Logarithmic Functions

$$f(x) = a\log_n(bx+c)+d \qquad f(x) = a\ln(bx+c)+d$$

In order to graph a logarithmic function, it is first necessary to find the domain of the logarithmic function. The range of a logarithmic function is all real numbers, so only the domain needs to be found. To find the domain of a logarithmic function evaluate $bx+c>0$. The inequality here is not $\geq$, because you cannot take the log of a negative number or zero. Once the domain is found, it will tell in which direction the function is moving. If you view the inequality symbol as an arrowhead, it will point in the direction in which the function is moving. This inequality will also help find the vertical asymptote for the function.

If the x inside the log does not have a negative coefficient, the curve will be on the right side of the vertical asymptote. If the coefficient in front of x is 1, begin with the key point of (1,0). From that point on, use the rules of transformations of functions to add or subtract to either the x or y values to find the new key point making the graph shift.

If the x inside the log has a negative coefficient, the curve will be on the left side of the vertical asymptote. If the coefficient in front of x is -1, begin with the key point of (-1,0) and shift from there.

*** The value of "a" affects the scale of the function. If the value of "a" is some number other that 1 or -1, <u>find the key point algebraically before you translate the function.</u>**

As the function shifts, it will be helpful to draw a broken line for both the horizontal and vertical asymptotes. It is OK to cross the horizontal asymptote, as you will find the key point always rests on it. The vertical asymptote, however, may never be crossed.

$$f(x) = a\log_n(bx+c)+d \qquad f(x) = a\ln(bx+c)+d$$

Solving for $bx+c=0$, will yield the equation for the vertical asymptote. The equation for the horizontal asymptote is $y=d$.

$$f(x) = \log_3(x-4)+2$$

Finding the domain.

$x-4>0$

$x>4$

Notice the similarity in the procedures.

Finding the vertical asymptote.

$x-4=0$

$x=4$

Finding the horizontal asymptote.

$y=2$

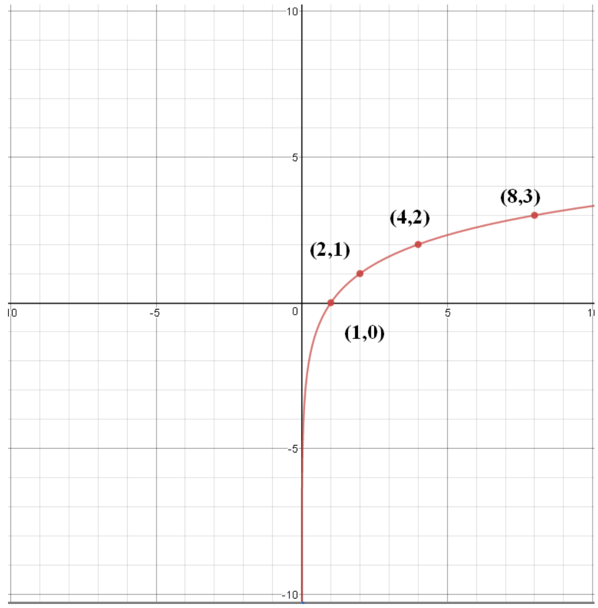
There is no real work involved with finding the horizontal asymptote. Identify the vertical shift. This is the equation of the horizontal asymptote.

***If the variable x inside the log has a coefficient other than 1 or -1, the key point will be different. <u>The key point must then be found algebraically</u>. To find the x value of the key point solve for $bx+c=1$. Substitute that solution back into the problem to find the y value.**

$$f(x) = a\log_n(bx+c)+d \qquad f(x) = a\ln(bx+c)+d$$

$$f(x) = \log_2 x$$

To the right is the graph of $f_{(x)} = \log_2 x$. Students sometimes struggle getting the points that produce an accurate curve. Keep in mind that this log is base 2. To make sure the curve is accurate, move in the following manner to find the appropriate coordinates.



From the origin, move:

- ***Over 1, up 0.***
- ***Over 2, up 1.***
- ***Over 4, up 2.***
- ***Over 8, up 3, and so on.***

Notice, the movement is in powers of 2.
2 to what power is 8? The answer is 3, so after we move over 8, move up 3.

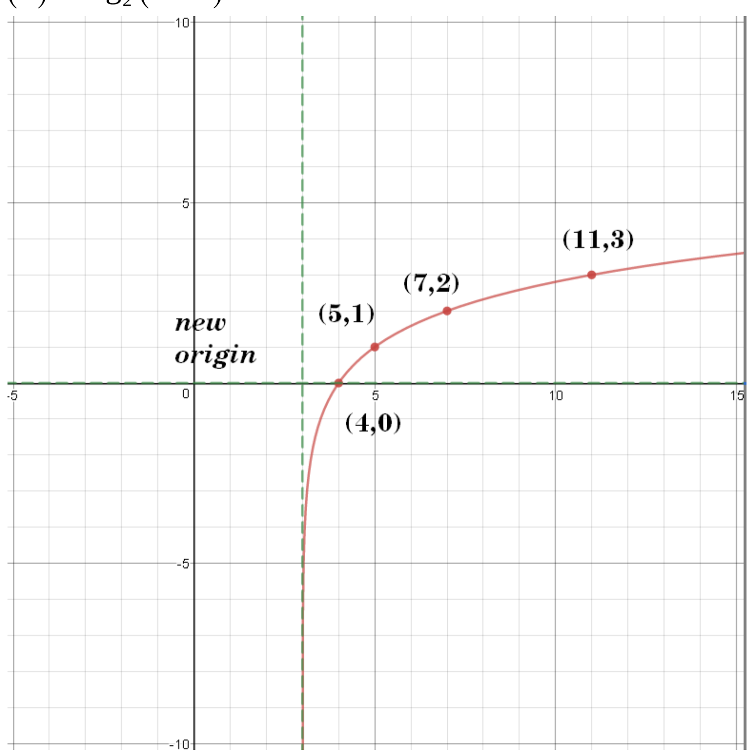
The parent function has the key point at (1, 0)

If there is a transformation involved in the function, draw your vertical asymptote using a broken line. You may also draw a horizontal line to indicate the vertical shift. In the graph below, the vertical asymptote is drawn and the horizontal line of reference is drawn. Notice it creates a "new origin" the graph of the parent function moves in concert with the new origin.

$$f(x) = \log_2(x-3)$$

The graph to the right shifts right 3 spaces. The vertical asymptote is responsible for a shift to the left or right.

If we move in the same manner as before from the "new origin" the graph to the right is the result.

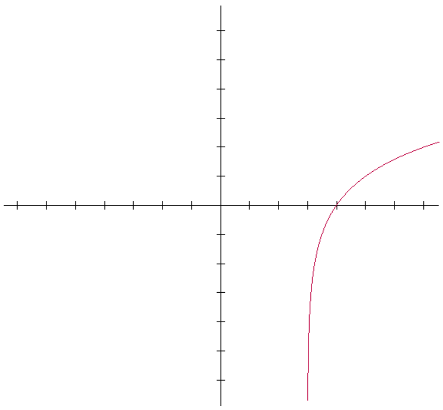


From the new origin, move:

- ***Over 1, up 0.***
- ***Over 2, up 1.***
- ***Over 4, up 2.***
- ***Over 8, up 3.***

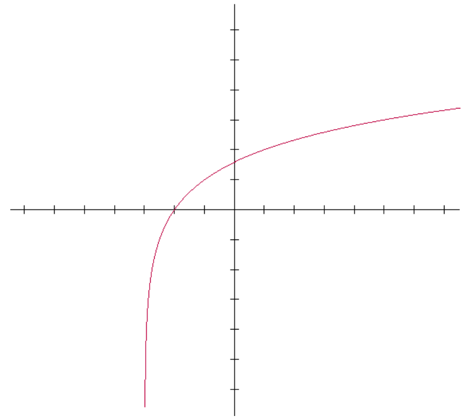
The four examples below demonstrate which values are responsible for the horizontal and vertical transformations of the function. Pay attention to the equation of function and its corresponding graph.

$f(x) = \log_2(x-3)$



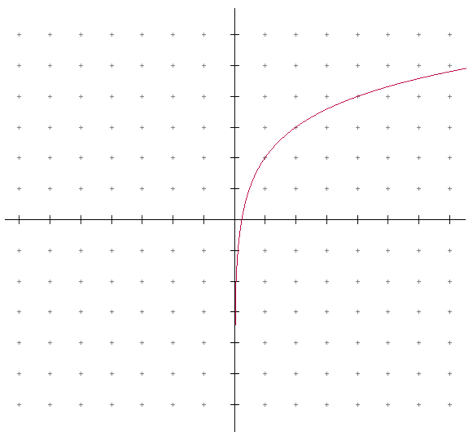
The graph of this function shifts right 3. Notice the key point moved to the right 3 places to (4,0).

$f(x) = \log_2(x+3)$



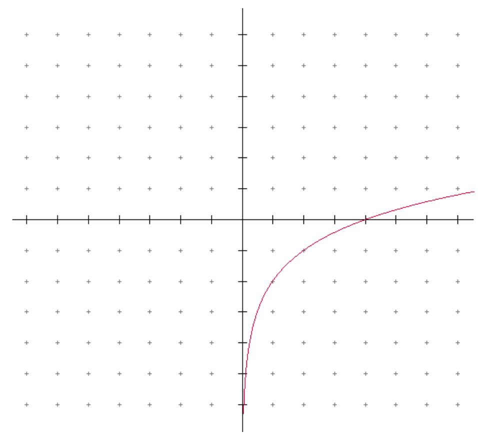
The graph of this function shifts to the left 3. The new key point is (-2,0).

$f(x) = \log_2 x + 2$



This function shifts up 2. Add 2 to the y value of the key point, and it is now at (1,2).

$f(x) = \log_2 x - 2$



This function shifts down 2. Subtracting 2 from the y value of the key point results in (1,-2).

$f(x) = a\log_n(bx+c)+d$ $\quad f(x) = a\ln(bx+c)+d$

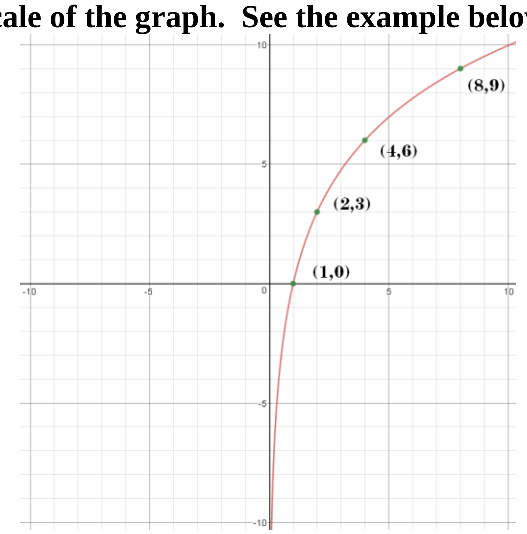
Any coefficient in front of the log function, will affect the scale of the graph. See the example below.

To the right is the graph of $f_{(x)} = 3\log_2 x$. This log is base 2, however the scale is increased by a factor of 3. Therefore, movement will follow the pattern below.

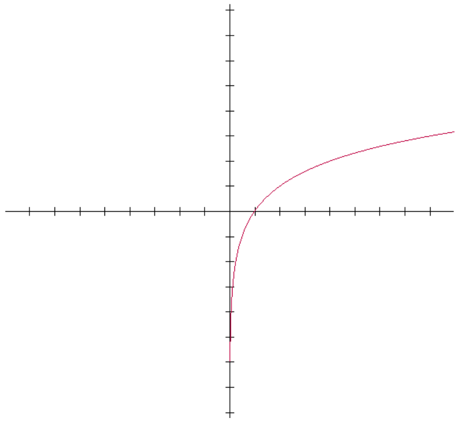
From the origin, move:

- ***Over 1, up 0(3)=0.***
- ***Over 2, up 1(3)=3.***
- ***Over 4, up 2(3)=6.***
- ***Over 8, up 3(3)=9, and so on.***

The initial movement is in powers of 2, however since the leading coefficient is 3, all y values will be multiplied by 3.

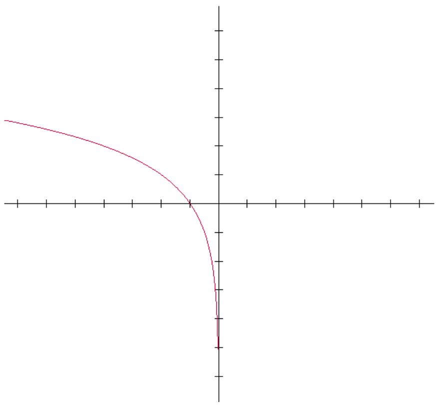


$$f_{(x)} = \log_2 x$$



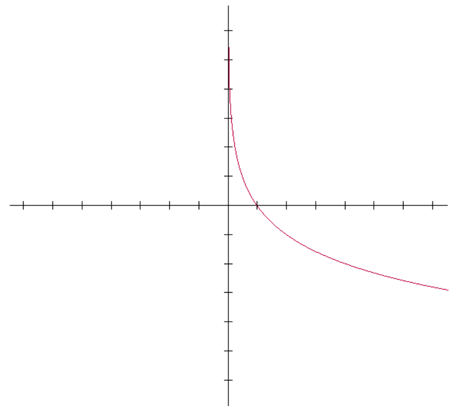
The parent function has the key point at (1, 0)

$$f_{(x)} = \log_2(-x)$$



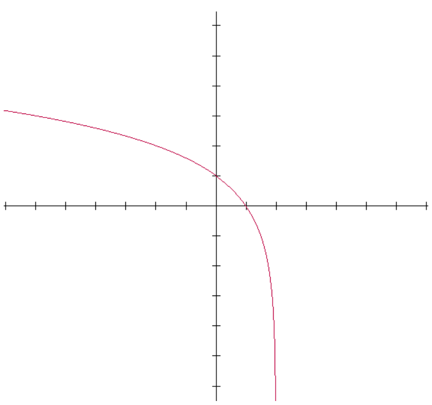
The graph of this function reflects about the vertical asymptote. Key point is now (-1,0).

$$f_{(x)} = -\log_2 x$$



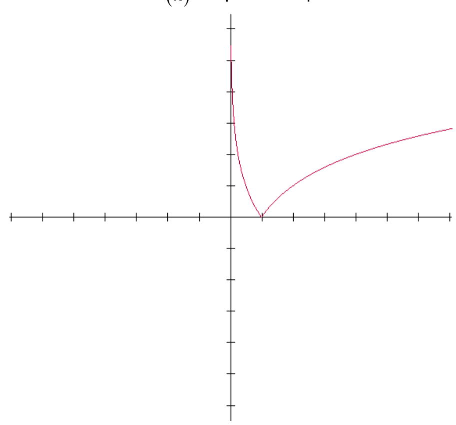
The graph of this function is reflected about the horizontal asymptote. Key point is still at (1,0).

$$f_{(x)} = \log_2(2-x)$$



Since the coefficient of x is -1, this graph will be on the left side of the vertical asymptote. Begin with the key point (-1,0), and shift right 2 because it is positive. Add 2 to the x value of the key point. The new key point is (1,0).

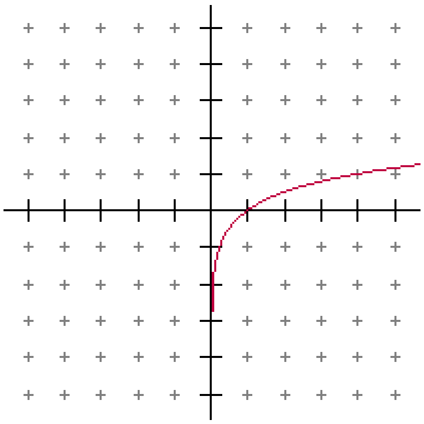
$$f_{(x)} = |\log_2 x|$$



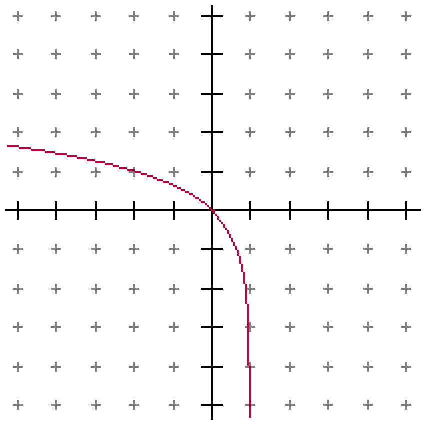
Notice the negative portion of the graph reflected above the x axis.

Match the appropriate graph with its equation below. Explain why each of your solutions is true.

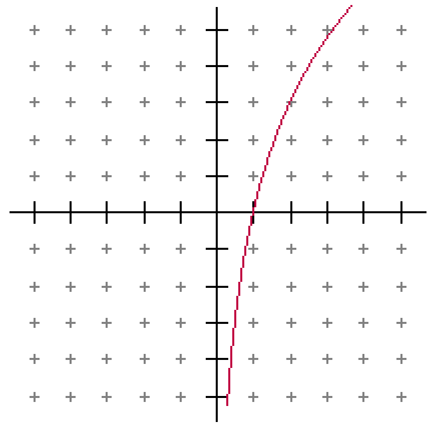
A



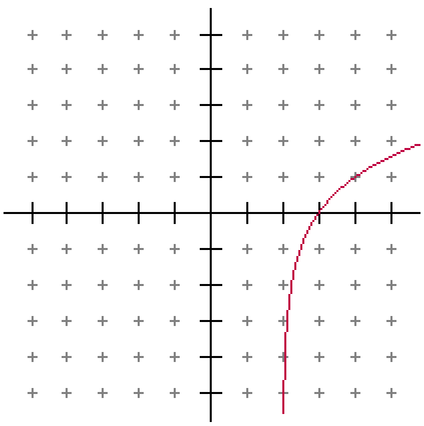
B



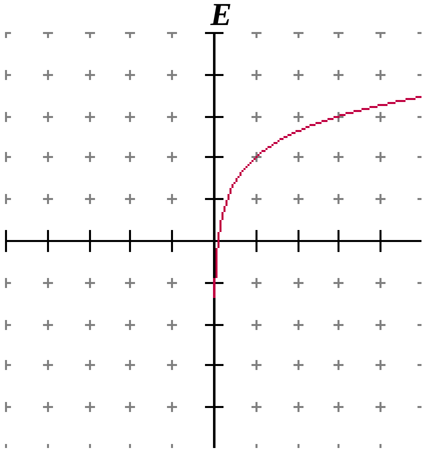
C



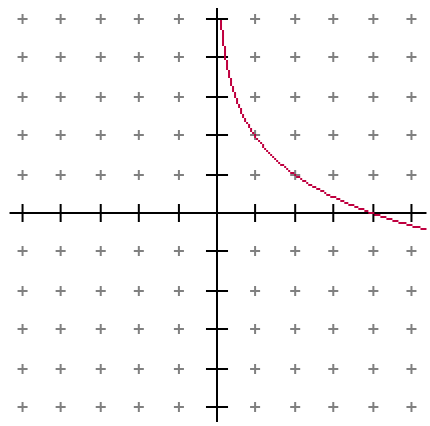
D



E



F



1) $f_{(x)} = \log_2 (x-2)$

2) $f_{(x)} = \log_3 (1-x)$

3) $f_{(x)} = -\log_2 x + 2$

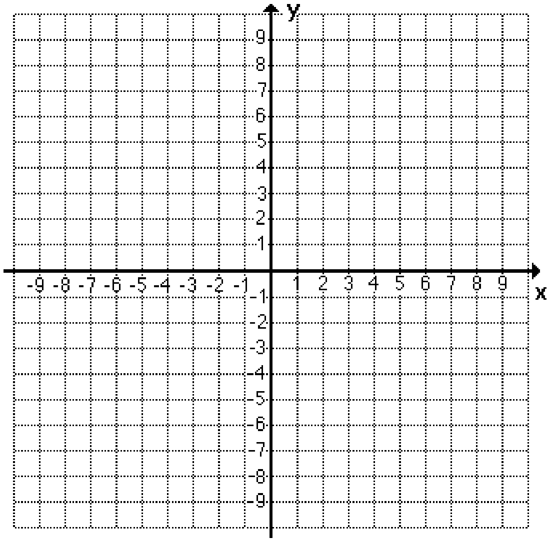
4) $f_{(x)} = \log_3 x + 2$

5) $f_{(x)} = \frac{1}{2}\log_2 x$

6) $f_{(x)} = 3\log_2 x$

Graph the following logarithmic functions finding all indicated values.

A) $f(x) = \log_2(x+2)$



Key point:

Vertical Asymptote:

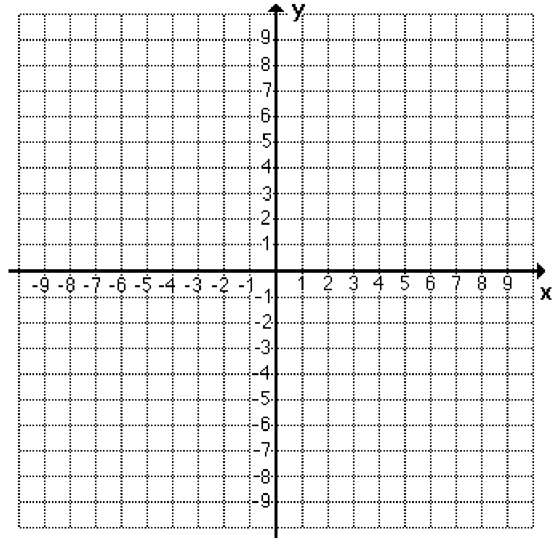
Y-intercept:

Range:

X-intercepts:

Domain:

B) $f(x) = \log_3(x-1)+2$



Key point:

Vertical Asymptote:

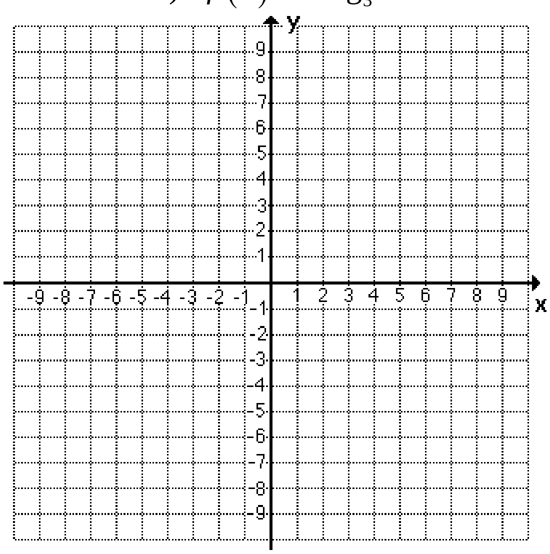
Y-intercept:

Range:

X-intercepts:

Domain:

C) $f(x) = 2\log_3 x$



Key point:

Vertical Asymptote:

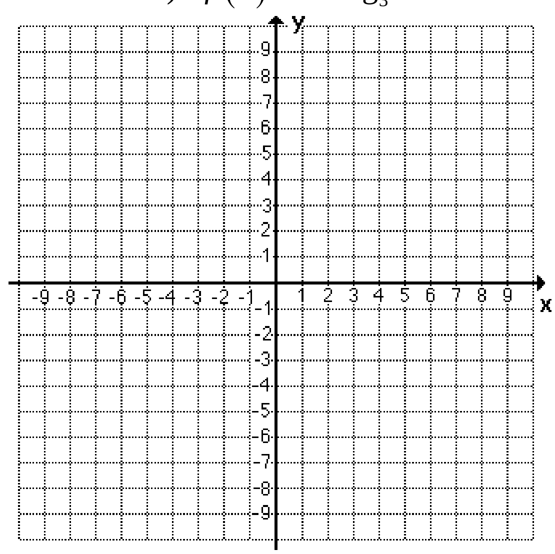
Y-intercept:

Range:

X-intercepts:

Domain:

D) $f(x) = -2\log_3 x$



Key point:

Vertical Asymptote:

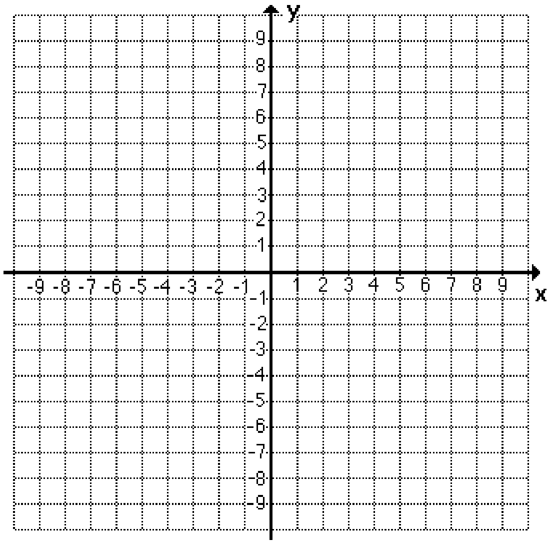
Y-intercept:

Range:

X-intercepts:

Domain:

E) $f(x) = \log_2(x+4)+6$



Key point:

Vertical Asymptote:

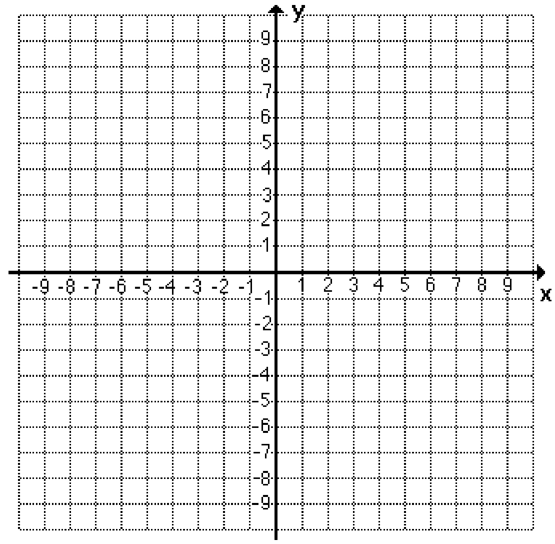
Y-intercept:

Range:

X-intercepts:

Domain:

F) $f(x) = 2\log_4(x+7)-5$



Key point:

Vertical Asymptote:

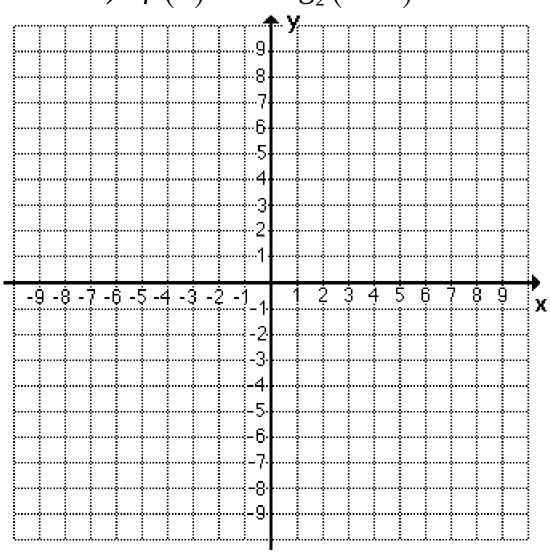
Y-intercept:

Range:

X-intercepts:

Domain:

G) $f(x) = -3\log_2(x-1)-2$



Key point:

Vertical Asymptote:

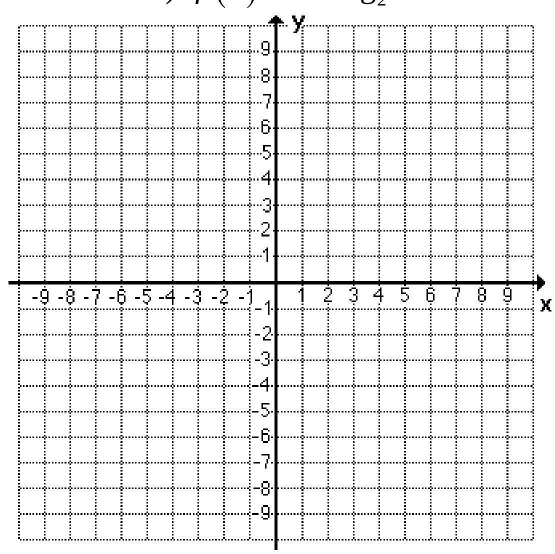
Y-intercept:

Range:

X-intercepts:

Domain:

H) $f(x) = 4-\log_2 x$



Key point:

Vertical Asymptote:

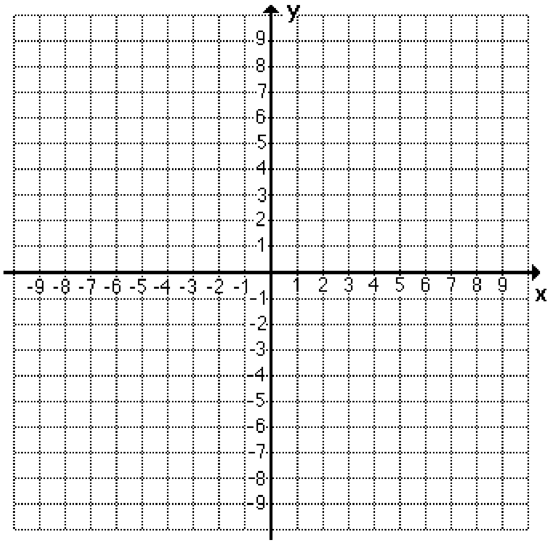
Y-intercept:

Range:

X-intercepts:

Domain:

I) $f(x) = \ln x$



Key point:

Vertical Asymptote:

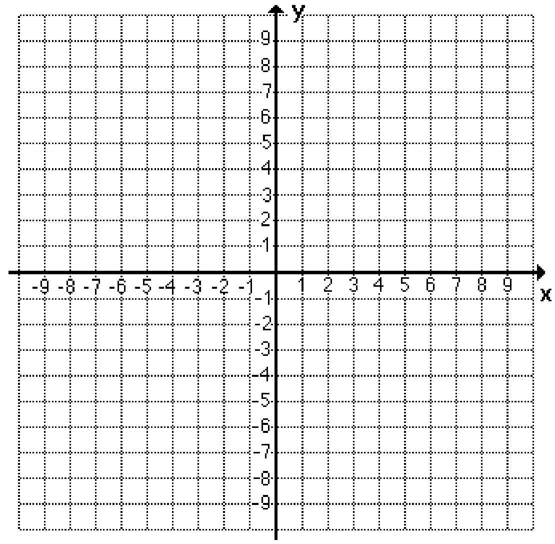
Y-intercept:

Range:

X-intercepts:

Domain:

J) $f(x) = -2\ln x$



Key point:

Vertical Asymptote:

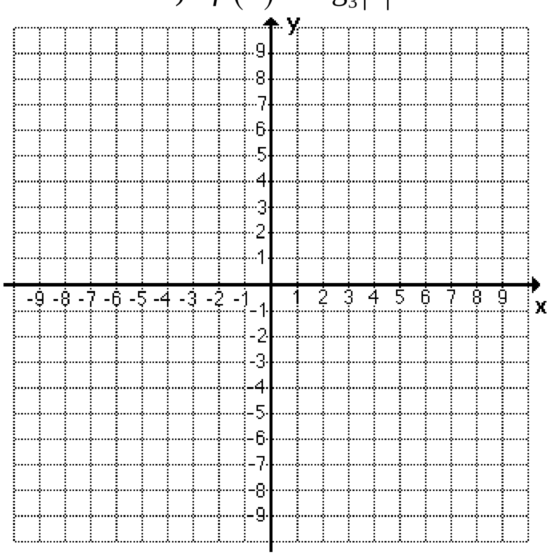
Y-intercept:

Range:

X-intercepts:

Domain:

K) $f(x) = \log_3 |x|$



Key point:

Vertical Asymptote:

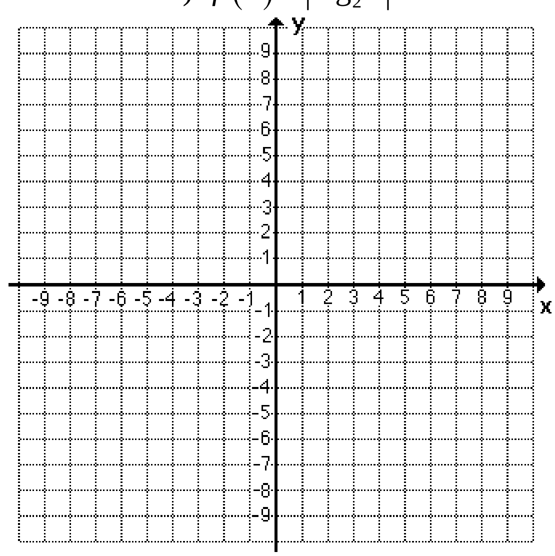
Y-intercept:

Range:

X-intercepts:

Domain:

L) $f(x) = |\log_2 x|$



Key point:

Vertical Asymptote:

Y-intercept:

Range:

X-intercepts:

Domain: