

Integrated Math 3

Chapter 6 Section 3 Study Guide and Intervention

Properties of Logarithms

Properties of Logarithms Properties of exponents can be used to develop the following properties of logarithms.

Product Property of Logarithms	For all positive numbers a , b , and x , where $x \neq 1$, $\log_x ab = \log_x a + \log_x b$.
Quotient Property of Logarithms	For all positive numbers a , b , and x , where $x \neq 1$, $\log_x \frac{a}{b} = \log_x a - \log_x b$.
Power Property of Logarithms	For any real number p and positive numbers m and b , where $b \neq 1$, $\log_b m^p = p \log_b m$.

Example: Use $\log_3 28 \approx 3.0331$ and $\log_3 4 \approx 1.2619$ to approximate the value of each expression.

a. $\log_3 36$

$$\begin{aligned}\log_3 36 &= \log_3 (3^2 \cdot 4) \\ &= \log_3 3^2 + \log_3 4 \\ &= 2 + \log_3 4 \\ &\approx 2 + 1.2619 \\ &\approx 3.2619\end{aligned}$$

b. $\log_3 7$

$$\begin{aligned}\log_3 7 &= \log_3 \left(\frac{28}{4}\right) \\ &= \log_3 28 - \log_3 4 \\ &\approx 3.0331 - 1.2619 \\ &\approx 1.7712\end{aligned}$$

c. $\log_3 256$

$$\begin{aligned}\log_3 256 &= \log_3 (4^4) \\ &= 4 \cdot \log_3 4 \\ &\approx 4(1.2619) \\ &\approx 5.0476\end{aligned}$$

Exercises

Use $\log_{12} 3 \approx 0.4421$ and $\log_{12} 7 \approx 0.7831$ to approximate the value of each expression.

1. $\log_{12} 21$

2. $\log_{12} \frac{7}{3}$

3. $\log_{12} 49$

4. $\log_{12} 36$

5. $\log_{12} 63$

6. $\log_{12} \frac{27}{49}$

7. $\log_{12} \frac{81}{49}$

8. $\log_{12} 16,807$

9. $\log_{12} 441$

Use $\log_5 3 \approx 0.6826$ and $\log_5 4 \approx 0.8614$ to approximate the value of each expression.

10. $\log_5 12$

11. $\log_5 100$

12. $\log_5 0.75$

13. $\log_5 144$

14. $\log_5 \frac{27}{16}$

15. $\log_5 375$

16. $\log_5 1.\bar{3}$

17. $\log_5 \frac{9}{16}$

18. $\log_5 \frac{81}{5}$

Integrated Math 3**Chapter 6 Section 3 Study Guide and Intervention** *(continued)***Properties of Logarithms**

Solve Logarithmic Equations You can use the properties of logarithms to solve equations involving logarithms.

Example: Solve each equation.

a. $2 \log_3 x - \log_3 4 = \log_3 25$

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Original equation

$$\log_3 x^2 - \log_3 4 = \log_3 25$$

Power Property

$$\log_3 \frac{x^2}{4} = \log_3 25$$

Quotient Property

$$\frac{x^2}{4} = 25$$

Property of Equality for Logarithmic Functions

$$x^2 = 100$$

Multiply each side by 4.

$$x = \pm 10$$

Take the square root of each side.

Since logarithms are undefined for $x < 0$, -10 is an extraneous solution. The only solution is 10.

b. $\log_2 x + \log_2 (x + 2) = 3$

$$\log_2 x + \log_2 (x + 2) = 3$$

Original equation

$$\log_2 x(x + 2) = 3$$

Product Property

$$x(x + 2) = 2^3$$

Definition of logarithm

$$x^2 + 2x = 8$$

Distributive Property

$$x^2 + 2x - 8 = 0$$

Subtract 8 from each side.

$$(x + 4)(x - 2) = 0$$

Factor.

$$x = 2 \text{ or } x = -4$$

Zero Product Property

Since logarithms are undefined for $x < 0$, -4 is an extraneous solution. The only solution is 2.

Exercises

Solve each equation. Check your solutions.

1. $\log_5 4 + \log_5 2x = \log_5 24$

2. $3 \log_4 6 - \log_4 8 = \log_4 x$

3. $\frac{1}{2} \log_6 25 + \log_6 x = \log_6 20$

4. $\log_2 4 - \log_2 (x + 3) = \log_2 8$

5. $\log_6 2x - \log_6 3 = \log_6 (x - 1)$

6. $2 \log_4 (x + 1) = \log_4 (11 - x)$

7. $\log_2 x - 3 \log_2 5 = 2 \log_2 10$

8. $3 \log_2 x - 2 \log_2 5x = 2$

9. $\log_3 (c + 3) - \log_3 (4c - 1) = \log_3 5$

10. $\log_5 (x + 3) - \log_5 (2x - 1) = 2$