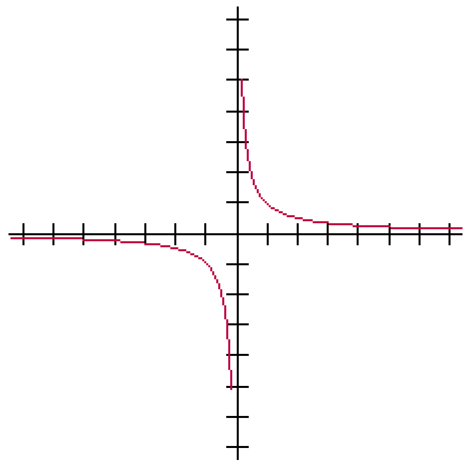
Graphing Rational Functions

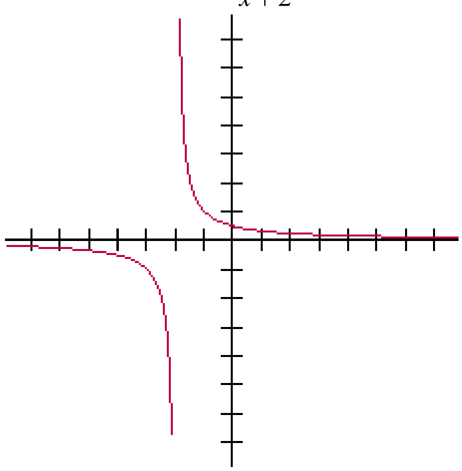
We really have no standard form of a rational function to look at, so we will concentrate on the parent function of $f_{(x)} = \frac{1}{x}$. The following pages illustrate the effects of the denominator, as well as the behavior of $-f_{(x)}$. A graphing calculator may be used to help get the overall shape of these functions. <u>DO NOT</u>, however, just copy the picture the calculator gives you.

$$f_{(x)} = \frac{1}{x}$$



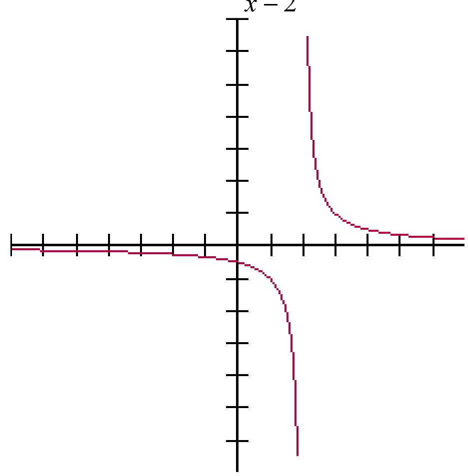
Here, the vertical asymptote is at x=0, and the horizontal asymptote is y=0.

$$f_{(x)} = \frac{1}{x+2}$$



The graph of this function shifts left 2.

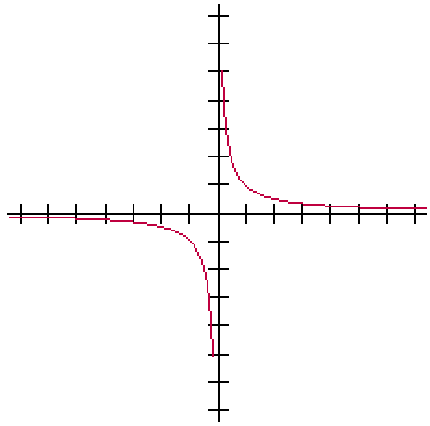
$$f_{(x)} = \frac{1}{x-2}$$



The graph of this function shifts right 2.

