

Integrated Math 3
Chapter 6 Section 2 Study Guide and Intervention
Solving Logarithmic Equations and Inequalities

Solving Logarithmic Equations

Property of Equality for Logarithmic Functions	If b is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$.
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Example 1: Solve $\log_2 2x = 3$.

$\log_2 2x = 3$	Original equation
$2x = 2^3$	Definition of logarithm
$2x = 8$	Simplify.
$x = 4$	Simplify.

The solution is $x = 4$.

Example 2: Solve the equation

$$\log_2 (x + 17) = \log_2 (3x + 23).$$

Since the bases of the logarithms are equal, $(x + 17)$ must equal $(3x + 23)$.

$$\begin{aligned} (x + 17) &= (3x + 23) \\ -6 &= 2x \\ x &= -3 \end{aligned}$$

Exercises

Solve each equation.

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|---|---|
| 1. $\log_2 32 = 3x$ | 2. $\log_3 2c = -2$ |
| 3. $\log_{2x} 16 = -2$ | 4. $\log_{25} \left(\frac{x}{2}\right) = \frac{1}{2}$ |
| 5. $\log_4 (5x + 1) = 2$ | 6. $\log_8 (x - 5) = \frac{2}{3}$ |
| 7. $\log_4 (3x - 1) = \log_4 (2x + 3)$ | 8. $\log_2 (x^2 - 6) = \log_2 (2x + 2)$ |
| 9. $\log_x + \log_4 27 = 3$ | 10. $\log_2 (x + 3) = 4$ |
| 11. $\log_x 1000 = 3$ | 12. $\log_8 (4x + 4) = 2$ |
| 13. $\log_2 x = \log_2 12$ | 14. $\log_3 (x - 5) = \log_3 13$ |
| 15. $\log_{10} x = \log_{10} (5x - 20)$ | 16. $\log_5 x = \log_5 (2x - 1)$ |
| 17. $\log_4 (x + 12) = \log_4 4x$ | 18. $\log_6 (x - 3) = \log_6 2x$ |

Integrated Math 3**Chapter 6 Section 2 Study Guide and Intervention** *(continued)***Solving Logarithmic Equations and Inequalities****Solving Logarithmic Inequalities**

Property of Inequality for Logarithmic Functions	If $b > 1$, $x > 0$, and $\log_b x > y$, then $x > b^y$. If $b > 1$, $x > 0$, and $\log_b x < y$, then $0 < x < b^y$. If $b > 1$, then $\log_b x > \log_b y$ if and only if $x > y$, and $\log_b x < \log_b y$ if and only if $x < y$.
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Example 1: Solve $\log_5 (4x - 3) < 3$.

$$\log_5 (4x - 3) < 3 \quad \text{Original equation}$$

$$0 < 4x - 3 < 5^3 \quad \text{Property of Inequality}$$

$$3 < 4x < 125 + 3 \quad \text{Simplify.}$$

$$\frac{3}{4} < x < 32 \quad \text{Simplify.}$$

The solution set is $\left\{x \mid \frac{3}{4} < x < 32\right\}$.

Example 2: Solve the inequality

$$\log_3 (3x - 4) < \log_3 (x + 1).$$

Since the base of the logarithms are equal to or greater than 1, $3x - 4 < x + 1$.

$$2x < 5$$

$$x < \frac{5}{2}$$

Since $3x - 4$ and $x + 1$ must both be positive numbers, solve $3x - 4 = 0$ for the lower bound of the inequality.

The solution is $\left\{x \mid \frac{4}{3} < x < \frac{5}{2}\right\}$.

Exercises

Solve each inequality.

1. $\log_2 2x > 2$

2. $\log_5 x > 2$

3. $\log_2 (3x + 1) < 4$

4. $\log_4 2x > -\frac{1}{2}$

5. $\log_3 (x + 3) < 3$

6. $\log_{27} 6x > \frac{2}{3}$

7. $\log_{10} 5x < \log_{10} 30$

8. $\log_{10} x < \log_{10} (2x - 4)$

9. $\log_{10} 3x < \log_{10} (7x - 8)$

10. $\log_2 (8x + 5) > \log_2 (9x - 18)$

11. $\log_{10} (3x + 7) < \log_{10} (7x - 3)$

12. $\log_2 (3x - 4) < \log_2 (2x + 7)$