

Integrated Math 3**Chapter 6 Section 4 Study Guide and Intervention****Base e and Natural Logarithms**

Base e and Natural Logarithms The irrational number $e \approx 2.71828\dots$ often occurs as the base for exponential and logarithmic functions that describe real-world phenomena.

Natural Base e	As n increases, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx 2.71828\dots$ $\ln x = \log_e x$
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The functions $f(x) = e^x$ and $f(x) = \ln x$ are inverse functions.

Inverse Property of Base e and Natural Logarithms	$e^{\ln x} = x$	$\ln e^x = x$
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Natural base expressions can be evaluated using the e^x and \ln keys on your calculator.

Example 1: Write a logarithmic equation equivalent to $e^{2x} = 7$.

$$e^{2x} = 7 \rightarrow \log_e 7 = 2x$$

$$2x = \ln 7$$

Example 2: Write each logarithmic equation in exponential form.

a. $\ln x \approx 0.3345$

$$\ln x \approx 0.3345 \rightarrow \log_e x \approx 0.3345$$

$$x \approx e^{0.3345}$$

b. $\ln 42 = x$

$$\ln 42 = x \rightarrow \log_e 42 = x$$

$$42 = e^x$$

Exercises

Write an equivalent exponential or logarithmic equation.

1. $e^{15} = x$

2. $e^{3x} = 45$

3. $\ln 20 = x$

4. $\ln x = 8$

5. $e^{-5x} = 0.2$

6. $\ln(4x) = 9.6$

7. $e^{8.2} = 10x$

8. $\ln 0.0002 = x$

Evaluate each logarithm to the nearest ten-thousandth.

9. $\ln 12,492$

10. $\ln 50.69$

11. $\ln 9275$

12. $\ln 0.835$

13. $\ln 943 - \ln 181$

14. $\ln 67 + \ln 103$

15. $\ln 931 \cdot \ln 32$

16. $\ln(139 - 45)$

Integrated Math 3**Chapter 6 Section 4 Study Guide and Intervention** *(continued)***Base e and Natural Logarithms**

Equations and Inequalities with e and \ln All properties of logarithms from earlier lessons can be used to solve equations and inequalities with natural logarithms.

Example: Solve each equation or inequality.

a. $3e^{2x} + 2 = 10$

$$3e^{2x} + 2 = 10$$

Original equation

$$3e^{2x} = 8$$

Subtract 2 from each side.

$$e^{2x} = \frac{8}{3}$$

Divide each side by 3.

$$\ln e^{2x} = \ln \frac{8}{3}$$

Property of Equality for Logarithms

$$2x = \ln \frac{8}{3}$$

Inverse Property of Exponents and Logarithms

$$x = \frac{1}{2} \ln \frac{8}{3}$$

Multiply each side by $\frac{1}{2}$

$$x \approx 0.4904$$

Use a calculator.

b. $\ln(4x - 1) < 2$

$$\ln(4x - 1) < 2$$

Original inequality

$$e^{\ln(4x - 1)} < e^2$$

Write each side using exponents and base e .

$$0 < 4x - 1 < e^2$$

Inverse Property of Exponents and Logarithms

$$1 < 4x < e^2 + 1$$

Addition Property of Inequalities

$$\frac{1}{4} < x < \frac{1}{4}(e^2 + 1)$$

Multiplication Property of Inequalities

$$0.25 < x < 2.0973$$

Use a calculator.

Exercises

Solve each equation or inequality. Round to the nearest ten-thousandth.

1. $e^{4x} = 120$

2. $e^x \leq 25$

3. $e^{x-2} + 4 = 21$

4. $\ln 6x \geq 4$

5. $\ln(x + 3) - 5 = -2$

6. $e^{-8x} \leq 50$

7. $e^{4x-1} - 3 = 12$

8. $\ln(5x + 3) = 3.6$

9. $2e^{3x} + 5 = 2$

10. $6 + 3e^{x+1} + 1 = 21$

11. $\ln(2x - 5) = 8$

12. $\ln 5x + \ln 3x > 9$