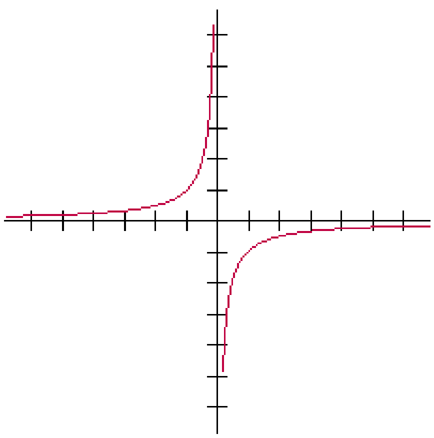
The range for each of these functions is $(-\infty, 0) \cup (0, \infty)$. There is no way to tell what the range of a rational function will be until it is graphedt. Remember, the curve may cross the horizontal axis.

$$f_{(x)} = \frac{1}{x}$$



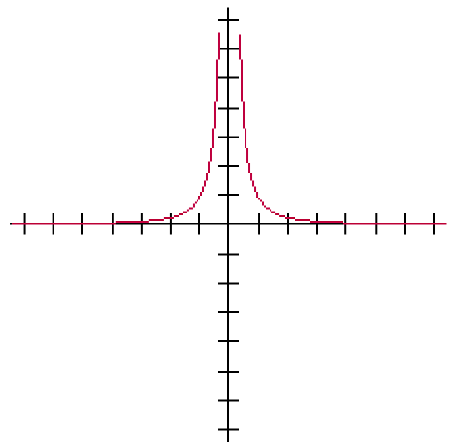
Here, the vertical asymptote is at x=0, and the horizontal asymptote is y=0.

$$f_{(x)} = -\frac{1}{x}$$



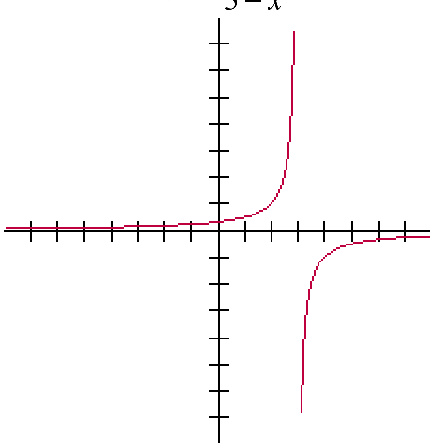
The graph of this function is a reflection of the parent function.

$$f_{(x)} = \frac{1}{x^2}$$



Notice how x^2 ***affects the function. Normally, one side of the function would go up, and the other would go down. Since there is no way to get a negative number in the denominator, both sides are going in the same direction.***

$$f_{(x)} = \frac{1}{3-x}$$



The graph of the function to the left flips upside down, similar to $f_{(x)} = -\frac{1}{x}$***, and shifts right 3. What happens here is a -1 is factored out of the denominator, changing the function to the following.***

$$f_{(x)} = \frac{1}{3-x} \Rightarrow \frac{1}{-(x-3)} \Rightarrow -\frac{1}{x-3}$$

As a result, this graph is a combination of shifting the graph and reflecting it about the horizontal asymptote.

Example

Graph the function $f_{(x)} = \dfrac{x-3}{x^2-x-12}$. Be sure to find all asymptotes, x and y intercepts, and the range and domain.

$$f_{(x)} = \frac{x-3}{(x+3)(x-4)}$$

The first step is to completely factor the rational function.

Vertical Asymptotes are $x=-3$ and $x=4$

The zeros of the denominator are the vertical asymptotes of the function.

Looking at the original function, the horizontal asymptote is $y=0$.

If the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y=0$.

The x intercept is $(3,0)$

The zero of the numerator is the x intercept of the function.

The y intercept is $\left(0,\frac{1}{4}\right)$.

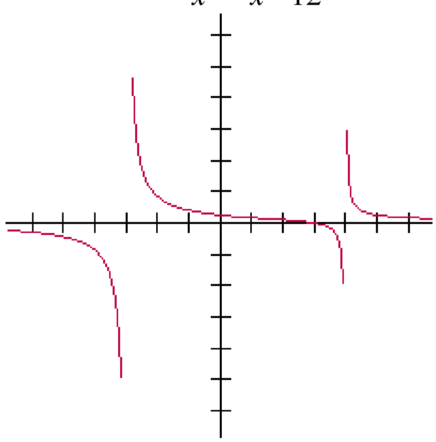
Substituting zero for x and evaluating the ratio of the two constants, -3 and -12. Yields a y intercept of $\frac{1}{4}$.

The domain of the function is $(-\infty,-3)\cup(-3,4)\cup(4,\infty)$

The domain is found using the vertical asymptotes. The domain here is all real numbers except -3 and 4.

The range should not be found without first graphing the function.

$$f_{(x)} = \frac{x-3}{x^2-x-12}$$



Due to limitations in the graphing software, the graph to the left is incomplete. Each of these lines is continuous. Before attempting to graph the functions, graph <u>all</u> asymptotes using broken lines. This will ensure that a vertical asymptote is not crossed. To get the general shape of the equation, use a combination of the x and y intercepts that were found, and plug in values for x close to the vertical asymptote. Based on the properties of all rational functions, it should be obvious how these curves behave on the outer intervals of these functions. They will always ride along the asymptotes in these areas.

V.A.: $x=-3$, $x=4$

H.A.: $y=0$

x-int: $(3,0)$

y-int: $\left(0,\frac{1}{4}\right)$

Range: $(-\infty,\infty)$

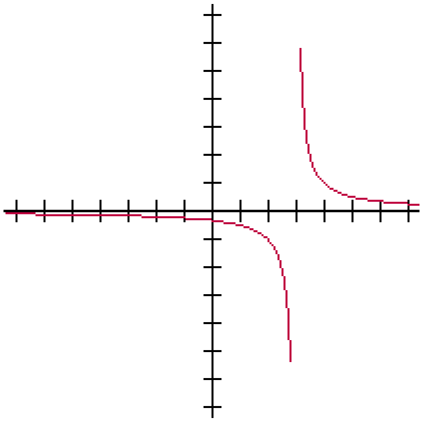
Domain: $(-\infty,-3)\cup(-3,4)\cup(4,\infty)$

Here is a list of all required information needed for each rational function. Since the graph of the function crossed the horizontal asymptote in the interval $(-3,4)$, the range of this function is all real numbers.

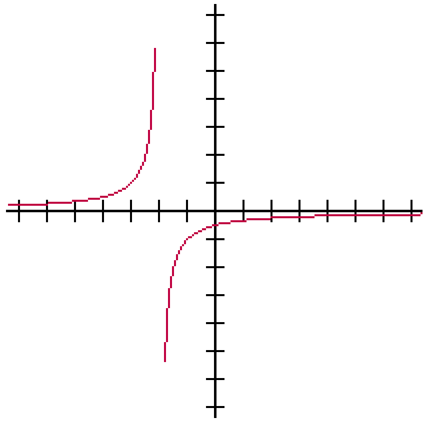
<u>These procedures must be used when graphing any rational function.</u>

Match the appropriate graph with its equation below. Explain why each of the solutions is true.

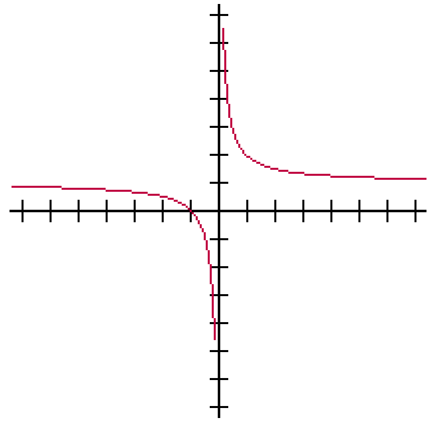
A



B



C



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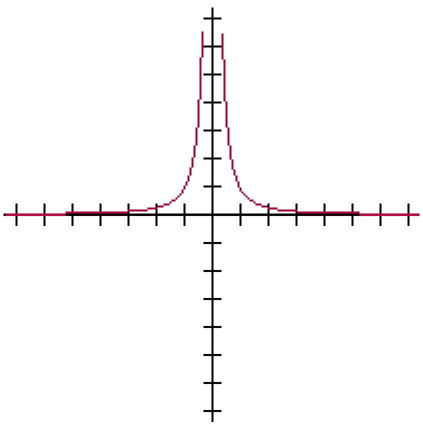
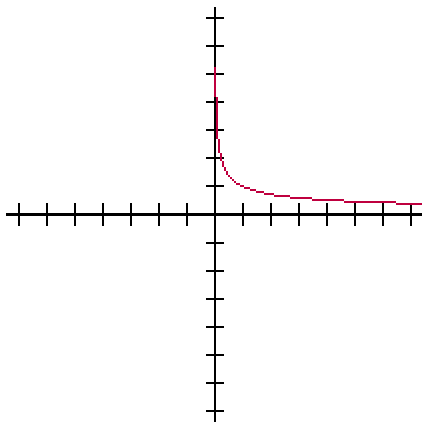
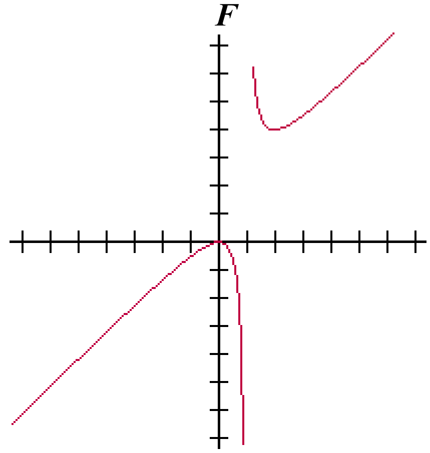


E



F



1) $f_{(x)} = \frac{1}{x^2}$

2) $f_{(x)} = \frac{1}{x-3}$

3) $f_{(x)} = \frac{1}{\sqrt{x}}$

4) $f_{(x)} = -\frac{1}{x+2}$

5) $f_{(x)} = \frac{x^2}{x-1}$

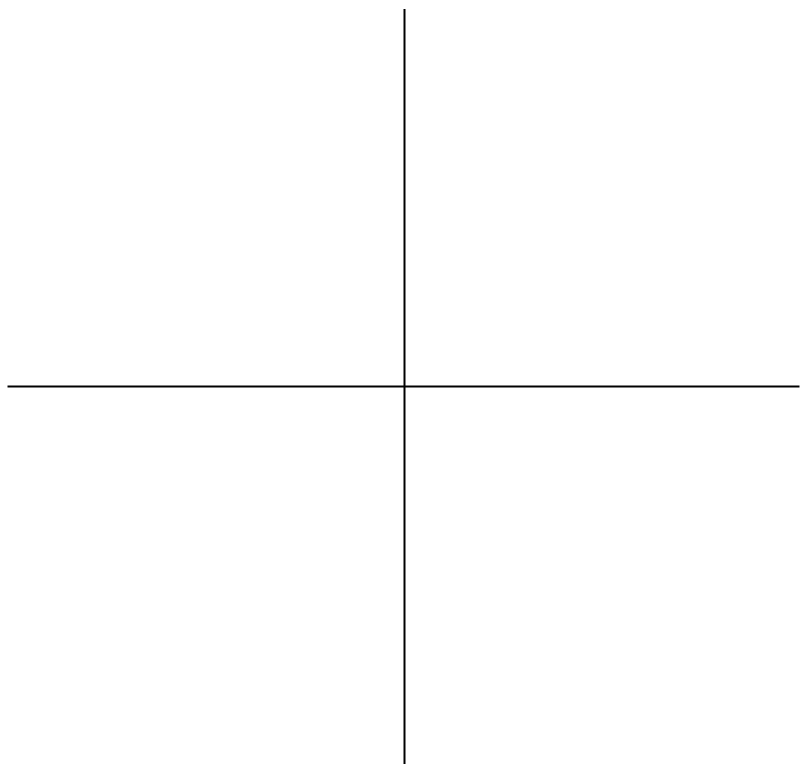
6) $f_{(x)} = \frac{1}{x}+1$

A graphing calculator may be used to help get a picture of the curve that will be created, but simply copying the picture shown in the calculator is unwise.

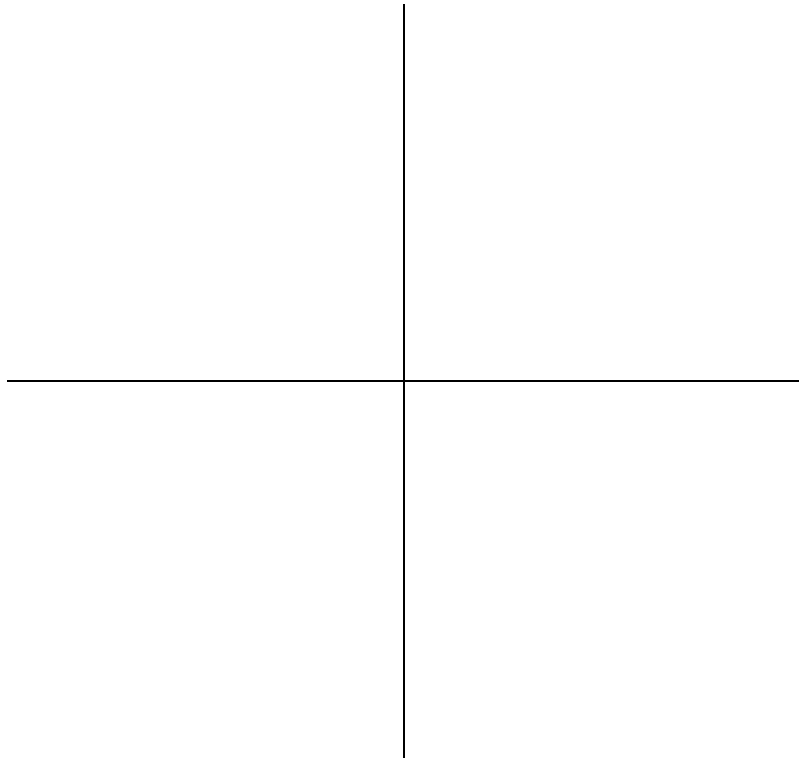
What is the problem with the picture of rational functions in graphing calculators?

Sketch the graph of each of the following functions. Be sure to find all asymptotes, x and y intercepts, and the range and domain of each of the following.

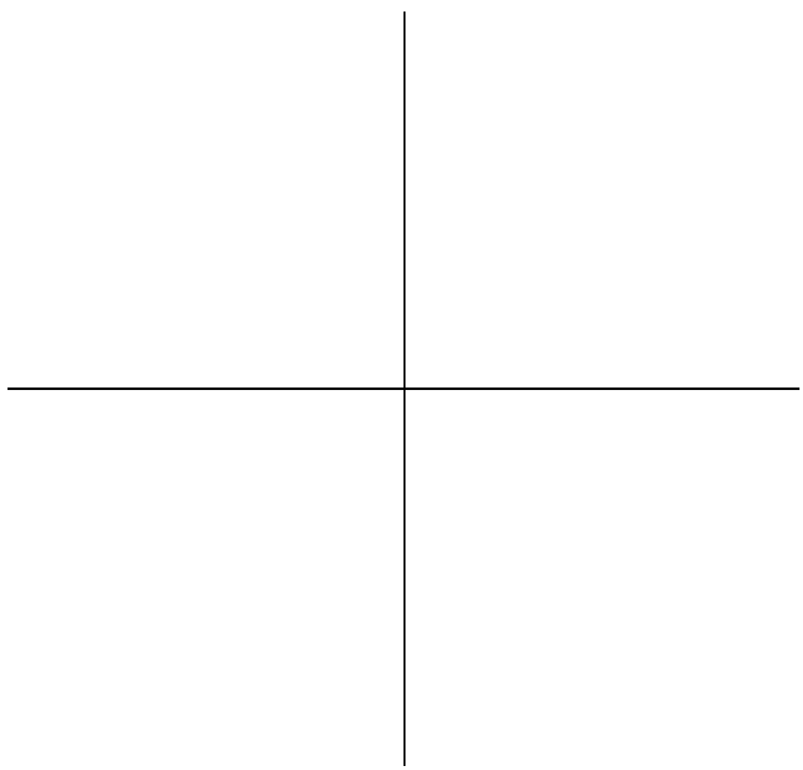
A) $f_{(x)} = \dfrac{1}{x-4}$



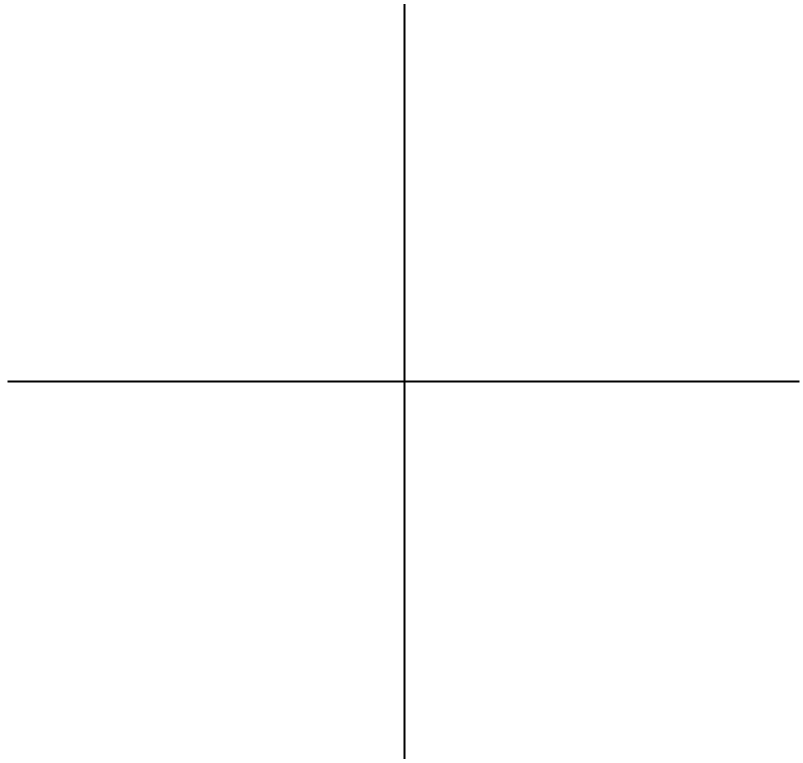
B) $f_{(x)} = -\dfrac{1}{x+2}$



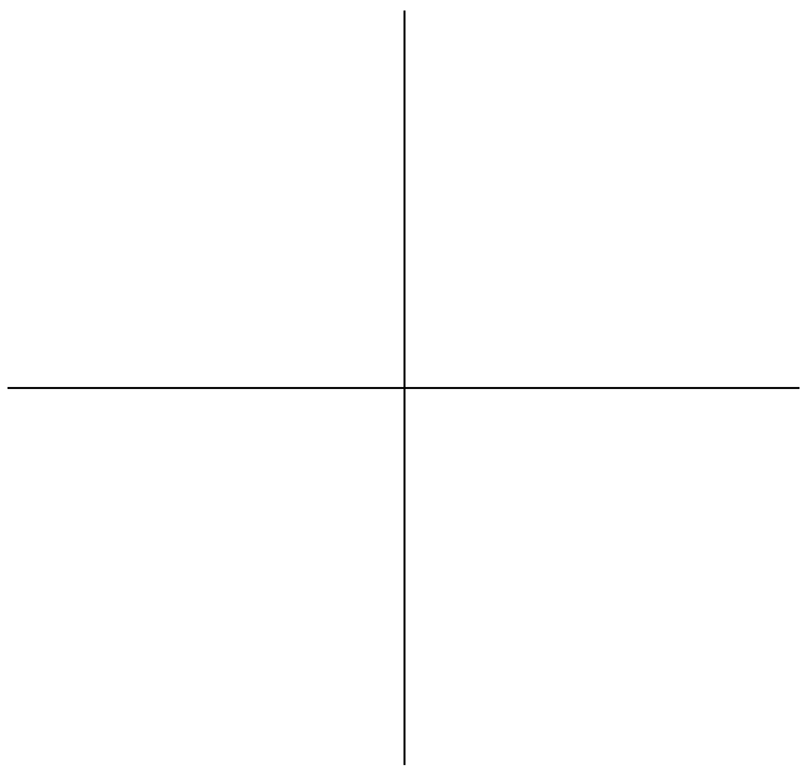
C) $f_{(x)} = \dfrac{x-3}{x+2}$



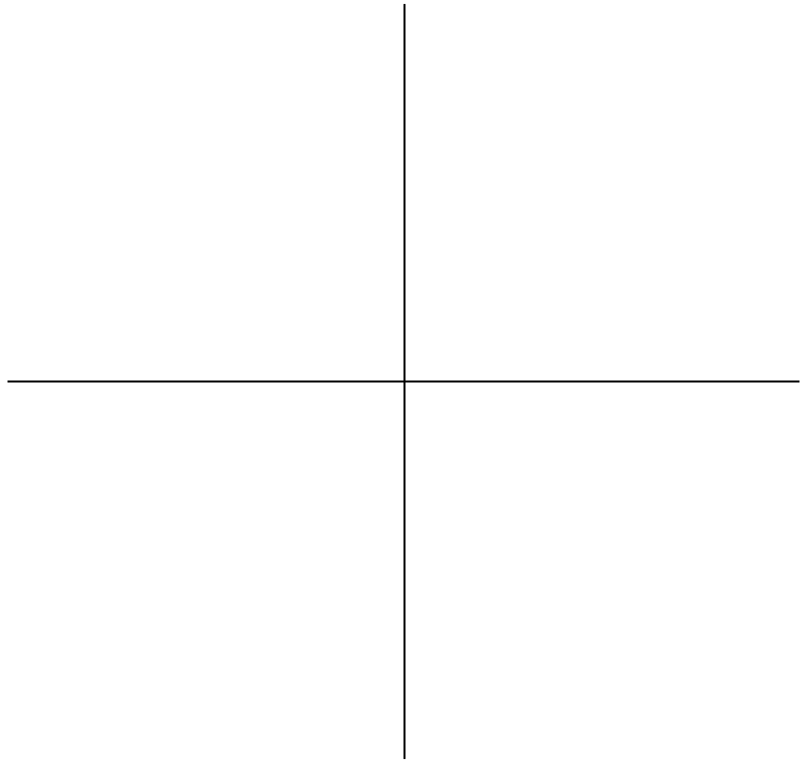
D) $f_{(x)} = \dfrac{x+2}{3x-9}$



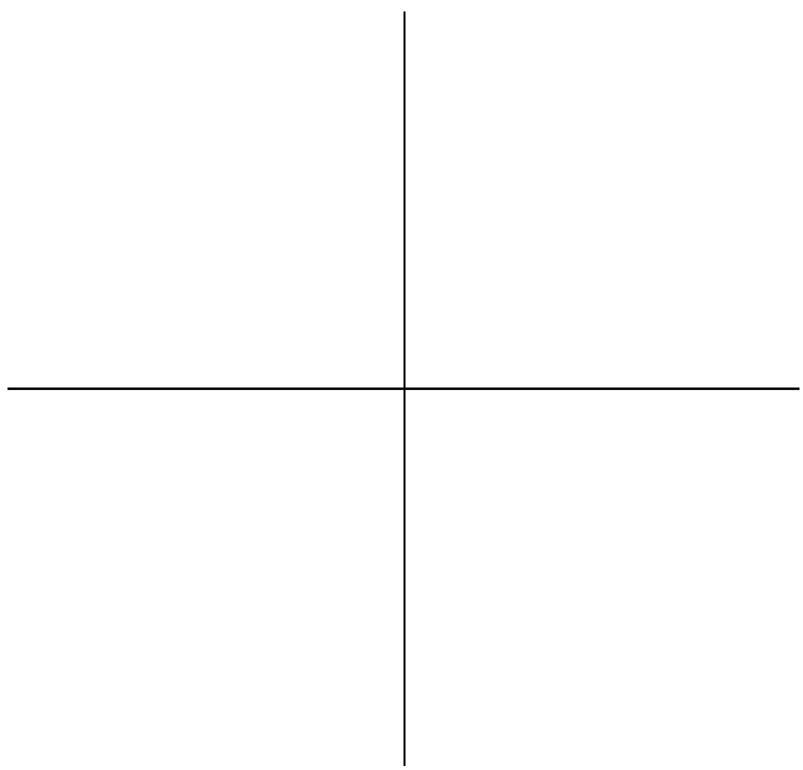
E) $f_{(x)} = \dfrac{2x^2+1}{x}$



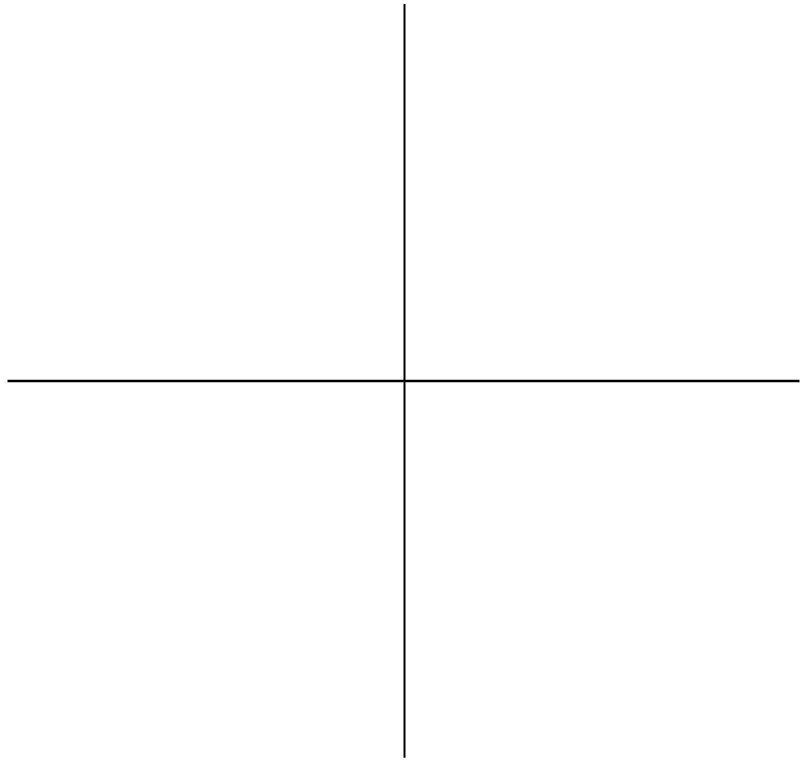
F) $f_{(x)} = \dfrac{x+2}{x^2-9}$



G) $f_{(x)} = \dfrac{x+2}{x-1}$



H) $f_{(x)} = \dfrac{2x^2}{x^2 - 4}$



I) $f_{(x)} = -\dfrac{x^3}{x^2 - 9}$