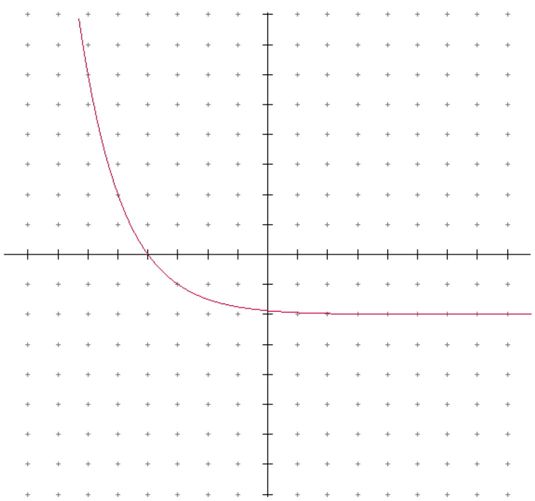
Exponential Decay

$$f(x) = ca^{x-h} + k$$

$$f(x) = \left(\frac{1}{2}\right)^x$$

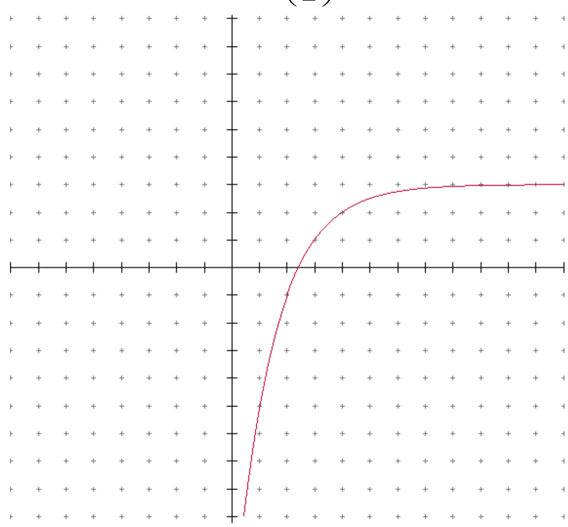


$$f(x) = \left(\frac{1}{2}\right)^{x+3} - 2$$



This is an exponential decay curve. The original graph will lie above the x axis. Therefore, begin with the key point (0,1). The key point will shift to the left 3, and down 2, so subtract 3 from the x value and 2 from the y value of the key point, and the coordinates of the new point will be at (-3,-1). The graph of this function shifts down 2, so the horizontal asymptote of this function is y = -2.

$$f(x) = -\left(\frac{1}{2}\right)^{x-4} + 3$$



This is an exponential decay function that is reflected and lies below the horizontal asymptote. The initial key point here is (0,-1). Since this graph will shift right 4 and up 3, add 4 to the x value of the key point and 3 to the y value. This yields a result of (4,2). Since the entire graph shifted upwards 3 spaces, the horizontal asymptote is y = 3. Once again, when graphing, do not cross the horizontal asymptote.

The translations of these functions are very similar to that of other functions we have seen. A point of reference with which to shift is all that is needed. Most important is to make sure to always use the appropriate key point to start with. <u>Draw the horizontal asymptote first</u>, that way the graph of the function does not accidentally cross it.

$$f(x) = ca^{x-h} + k$$

The domain of any exponential function is $(-\infty, \infty)$***. The values of c and k terms will determine the range of the function. Since the horizontal asymptote of an exponential function is given by y=k, the value of k will determine where the horizontal asymptote of the function lies, whereas the value of*** c ***will determine if the function is above or below that asymptote. Be careful not to use brackets when describing the range of an exponential function. The horizontal asymptote <u>must not be touched</u>, so only parenthesis may be used to describe the range in interval notation.***

Find the range and domain of each of the following exponential functions.

A) $f(x) = 2^{x+6} - 4$

B) $f(x) = -\left(\frac{1}{2}\right)^{x-1} + 3$

C) $f(x) = 2(3)^{x+1} - 5$

D) $f(x) = 5^{-x} - 3$

E) $f(x) = -2(5)^{x+2} - 3$

F) $f(x) = e^{x+2} - 3$

G) $f(x) = \left(\frac{5}{4}\right)^{x-8} + 2$

H) $f(x) = -2^{x-3} - 7$

I) $f(x) = -4^{3-x} + 2$

J) $f(x) = 2\left(\frac{1}{3}\right)^{x-5} + 1$

K) $f(x) = -6^{x-7} - 1$

L) $f(x) = -e^{x-2} + 3$

Here is an example of finding the x and y intercept of an exponential function.

$$f(x) = 3^{x+2} - 4$$

<u>Finding the x intercept.</u>

Begin by substituting 0 for $f(x)$

$$0 = 3^{x+2} - 4$$
$$4 = 3^{x+2}$$
$$\log 4 = \log 3^{x+2}$$
$$\log 4 = (x+2)\log 3$$
$$\log 4 = x\log 3 + 2\log 3$$
$$\log 4 - 2\log 3 = x\log 3$$

Now divide both sides by log 3.

$$\frac{\log 4 - 2\log 3}{\log 3} = \frac{x\log 3}{\log 3}$$
$$x = \frac{\log 4 - 2\log 3}{\log 3}$$
$$x \approx -0.7381$$

<u>Finding the y intercept.</u>

Begin by evaluating $f(0)$. In other words, find the value of the function when x is zero.

$$f(0) = 3^{0+2} - 4$$
$$f(0) = 3^2 - 4$$
$$f(0) = 9 - 4$$
$$f(0) = 5$$

As you can see, this function has an x intercept of approximately (-0.74,0), and a y intercept of (0,5).

Find the key point to each of the following functions.

A) $f(x) = 3^{x+4} - 2$

B) $f(x) = -4^{x-2} + 1$

C) $f(x) = 2^{4-x} + 5$

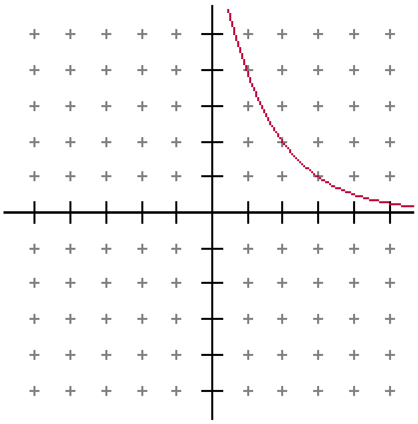
D) $f(x) = 3(2)^{x+1} - 5$

E) $f(x) = 2\left(\frac{1}{2}\right)^{x+4} - 3$

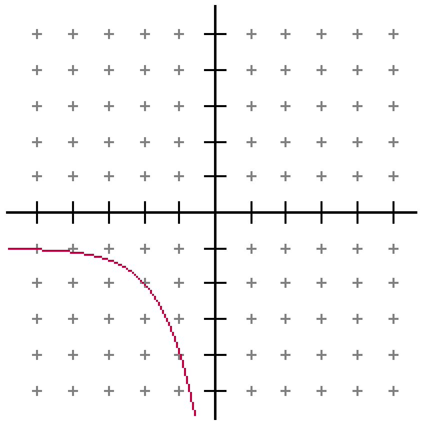
F) $f(x) = -3^{x+2} - 4$

Match the appropriate graph with its equation below. Explain why each of your solutions is true.

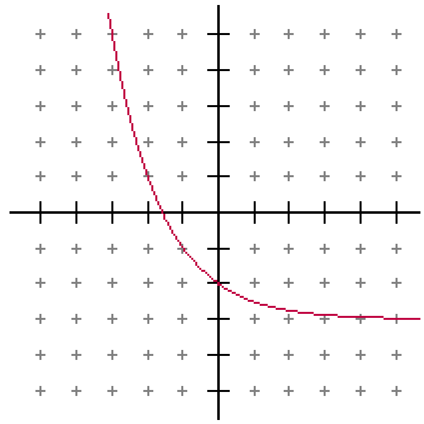
A



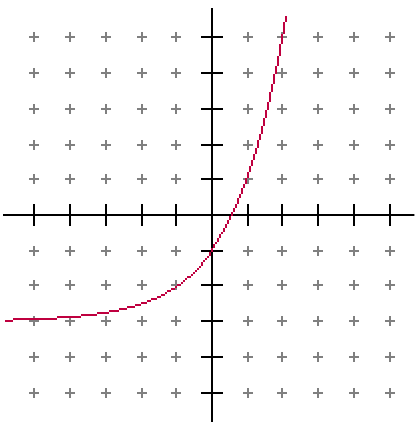
B



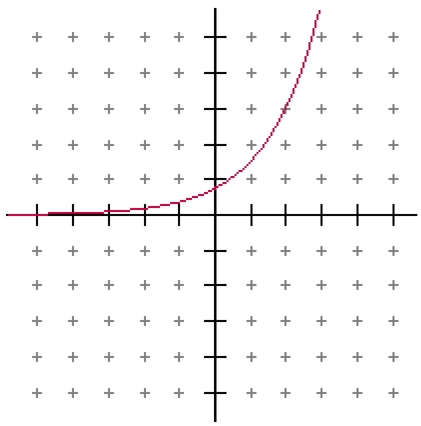
C



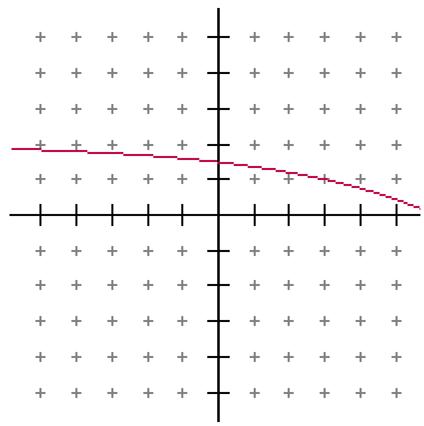
D



E



F



1) $f(x) = 2^{x+1} - 3$

2) $f(x) = \left(\frac{1}{2}\right)^{x} - 3$

3) $f(x) = 3(2)^{x-2}$

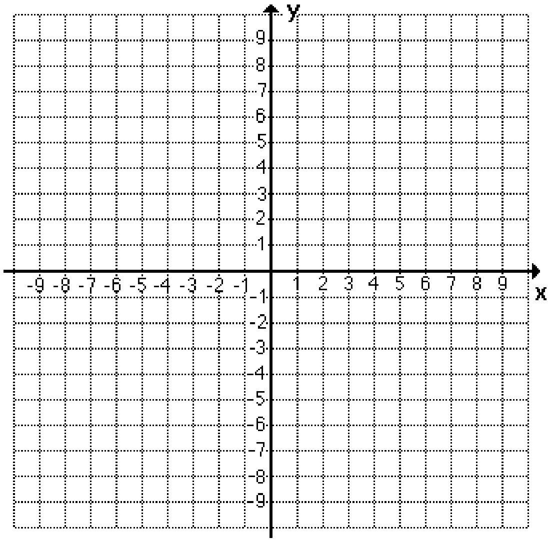
4) $f(x) = -3^{x+2} - 1$

5) $f(x) = -\left(\frac{5}{4}\right)^{x-3} + 2$

6) $f(x) = 2^{3-x}$

Graph each of the following exponential functions. Be sure to label the key point of the function. Find the x intercept (if it exists) and y intercept of each function.

A) $f(x) = 3^{x-2} - 1$



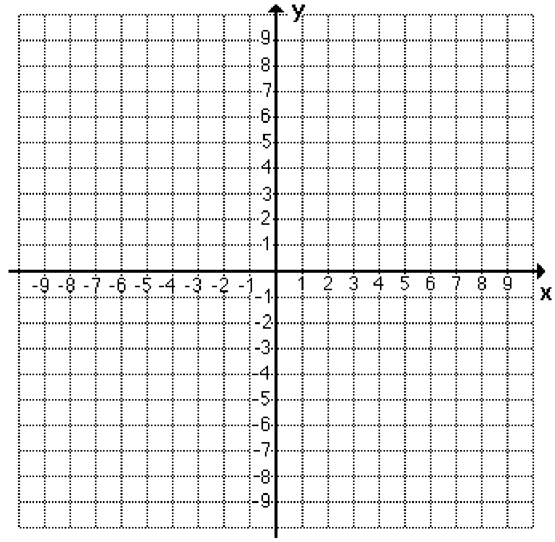
Y-intercept:

X-intercepts:

Range:

Domain:

B) $f(x) = -2^{x+3} - 4$



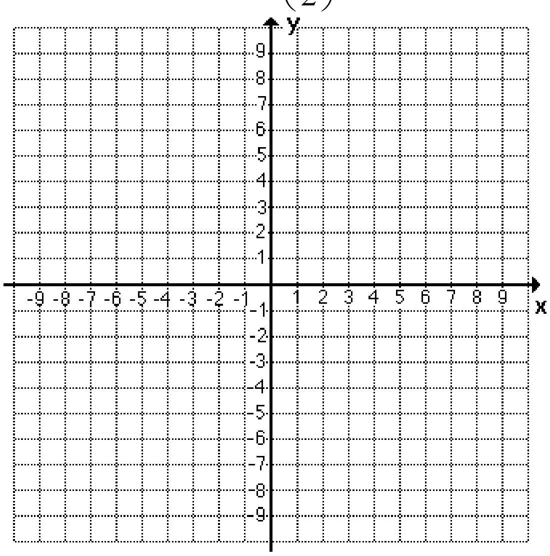
Y-intercept:

X-intercepts:

Range:

Domain:

C) $f(x) = \left(\frac{1}{2}\right)^{x+2} - 3$



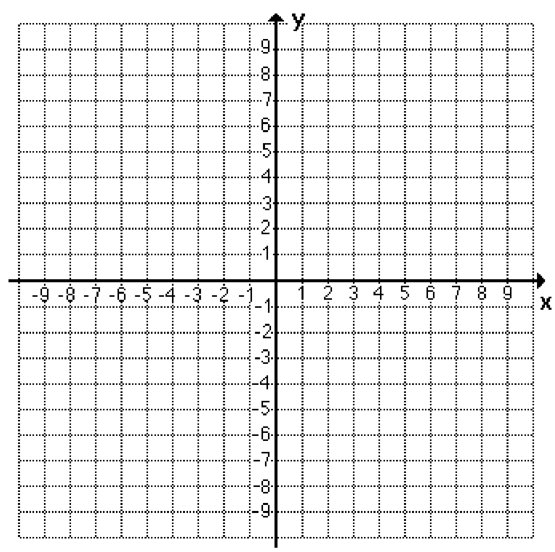
Y-intercept:

X-intercepts:

Range:

Domain:

D) $f(x) = 2^{4-x}$



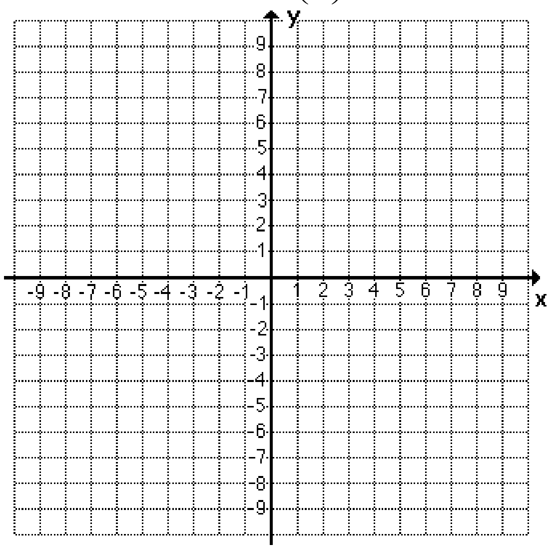
Y-intercept:

X-intercepts:

Range:

Domain:

E) $f(x) = -\left(\frac{1}{3}\right)^{x-2} - 3$



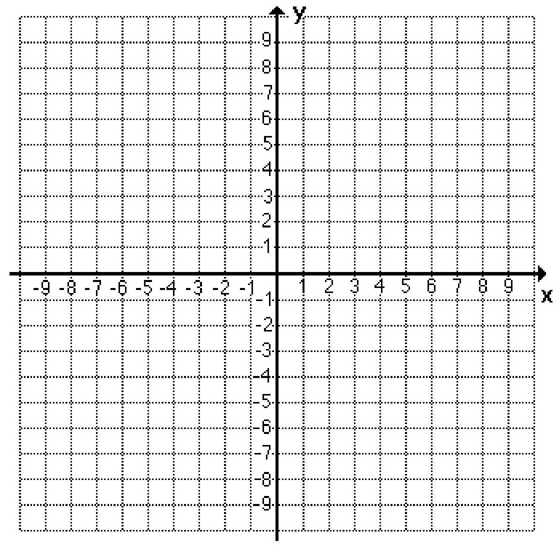
Y-intercept:

X-intercepts:

Range:

Domain:

F) $f(x) = 2(3)^{x-2} + 1$



Y-intercept:

X-intercepts:

Range:

Domain: