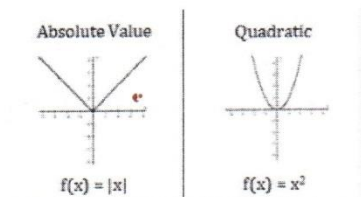


Name: Key Period: _____ Date: _____

Integrated Math 3- End of Course Final Review Packet

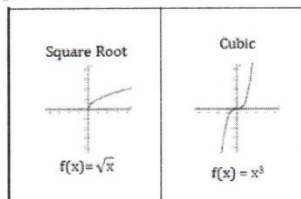
Part 1: Functions

Transformations of Parent Functions



Transformation	Appearance in Function
Vertical Translation	$f(x) \rightarrow f(x) + d$ $\rightarrow$
Horizontal Translation	$f(x) \rightarrow f(x - c)$ $\rightarrow$
Vertical Stretch/Compression	$f(x) \rightarrow af(x)$
Reflection in x-axis	$f(x) \rightarrow -f(x)$
Horizontal Stretch/Compression	$f(x) \rightarrow f(kx)$
Reflection in y-axis	$f(x) \rightarrow f(-x)$

+d = up d
-d = down d
-c = right c
+c = left c



#1-4. Describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.

1. $f(x) = x^2, g(x) = -(x + 1)^2 - 3$

*flips upside down and shifts
left 1, down 3.*

2. $f(x) = |x|, g(x) = |x - 4| + 6$

shifts right 4, up 6

3. $f(x) = \sqrt{x}, g(x) = -\sqrt{x + 3} - 1$

*goes downward and shifts
left 3, down 1*

4. $f(x) = x^3, g(x) = (-x)^3 + 2$

*flips from left to right and
shifts up 2*

Domain and Range

Parent Function	Domain (x-values)	Range (y-values)
$y = x^2$	$x = \text{All Real Numbers}$	$y \geq 0$
$y = \sqrt{x}$	$x \geq 0$	$y \geq 0$
$y = x $	$x = \text{All Real Numbers}$	$y \geq 0$
$y = \frac{1}{x}$	$x = \text{All Real Numbers, except } x \neq 0$	$y > 0$

5. Find the domain of $f(x) = \sqrt{x - 1} + 6$.

*$x - 1 \geq 0$
 $x \geq 1$ $[1, \infty)$*

6. Find the domain of $f(x) = -(x + 4)^2 - 3$.

$(-\infty, \infty)$

7. Find the range of $f(x) = (x + 2)^2 + 3$

$(-\infty, \infty)$

8. Find the range of $f(x) = |x + 1| - 2$

$(-\infty, \infty)$

Composing Functions

9. Given $f(x) = 2x - 1$ and $g(x) = \frac{x+3}{2}$, find

$f(g(x))$. *$f(g(x)) = f(\frac{x+3}{2})$*

$= 2(\frac{x+3}{2}) - 1$

$= x + 3 - 1$

$f(g(x)) = x + 2$

10. Given $f(x) = x^2$ and $g(x) = x + 4$, find $f(g(x))$.

$f(g(x)) = f(x + 4)$

$= (x + 4)^2$

$f(g(x)) = x^2 + 8x + 16$

11. Given $f(x) = 3x + 1$ and $g(x) = 2x - 4$, find $g(f(x))$.

$$g(f(x)) = g(3x+1)$$

$$= 2(3x+1) - 4$$

$$= 6x + 2 - 4$$

$$g(f(x)) = 6x - 2$$

12. Given $f(x) = \frac{x+6}{3}$ and $g(x) = 3x + 1$, find $g(f(x))$.

$$g(f(x)) = g\left(\frac{x+6}{3}\right)$$

$$= 3\left(\frac{x+6}{3}\right) + 1$$

$$= x + 6 + 1$$

$$g(f(x)) = x + 7$$

Inverse Functions

Find the inverse of $f(x) = 3x - 2$

$$f(x) = 3x - 2$$

$$y = 3x - 2 \rightarrow x = \frac{y+2}{3}$$

$$\frac{x+2}{3} = y$$

$$f^{-1}(x) = \frac{x+2}{3}$$

Step #1: Switch x and y

Step #2: Isolate y.

If $f(x)$ and $g(x)$ are inverse functions, then $f(g(x)) = x$.

13. Find the inverse of $f(x) = 3x^2 + 1$.

$$x = 3y^2 + 1$$

$$\frac{x-1}{3} = y^2$$

$$y = \sqrt{\frac{x-1}{3}}$$

$$f^{-1}(x) = \sqrt{\frac{x-1}{3}}$$

14. Find the inverse of $f(x) = \sqrt[3]{x+3} - 2$.

$$x = \sqrt[3]{y+3} - 2$$

$$(x+2)^3 = (\sqrt[3]{y+3})^3$$

$$y+3 = (x+2)^3$$

$$y = (x+2)^3 - 3$$

$$f^{-1}(x) = (x+2)^3 - 3$$

15. Are $f(x)$ and $g(x)$ inverse functions?

$$f(x) = 2x - 1, \quad g(x) = \frac{x}{2} + 1$$

$$f(g(x)) = 2\left(\frac{x}{2} + 1\right) - 1$$

$$f(g(x)) = x + 2 - 1$$

$$f(g(x)) = x + 1$$

no, $f(x)$ and $g(x)$ are not inverse of each other

16. Are $f(x)$ and $g(x)$ inverse functions?

$$f(x) = 5x + 1, \quad g(x) = \frac{x-1}{5}$$

$$f(g(x)) = 5\left(\frac{x-1}{5}\right) + 1$$

$$f(g(x)) = x - 1 + 1$$

$$f(g(x)) = x$$

$$g(f(x)) = \frac{(5x+1)-1}{5}$$

$$g(f(x)) = \frac{5x}{5}$$

$$g(f(x)) = x$$

yes, $f(x)$ and $g(x)$ are inverses

Even & Odd Functions

Even Functions	Odd Functions
$f(-x) = f(x)$	$f(-x) = -f(x)$

#17-20. Determine if the functions are even or odd, or neither.

17. $f(x) = 3x^4 - 2x^2 + 8$

$$f(-x) = 3(-x)^4 - 2(-x)^2 + 8$$

$$f(-x) = 3x^4 - 2x^2 + 8$$

Even

19. $f(x) = x^2 - 2x + 1$

$$f(-x) = (-x)^2 - 2(-x) + 1$$

$$f(-x) = x^2 + 2x + 1$$

neither

18. $f(x) = 2x^3 - 4x$

$$f(-x) = 2(-x)^3 - 4(-x)$$

$$f(-x) = -2x^3 + 4x$$

odd

20. $f(x) = 4x^2 + 5$

$$f(-x) = 4(-x)^2 + 5$$

$$f(-x) = 4x^2 + 5$$

Even

Rate of Change

The rate of change of a function from $x = a$ to $x = b$ is $\frac{f(b) - f(a)}{b - a}$.

21. Find the rate of change of $f(x) = x^2 + 2$ from $x = -1$ to $x = 3$.

$$f(-1) = (-1)^2 + 2 = 3 \quad f(3) = 3^2 + 2 = 11$$

$$\frac{11 - 3}{3 - (-1)} = \frac{8}{4} = \boxed{2}$$

22. Find the rate of change of $f(x) = \sqrt{3x - 2}$ from $x = 1$ to $x = 6$.

$$f(1) = \sqrt{3(1) - 2} = \sqrt{1} = 1 \quad f(6) = \sqrt{3(6) - 2} = \sqrt{16} = 4$$

$$\frac{4 - 1}{6 - 1} = \frac{3}{5} = \boxed{\frac{3}{5}}$$

23. Find the rate of change of $f(x) = \sqrt{1 - 2x}$ from $x = -12$ to $x = -4$.

$$f(-12) = \sqrt{1 - 2(-12)} = \sqrt{25} = 5 \quad f(-4) = \sqrt{1 - 2(-4)} = \sqrt{9} = 3$$

$$\frac{3 - 5}{-4 - (-12)} = \frac{-2}{8} = -\frac{1}{4} = \boxed{-\frac{1}{4}}$$

24. Find the rate of change of $f(x) = 2x - 3$ from $x = -2$ to $x = 2$.

$$f(-2) = 2(-2) - 3 = -7 \quad f(2) = 2(2) - 3 = 1$$

$$\frac{1 - (-7)}{2 - (-2)} = \frac{8}{4} = \boxed{2}$$

Part 2: Logarithmic and Exponential Functions

Evaluating Logarithms

Rewrite as an Exponential Function
 $\log_b y = x \rightarrow b^x = y$

Use Change of Base Formula (Calculator)

$$\log_b y = \frac{\log y}{\log b}$$

#25-28. Evaluate the Logarithms.

25. $\log_3 27 = 3$ $3^x = 27$
 $3^x = 3^3 \quad x = 3$

26. $\log_5 25 = 2$ $5^x = 25$
 $5^x = 5^2 \quad x = 2$

27. $\log_4 16 = 2$ $4^x = 16$
 $4^x = 4^2 \quad x = 2$

28. $\log_2 16 = 4$ $2^x = 16$
 $2^x = 2^4 \quad x = 4$

Expanding Logarithms

1. $\log_b mn = \log_b m + \log_b n$

2. $\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$

3. $\log_b m^r = r \log_b m$

29. Simplify and Expand $\log_2 \frac{4x^2y}{2x}$.

$$\log_2 2xy$$

$$\boxed{\log_2 2 + \log_2 x + \log_2 y}$$

$\log_2 2 = 1$

30. Simplify and Expand $\log_3 \frac{x^5y^2}{x^3y^3}$.

$$\log_3 x^2 - \log_3 y$$

$$\boxed{2 \log_3 x - \log_3 y}$$

31. Expand $\log 24$ in two different ways.

$$\log 24 = \log 2^3 \cdot 3$$

$$\boxed{3 \log 2 + \log 3}$$

or

$$\boxed{\log 8 + \log 3}$$

32. Expand $\log 36$ in two different ways.

$$\log 36 = \log 2^2 \cdot 3^2$$

$$\boxed{2 \log 6}$$

or

$$\boxed{2 \log 2 + 2 \log 3}$$

Solving Logarithms & Exponential Equations

33. $4^{x-2} = 16^{x+1}$

$4^{x-2} = (4^2)^{x+1}$
 $4^{x-2} = 4^{2x+2}$
 $x-2 = 2x+2$
 $-x-2 = -x-2$
 $-4 = x$
 $x = -4$

34. $27^{2x} = 3^{x+10}$

$(3^3)^{2x} = 3^{x+10}$
 $3^{6x} = 3^{x+10}$
 $6x = x+10$
 $5x = 10$
 $x = 2$

35. $\log_3(x+2) - \log_3 5 = 3$

$\log_3 \frac{x+2}{5} = 3$
 $3^3 = \frac{x+2}{5}$
 $27 = \frac{x+2}{5}$
 $135 = x+2$
 $x = 133$

36. $\log_6 2x + \log_6 3 = 4$

$\log_6 6x = 4$
 $6^4 = 6x$
 $1296 = 6x$
 $x = 216$

37. $\log_2(7-3x) = 4$

$2^4 = 7-3x$
 $16 = 7-3x$
 $9 = -3x$
 $x = -3$

38. $\log_5(2x+1) = 2$

$5^2 = 2x+1$
 $25 = 2x+1$
 $24 = 2x$
 $x = 12$

39. $4e^{3x} = 12$

$\ln e^{3x} = \ln 3$
 $3x = \ln 3$
 $x = \frac{1}{3} \ln 3$

40. $-6e^{2x} = -36$

$e^{2x} = 6$
 $\ln e^{2x} = \ln 6$
 $2x = \ln 6$
 $x = \frac{1}{2} \ln 6$

41. $e^{4x} - 2 = 6$

$e^{4x} = 8$
 $\ln e^{4x} = \ln 8$
 $4x = \ln 8$
 $x = \frac{1}{4} \ln 8$

42. $e^{2x} = 1.5$

$\ln e^{2x} = \ln 1.5$
 $2x = \ln 1.5$
 $x = \frac{1}{2} \ln 1.5$

Exponential Growth vs. Exponential Decay

$f(x) = a(b)^x$

Exponential Growth: $b > 1$.

Exponential Decay: $0 < b < 1$.

#43-46. Classify the following as exponential growth or exponential decay.

43. $f(x) = 12(1.25)^x$

Exp Growth

44. $f(x) = 4\left(\frac{5}{4}\right)^x$

Exp. Growth

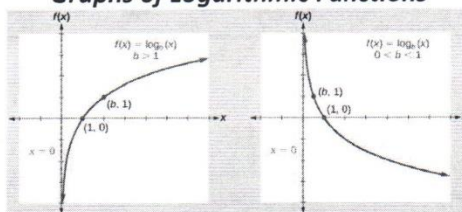
45. $f(x) = 15(0.5)^x$

Exp Decay

46. $f(x) = 2.3^x$

Exp. Growth

Graphs of Logarithmic Functions



#47-50. Find the x-intercept and Vertical Asymptote for the graph of each function.

47. $f(x) = \log_2(x - 3)$

$\log_2(x-3) = 0 \implies 1 = x-3 \implies x = 4$
 $2^0 = x-3 \implies 4 = x$

VA: $x = 3$
 x int: $(4, 0)$

48. $f(x) = \log_5 x - 2$

$\log_5 x - 2 = 0 \implies \log_5 x = 2 \implies x = 25$
 $5^2 = x$

VA: $x = 0$
 x int: $(25, 0)$

49. $f(x) = \log_3(x + 2)$

$\log_3(x+2) = 0 \implies x+2 = 1 \implies x = -1$
 $3^0 = x+2 \implies x = -1$

VA: $x = -2$
 x int: $(-1, 0)$

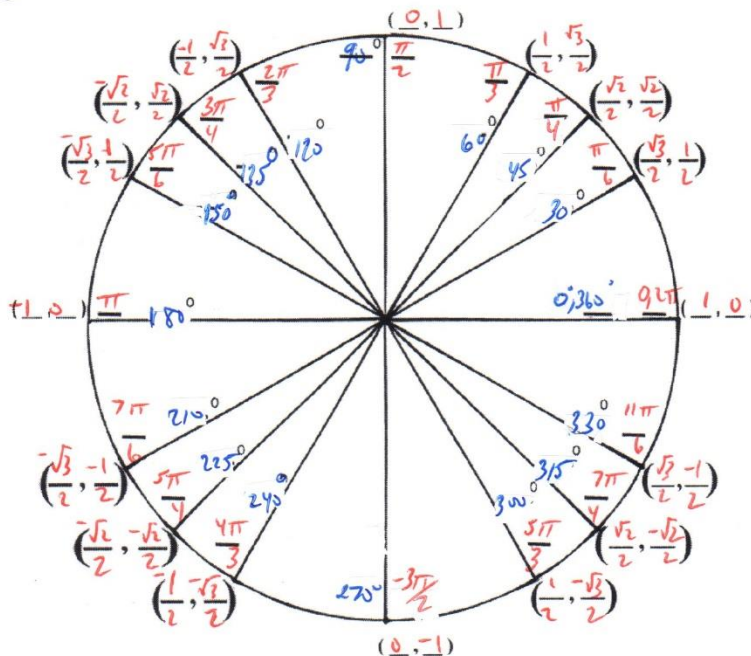
50. $f(x) = \log_2 x + 1$

$\log_2 x + 1 = 0 \implies \log_2 x = -1 \implies x = \frac{1}{2}$
 $2^{-1} = x$

VA: $x = 0$
 x int: $(\frac{1}{2}, 0)$

Part 3: Trigonometry

Fill in the Unit Circle



Evaluating Trigonometric Functions

51. $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

52. $\sin(120^\circ) = \frac{\sqrt{3}}{2}$

53. $\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

54. $\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

55. $\sin \pi = 0$

56. $\cos 0 = 1$

57. $\cos(-45^\circ) = \frac{\sqrt{2}}{2}$

58. $\sin(-60^\circ) = -\frac{\sqrt{3}}{2}$

59. $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$. Find another value for θ where $\sin \theta = \frac{1}{2}$.

$\theta = \frac{5\pi}{6}$

Name two angles that have the opposite value of

$\sin\left(\frac{\pi}{6}\right) \rightarrow \frac{1}{2}$

want y value $-\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

60. $\cos\left(\frac{3\pi}{4}\right) = \frac{-\sqrt{2}}{2}$. Find another value for θ where $\cos \theta = \frac{-\sqrt{2}}{2}$.

$\theta = \frac{5\pi}{4}$

Name two angles that have the opposite value of

$\cos\left(\frac{3\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

want $\sqrt{2}/2$

$\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

Converting Between Radians and Degrees

Degrees to Radians

$$x^\circ \cdot \frac{\pi}{180^\circ}$$

Radians to Degrees

$$x \cdot \frac{180^\circ}{\pi}$$

#61-64. Convert the angle measure from radians to degrees.

61. $\frac{5\pi}{4}$

$\frac{5\pi}{4} \cdot \frac{180}{\pi} = 225^\circ$

62. $\frac{\pi}{2}$

$\frac{\pi}{2} \cdot \frac{180}{\pi} = 90^\circ$

63. $\frac{11\pi}{6}$

$\frac{11\pi}{6} \cdot \frac{180}{\pi} = 330^\circ$

64. $\frac{9\pi}{4}$

$\frac{9\pi}{4} \cdot \frac{180}{\pi} = 405^\circ$

#65-68. Convert the angle measure from degrees to radians.

65. 120°

$120^\circ \cdot \frac{\pi}{180} = \frac{2\pi}{3}$

66. 480°

$480^\circ \cdot \frac{\pi}{180} = \frac{48\pi}{18} = \frac{8\pi}{3}$

67. 225°

$225^\circ \cdot \frac{\pi}{180} = \frac{5\pi}{4}$

68. 300°

$300^\circ \cdot \frac{\pi}{180} = \frac{30\pi}{18} = \frac{5\pi}{3}$

Graphing Trigonometric Functions

$$y = A \sin B(\theta - C) + D$$

A	B	C (Inside Parentheses)	D
A= Amplitude Tells you the Maximums and Minimums If A is negative, there is a reflection over the x-axis (Maximums and Minimums change places- First "M" is a minimum)	B= Frequency Used to find the Period Period = $\frac{2\pi}{B}$	Phase Shift = $\frac{C}{B}$ If $\frac{C}{B}$ is positive , the graph shifts left . If $\frac{C}{B}$ is negative , the graph shifts right . $\frac{\pi}{2}$ is 2 units, $\frac{\pi}{4}$ is 1 unit	Vertical Shift = D If D is positive, the graph shifts up. If D is negative, the graph shifts down.

#69-72. Identify the Amplitude, Period, and Shifts for the graphs below.

69. $f(x) = 3\sin(\pi x)$

A= 3 P= 2 PS= none VS= none

70. $f(x) = -\cos 2(x - 1)$

A= 1 P= π PS= 1 VS= none

71. $k(t) = 20 + 2\cos(\pi t)$

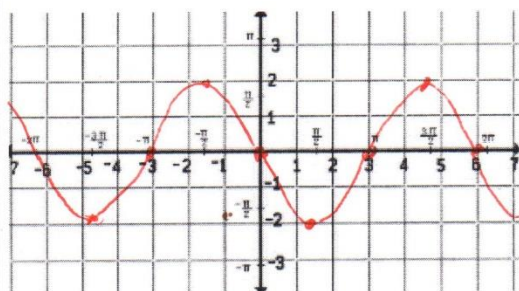
A= 2 P= 2 PS= none VS= +20

72. $h(t) = 2\sin(\pi t + 2) - 3$

A= 2 P= 2 PS= $-\frac{2}{\pi}$ VS= -3

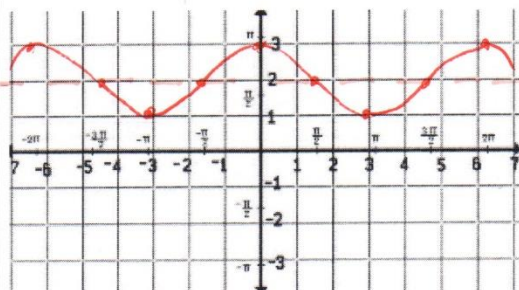
#73-76. Graph the trigonometric functions. *Amp: 2*

73. $f(x) = -2\sin(2x)$ *per: π
p.s. $2x$*



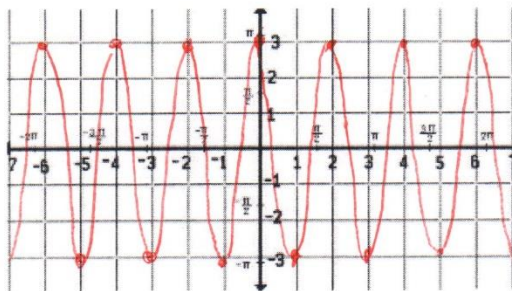
75. $f(x) = \cos(x) + 2$

Amp: 1 per 2π



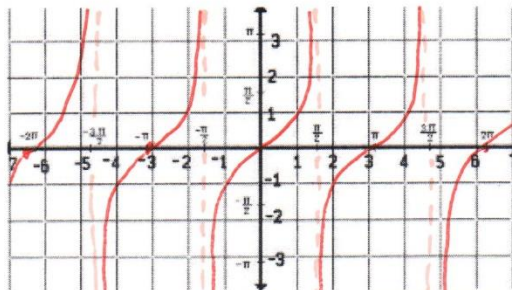
74. $f(x) = 3\cos(\pi x)$

Amp 3 per 2



76. $f(x) = \tan x$

Amp: 1 per: π

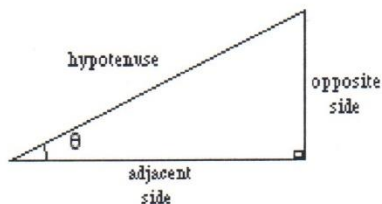


Using Right Triangles to Find Trigonometric Values

SOH $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$

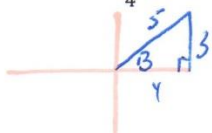
CAH $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

TOA $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$



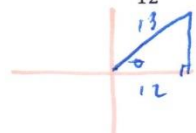
Pythagorean Theorem:
 $a^2 + b^2 = c^2$

77. If $\tan B = \frac{3}{4}$ and $\sin B = \frac{3}{5}$, find $\cos B$.



$\cos B = \frac{4}{5}$

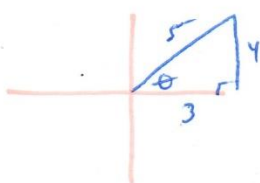
78. If $\tan \theta = \frac{5}{12}$, find $\sin \theta$.



$(5)^2 + (12)^2 = c^2$
 $25 + 144 = c^2$
 $169 = c^2$
 $c = 13$

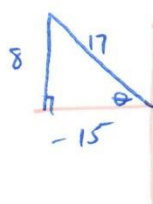
$\sin \theta = \frac{5}{13}$

79. If $\cos \theta = \frac{3}{5}$ and $\sin \theta$ is positive, find $\tan \theta$.



$\tan \theta = \frac{4}{3}$

80. If $\sin \theta = \frac{8}{17}$ and $\cos \theta$ is negative, find $\tan \theta$.



$a^2 + (8)^2 = (17)^2$
 $a^2 + 64 = 289$
 $-64 -64$
 $a^2 = 225$
 $a = 15$

$\tan \theta = -\frac{8}{15}$

Types of Statistical Studies

#81-83. Classify the study as a survey, an observational study, or an experiment.

81. Apple gathers 100 college students and monitors their reactions while completing an assignment on the newest MacBook Air.

Observational Study

82. A store emails a questionnaire to 1,000 randomly chosen online customers.

Survey

83. In-n-out is thinking about adding more relish to their spread. They put the new spread on half of their orders for one day, and then ask the customers their opinion.

Experiment

Interpreting Survey Results.

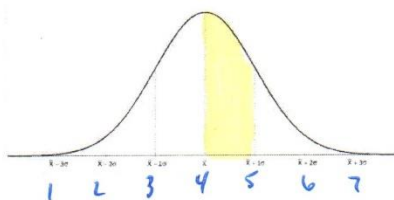
84. When surveying students on their Snapchat Scores, 2 out of 25 students reported not having a Snapchat. What proportion of the sample population did have a Snapchat?

$\frac{23}{25}$ had Snapchat

85. 150 out of 250 students reported spending more than \$300 on prom. What proportion of the population of prom attendees spent more than \$300?

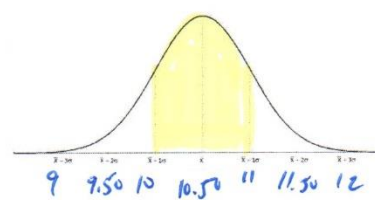
$\frac{150}{250} = \frac{15}{25} = \frac{3}{5}$

Normal Distribution



86. The number of years that people spend on their college education is normally distributed with a mean of 4 and a standard deviation of 1. What percentage of the population attends college between 4 and 5 years?

34%

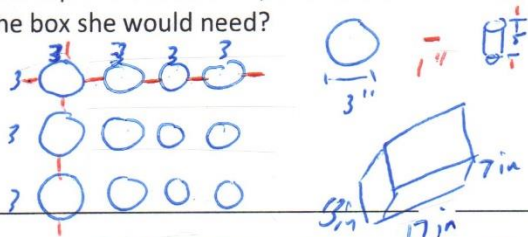


87. Movie prices at a theater are normally distributed with a mean of \$10.50 and a standard deviation of \$0.50. What price range represents the middle 68% of movie prices?

\$10.00 to \$11.00

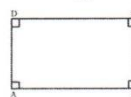
Part 5: Solids

88. Mrs. Pots wants to mail her son Chip some of his favorite soup. She has 12 cans and plans to pack them in a 3x4x1 formation. Each can has a diameter of 3 inches and is 5 inches tall. If she wants to put 1 inch of packing between each can, on all sides, and on the top and the bottom, what are the dimensions of the box she would need?



13 in x 17 in x 7 in

89. What solid would be formed if you rotated Rectangle ABCD around side CD?



cylinder

90. What solid would be formed if you rotated Triangle ABC around side BC?



Cone