FINDING THE DETERMINANT OF A SQUARE MATRIX

Only square matrices have determinants.

When asked to find a determinant, very rarely will words be used. Rather, you will see symbols that tell you what is expected. Much like ||v|| tells you to find the magnitude of the vector v, symbols will be used asking you to find the determinant of the matrix. The problem will look like the following.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \qquad \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

FINDING THE DETERMINANT OF A 2X2

If matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $|A|$ and be found

using the procedure: $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Evaluate the following:

1. $\begin{vmatrix} 3 & 5 \\ -6 & -2 \end{vmatrix}$

2. $\begin{vmatrix} 4 & 7 \\ -2 & 5 \end{vmatrix}$

FINDING THE DETERMINANT OF A 3X3 (using the diagonal method).

$$\begin{vmatrix} -2 & 9 & 4 \\ 7 & -6 & 0 \\ 6 & 7 & -6 \end{vmatrix}$$

Step 1, copy the 1st two columns over on the right side.

$$\begin{vmatrix} -2 & 9 & 4 & -2 & 9 \\ 7 & -6 & 0 & 7 & -6 \\ 6 & 7 & -6 & 6 & 7 \end{vmatrix}$$

Now multiply across the diagonals find the product of threes.

The determinant is: $\frac{\text{Bottom}}{\text{Total}} - \frac{\text{Top}}{\text{Total}}$

FINDING THE DETERMINANT BY EXPANSION OF MINORS AND CO-FACTORS

TO BE USED FOR ANY SQUARE MATRIX 3X3 OR LARGER.

Example:

$$\begin{vmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & 0 & 1 \end{vmatrix}$$

Any row or column may be chosen for the expansion of minors and co-factors

Once you have chosen a column or row to work with, write the matrix 3 more times. If dealing with a 4x4 (4 times), if a 5x5 (5 times) and so forth.

$$\begin{vmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & 0 & 1 \end{vmatrix} \qquad \begin{vmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & 0 & 1 \end{vmatrix} \qquad \begin{vmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & 0 & 1 \end{vmatrix}$$

Circle each minor in the column or row you are working with and delete the cofactors that line up vertically and horizontally with that minor.

Now use the minors as scalar multipliers that will be multiplied to the determinant of the remaining cofactors.

Fill in the appropriate signs and evaluate.

4x4

$$\begin{vmatrix} 3 & 6 & -5 & 4 \\ -2 & 0 & 6 & 0 \\ 1 & 1 & 2 & 2 \\ 0 & 3 & -1 & -1 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 6 & -5 & 4 \\ -2 & 0 & 6 & 0 \\ 1 & 1 & 2 & 2 \\ 0 & 3 & -1 & -1 \end{vmatrix} \quad \begin{vmatrix} 3 & 6 & -5 & 4 \\ -2 & 0 & 6 & 0 \\ 1 & 1 & 2 & 2 \\ 0 & 3 & -1 & -1 \end{vmatrix} \quad \begin{vmatrix} 3 & 6 & -5 & 4 \\ -2 & 0 & 6 & 0 \\ 1 & 1 & 2 & 2 \\ 0 & 3 & -1 & -1 \end{vmatrix} \quad \begin{vmatrix} 3 & 6 & -5 & 4 \\ -2 & 0 & 6 & 0 \\ 1 & 1 & 2 & 2 \\ 0 & 3 & -1 & -1 \end{vmatrix}$$