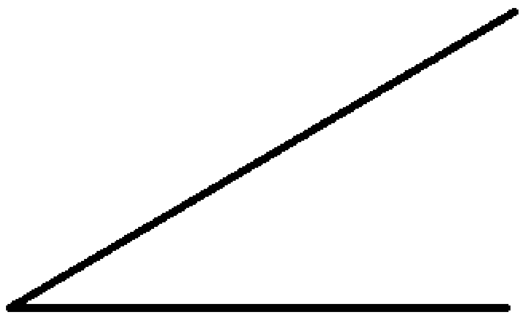
Introduction to Trigonometry

I can give a graphical representation of an angle.

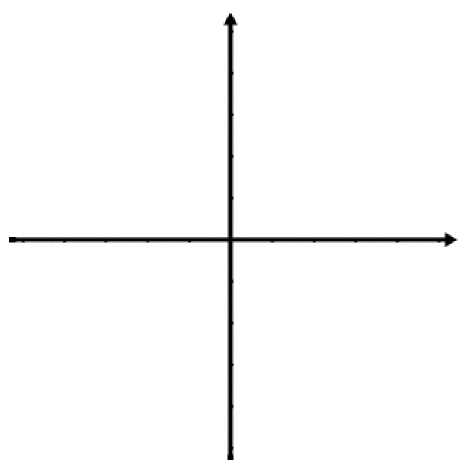
I can convert angle measurements from radians to degrees and degrees to radians.

I can determine the quadrant in which the terminal side of an angle resides.

Angles: An angle is a figure created by two rays that share a common endpoint. The two rays are the sides of the angle and the common endpoint is the vertex of the angle.

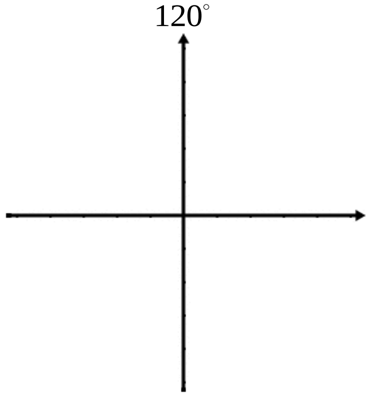


*If the initial side of an angle is on the positive side of the x-axis, the angle is in Standard Position.

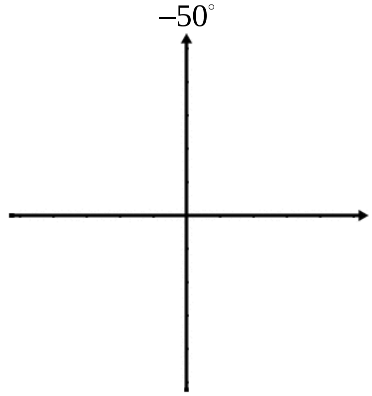


Give a graphical representation of each angle.

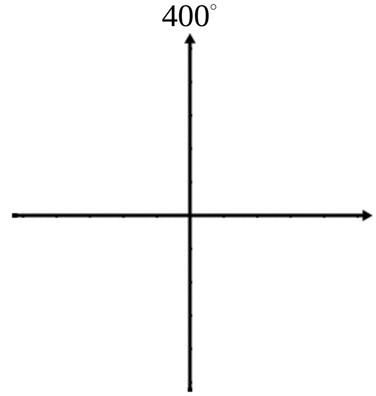
120°



-50°

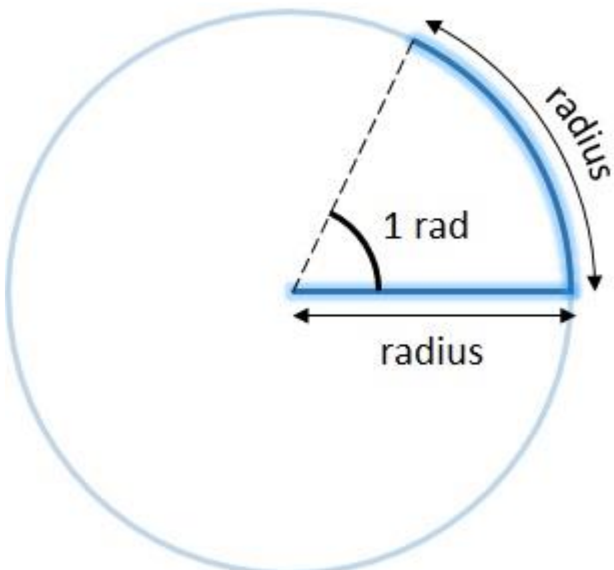


400°

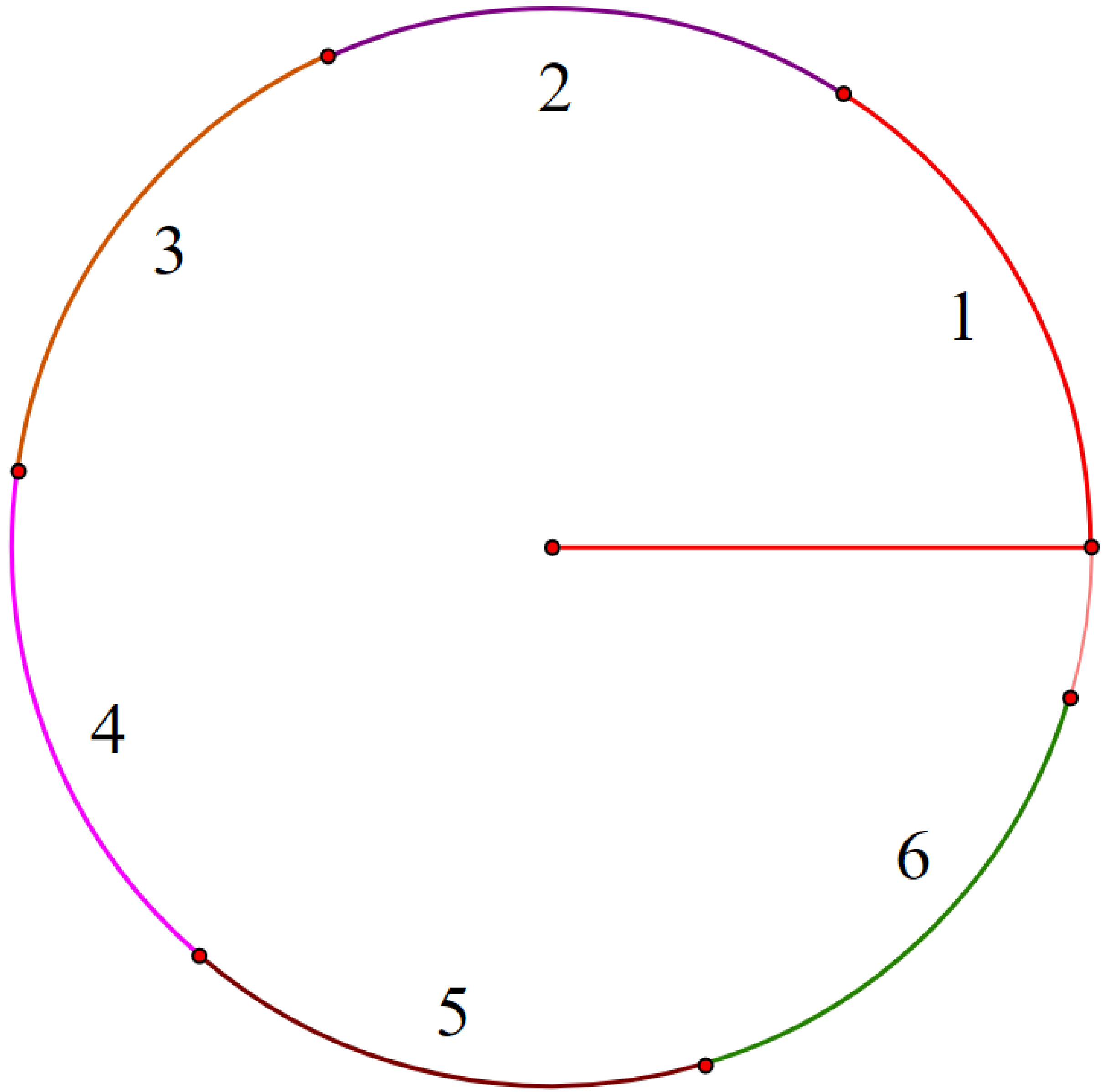


Radian Measure

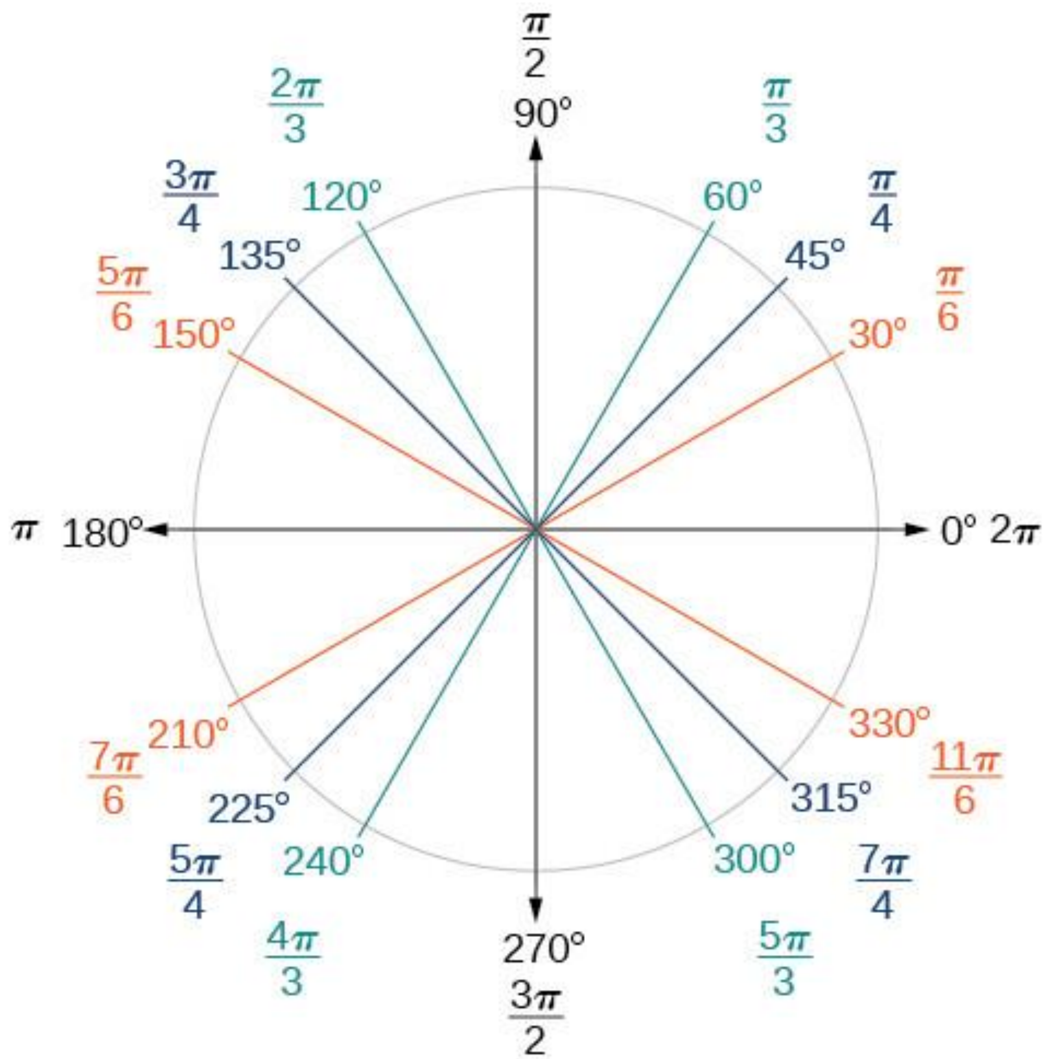
What is a Radian? **One radian is the measure of a central angle, θ, that intercepts an arc S, equal in length to the radius, r, of the circle.**



If an arc is subtended by an angle of 20°, where the radius of the circle is 6 in, find arc length.


2
3
1
4
6
5

Angle measurements is radians and degrees.



Conversion Formulas

Radians → Degrees

$$\boxed{\text{Radians} \cdot \frac{180^\circ}{\pi}}$$

Degrees → Radians

$$\boxed{\text{Degree} \cdot \frac{\pi}{180^\circ}}$$

In which quadrant does the terminal side of $\frac{17\pi}{3}$ lie?

State the quadrant in which the terminal side of each angle lies.

1) $-\frac{\pi}{3}$

2) $-665°$

3) $305°$

4) $\frac{5\pi}{3}$

5) $-525°$

6) $\frac{\pi}{12}$

7) $-380°$

8) $\frac{11\pi}{4}$

9) $-500°$

10) $45°$

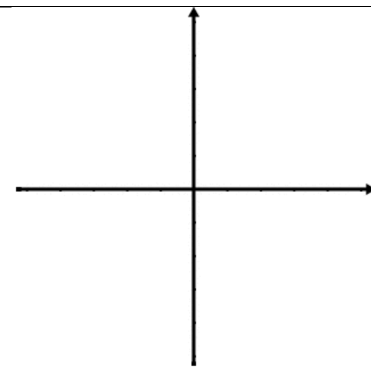
11) $-\frac{11\pi}{3}$

12) $\frac{4\pi}{3}$

Co-Terminal Angles

I can find coterminal angles in both degrees and radians.

Co-Terminal angles are angles who share the same initial and terminal sides.

Find a positive angle that is coterminal to 50°.	

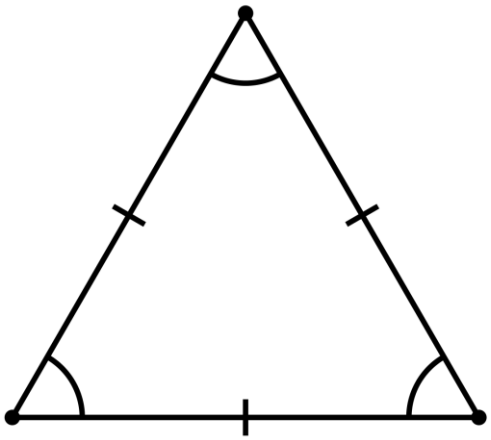
Find two positive and two negative co-terminal angles to the following.

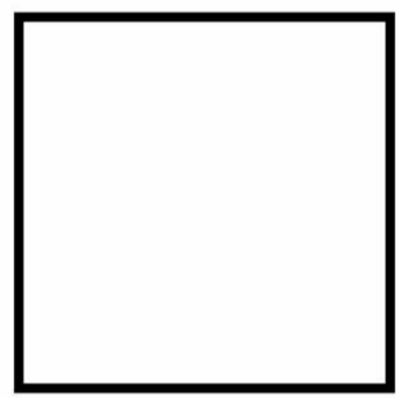
A) 20° **B)** -100° **C)** 800°

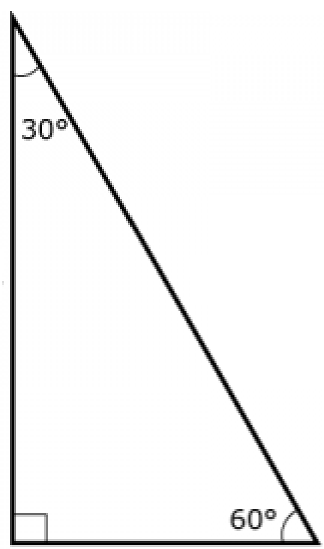
D) $\frac{\pi}{3}$ **E)** $\frac{5\pi}{4}$ **F)** $-\frac{5\pi}{6}$

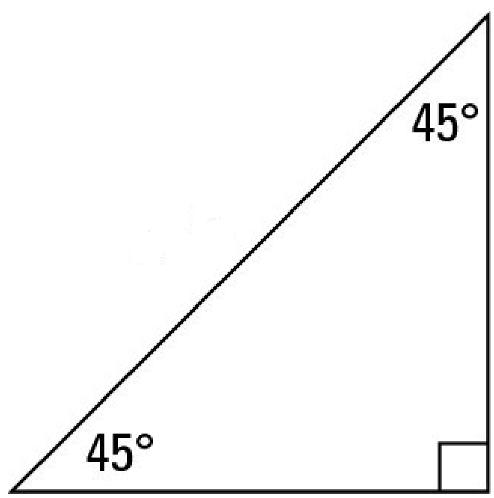
The Unit Circle

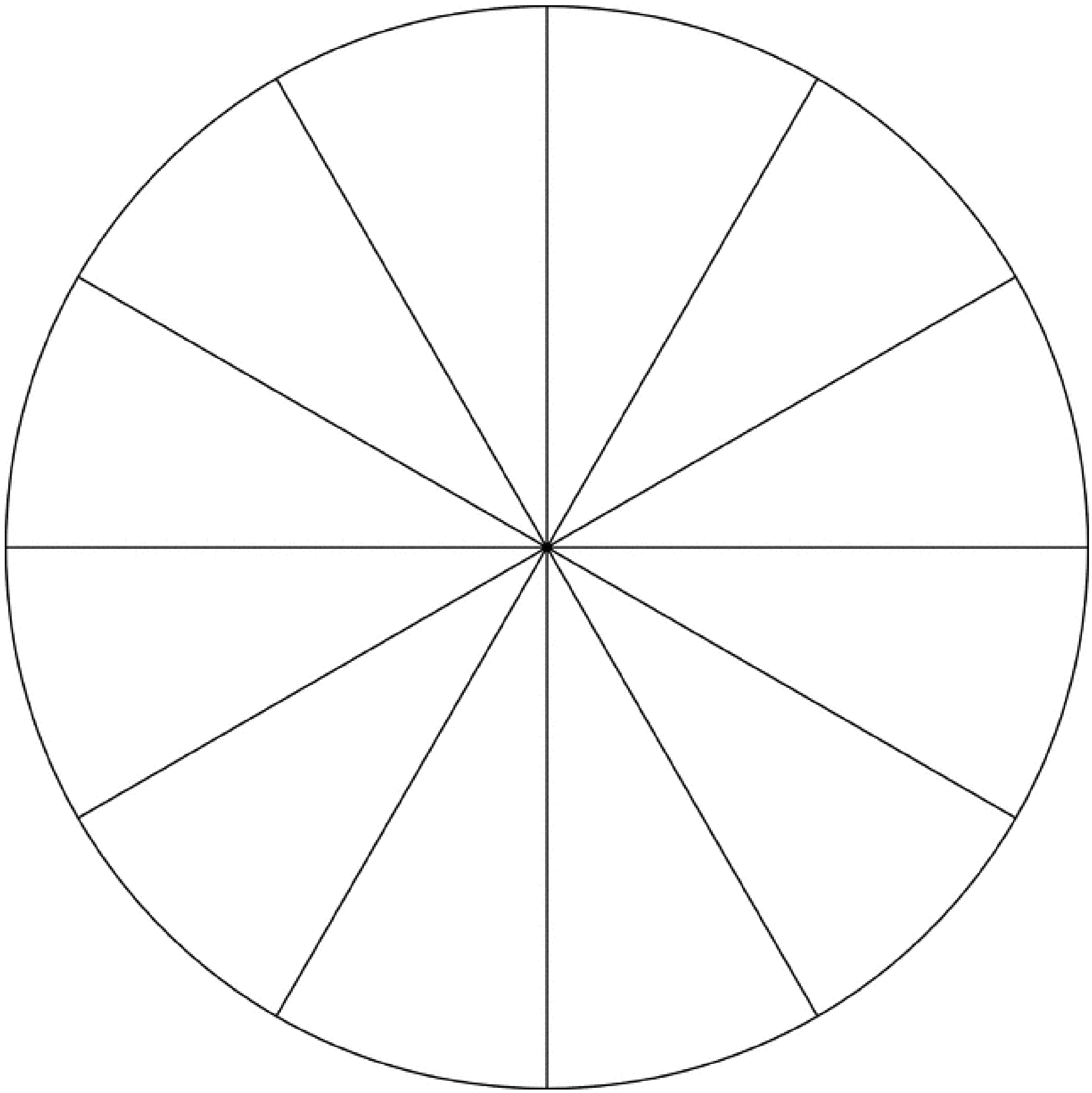
I can build the unit circle.

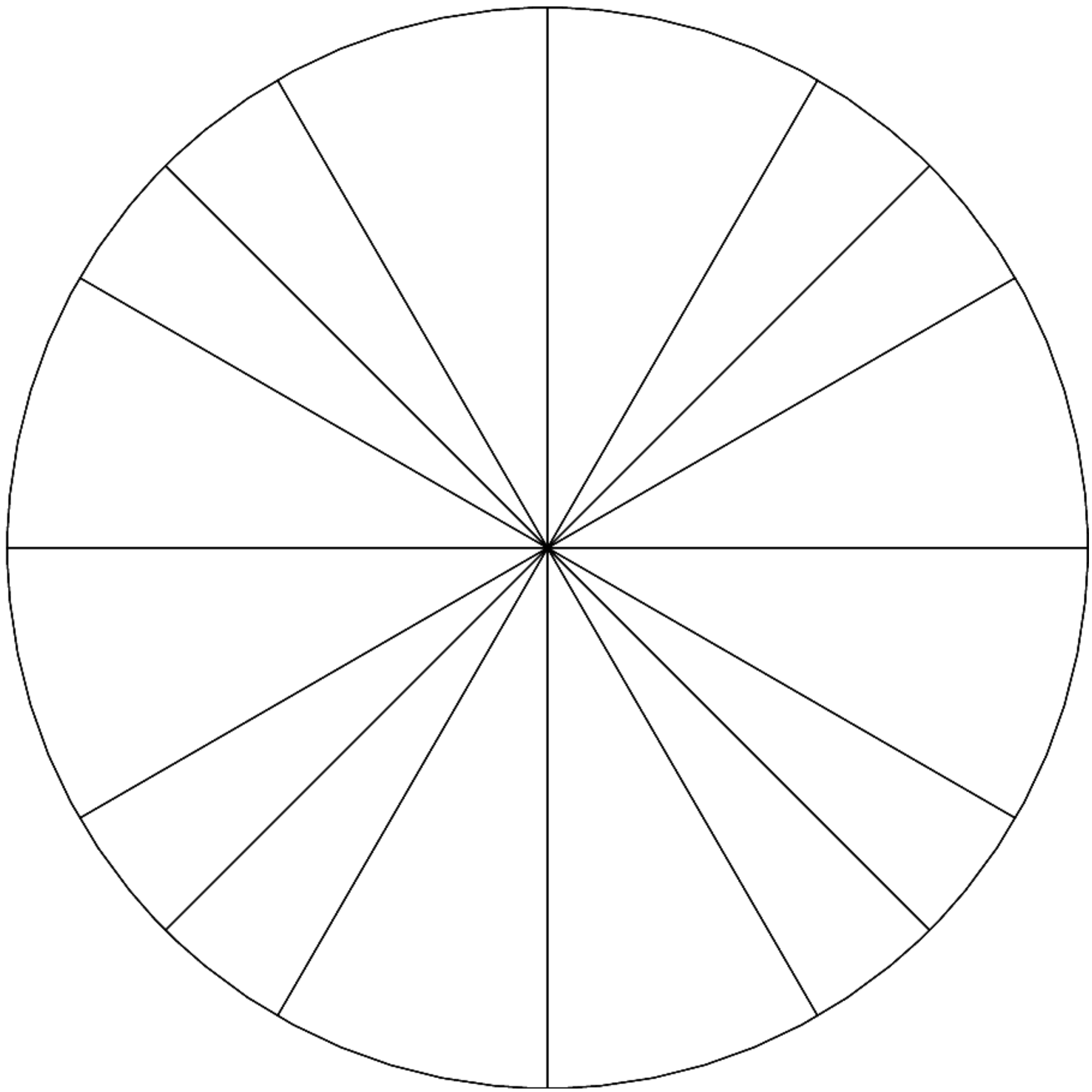




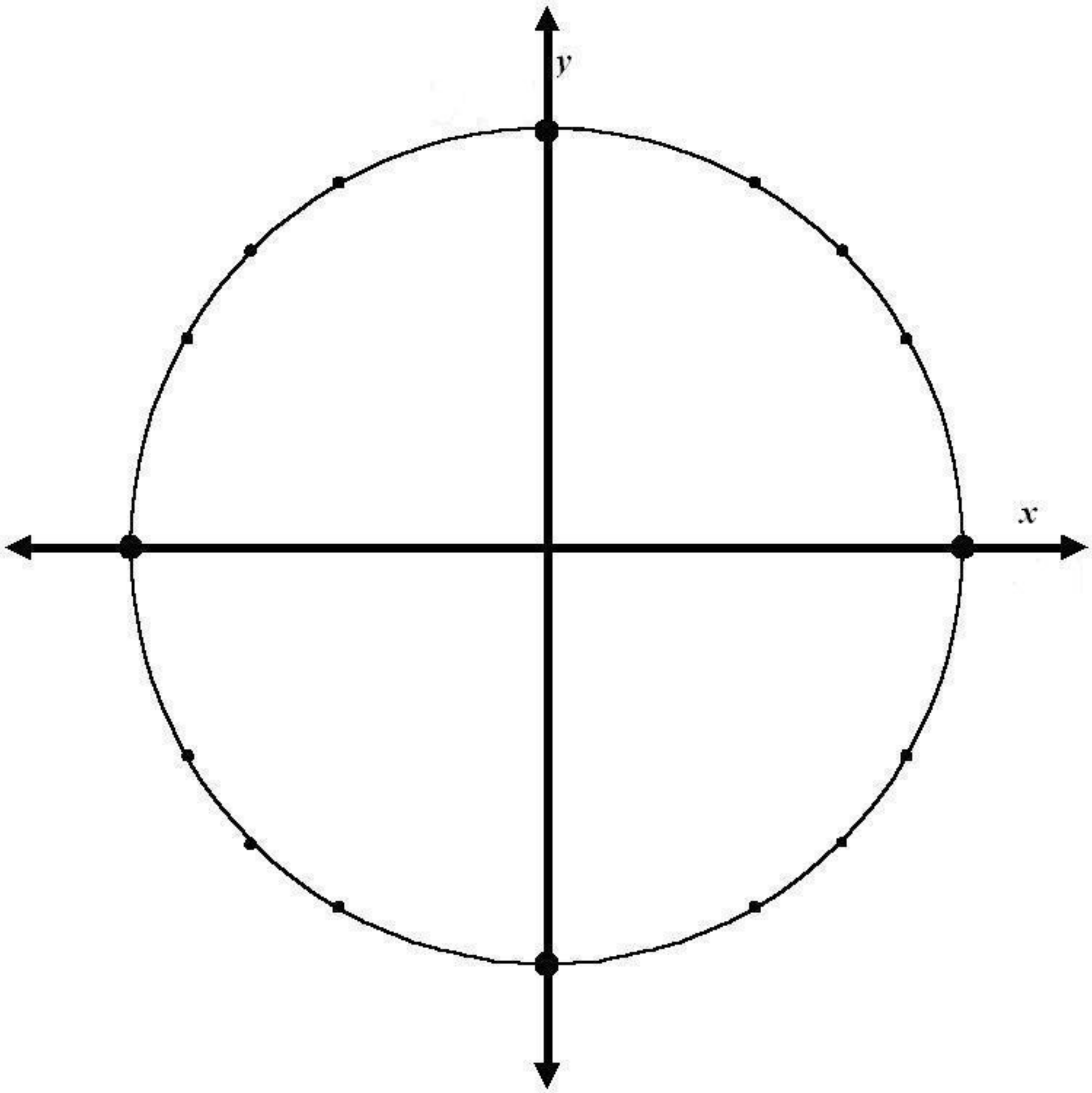








Create the unit circle, label all radian measure and coordinates.



I can use the unit circle to evaluate trig functions.

The 6 Fundamental Trigonometric Functions

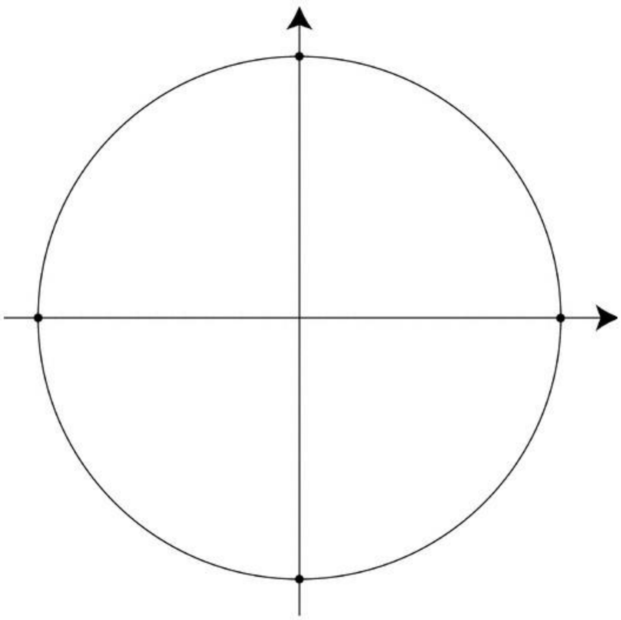
Sine	Cosecant
Cosine	Secant
Tangent	Cotangent

The Reciprocal Identities

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$



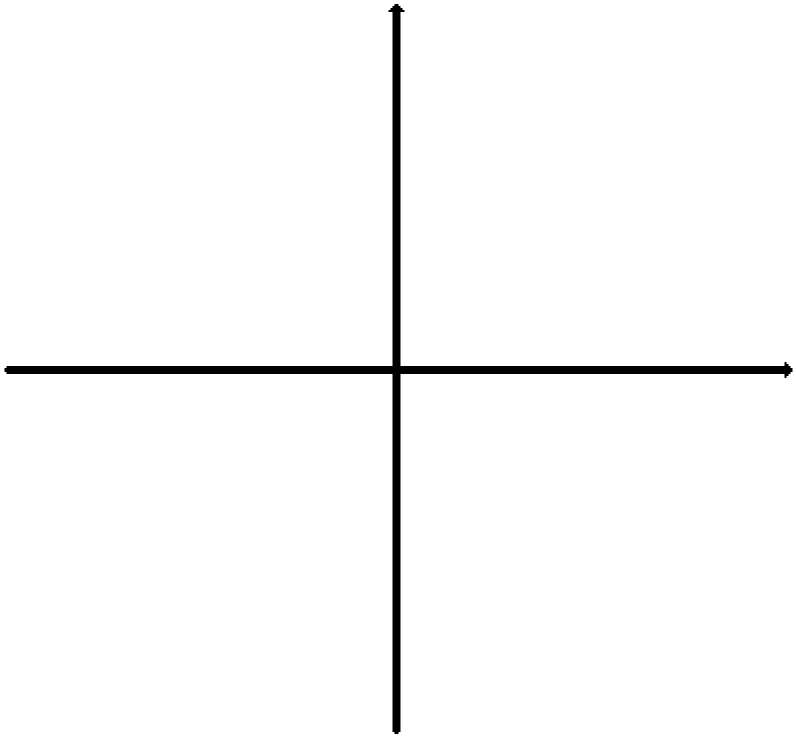
θ is any angle on the unit circle.

Soh-Cah-Toa

$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \cos\theta = \frac{\text{adj}}{\text{hyp}} \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$

<u>The Definition of Trig Functions</u>

Question: Evaluate $\sin\frac{\pi}{6} =$



Using the Unit Circle to Evaluate Trig Functions

Evaluate the 6 trig functions for the angle θ.

A) $\theta = \frac{2\pi}{3}$

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

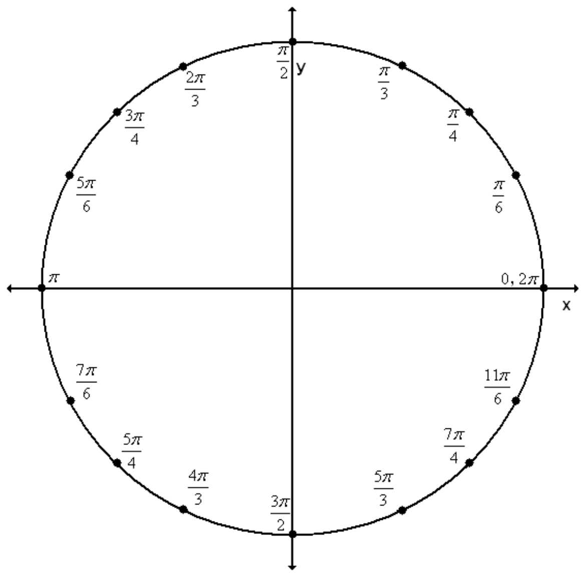
$\tan\theta =$ $\cot\theta =$

B) $\theta = \frac{5\pi}{3}$

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

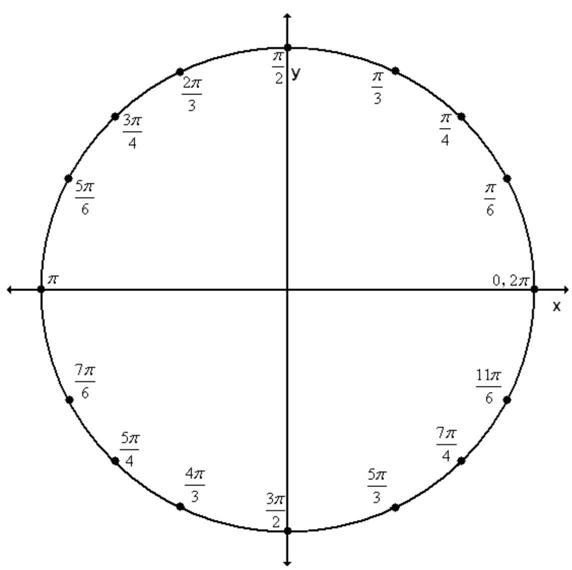


C) $\theta = -\frac{4\pi}{3}$

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

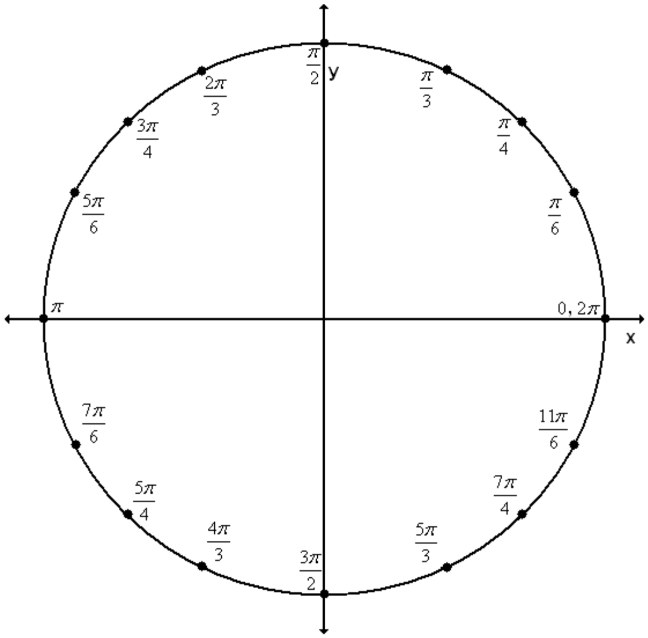


D) $\theta = -\frac{\pi}{3}$

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$



E) $\theta = \frac{41\pi}{6}$

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

F) $\theta = -\frac{14\pi}{3}$

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

THE FUNDAMENTAL TRIGONOMETRIC IDENTITIES

The reciprocal Identities

$$\sin\theta = \qquad\qquad \csc\theta =$$

$$\cos\theta = \qquad\qquad \sec\theta =$$

$$\tan\theta = \qquad\qquad \cot\theta =$$

The Quotient Identities

The Pythagorean Identities

Simplifying Trigonometric Expressions

Using The Fundamental Trigonometric Identities

Use the fundamental identities to simplify the expression. There is more than one correct form of each answer. (Show all work)

1. $\cot\theta\sec\theta$

2. $\sin\theta\left(\csc\theta-\sin\theta\right)$

3. $\dfrac{\cot x}{\csc x}$

4. $\dfrac{1-\sin^2 x}{\csc^2 x-1}$

5. $\sec\alpha\cdot\dfrac{\sin\alpha}{\tan\alpha}$

6. $\dfrac{\sec^2 x-1}{\sin^2 x}$

7. $\cot\theta\sin\theta+\tan\theta\cos\theta$

8. $\sin\beta\tan\beta+\cos\beta$

9. $\dfrac{\cos^2\theta}{1-\sin\theta}$

In exercises 10-15, factor the expression and use the fundamental identities to simplify. There is more than one form of each answer. (Show all work)

10. $\tan^2 x - \tan^2 x \sin^2 x$

11. $\sin^2 \theta \sec^2 \theta - \sin^2 \theta$

12. $\dfrac{\sec^2 x - 1}{\sec x - 1}$

13. $\tan^4 x + 2\tan^2 x + 1$

14. $\csc^3 x - \csc^2 x - \csc x + 1$

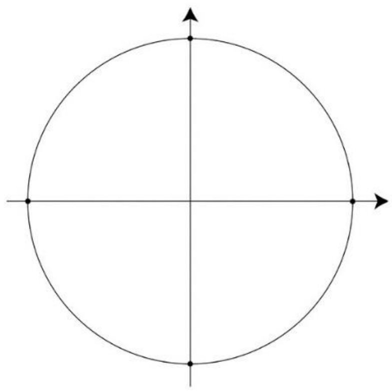
15. $\sin^4 x - \cos^4 x$

In exercises 16-17, perform the multiplication and use the fundamental identities to simplify. There is more than one form of each answer. (Show all work)

16. $(\sin x + \cos x)^2$

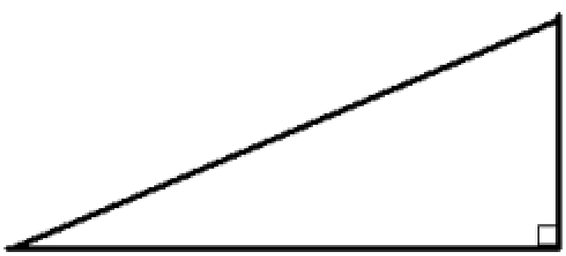
17. $(2\csc x + 2)(2\csc x - 2)$

TRIG FUNCTIONS OF ANY ANGLE



Evaluate the 6 Trig functions for the angle θ.

A)

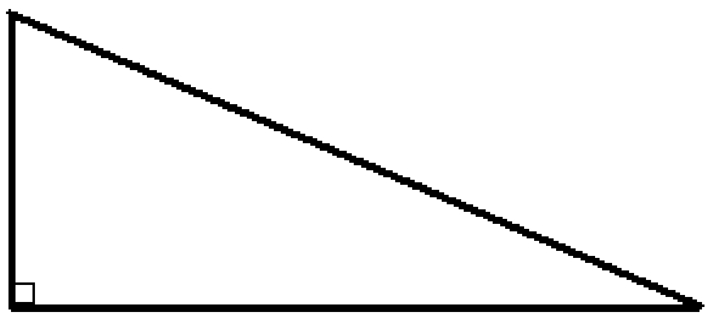


$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

B)

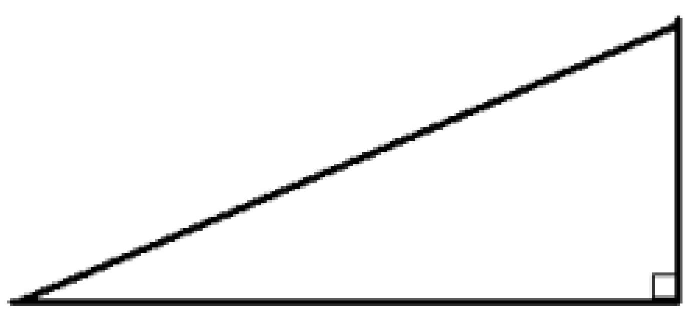


$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

C)



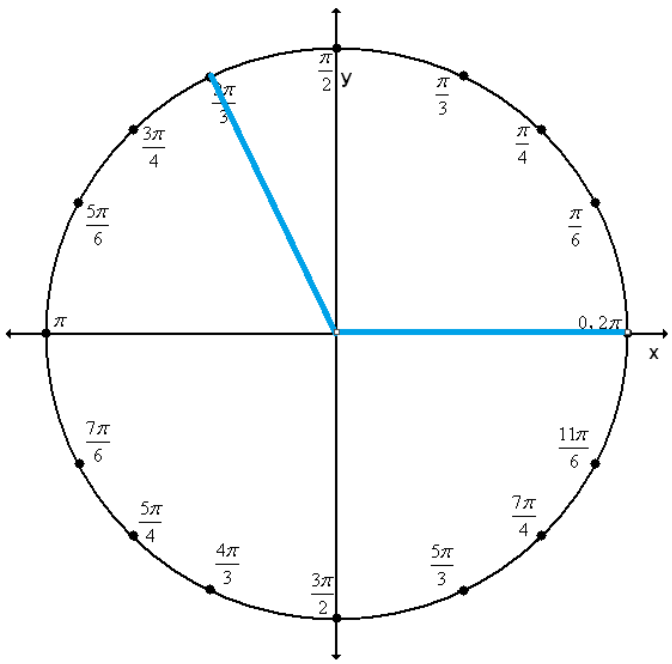
$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

FINDING REFERENCE ANGLES

Why do we need reference angles?



Finding Reference Angles

If θ is in Quadrant I,   then…

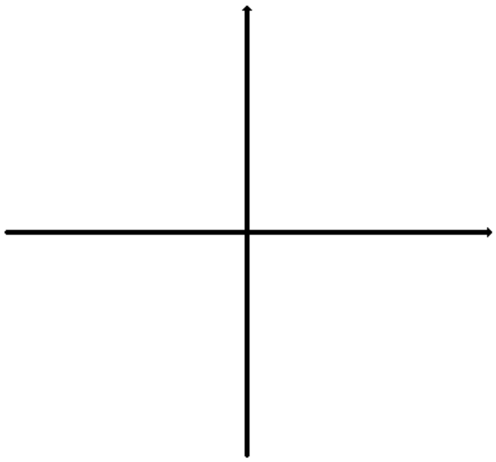
If θ is in Quadrant II,   then…

If θ is in Quadrant III,   then…

If θ is in Quadrant IV,   then…

Evaluate the 6 trigonometric functions given the following:

Evaluate the 6 trig functions of an angle θ that has a terminal side with endpoint of $(-5,6)$.



$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

A) the terminal side of the angle has endpoint of $(-5,7)$.

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

Same Question Disguised in many ways

Evaluate the 6 trig functions for the angle θ given….

B) the terminal side of the angle has endpoint of $(5,-3)$

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

C) The $\tan\theta = -\frac{5}{2}$, and θ lies in $\frac{\pi}{2} \le \theta \le \pi$.

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

D) The $\csc\theta = 2$, and $0 < \theta < \frac{\pi}{2}$

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

E) The $\cos\theta = \frac{8}{17}$, and $\tan\theta < 0$

$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

F) The $\csc\theta = -\frac{7}{3}$, and $\cos\theta > 0$

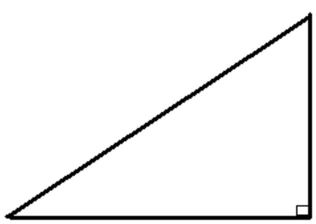
$\sin\theta =$ $\csc\theta =$

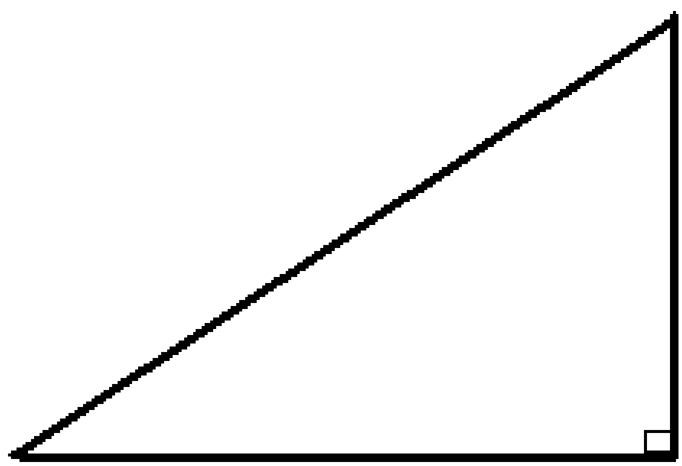
$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

Keep in mind, proportions can help.

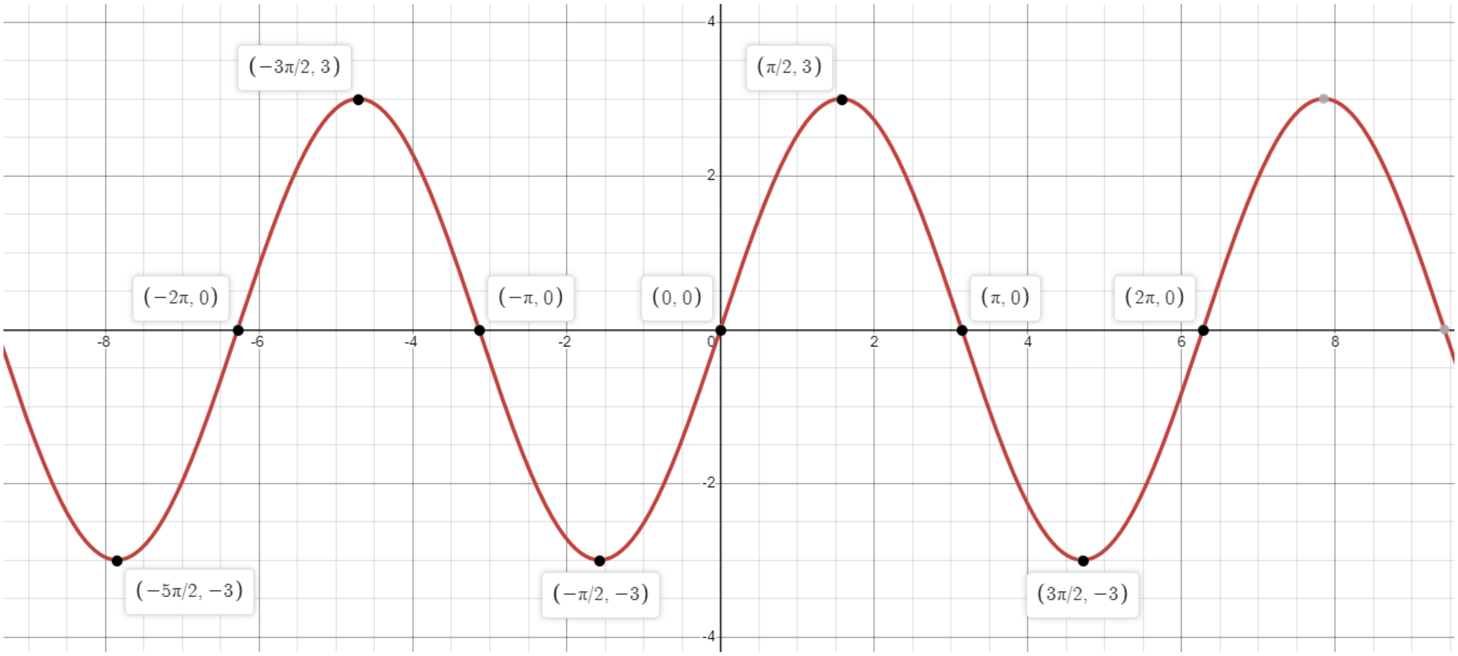
Below on the left is a standard 3-4-5 right triangle. 5 times larger





Using proportions will allow you to evaluate the trig functions of an angle who's side measurements are not integers.

GRAPHING TRIG FUNCTIONS



Amplitude:

Period:

Phase Shift:

Initial Interval:

How do you find each.

Given the standard form of a trigonometric function:

$$y = a\sin(bx + c) + d \qquad y = a\csc(bx + c) + d$$

$$y = a\cos(bx + c) + d \qquad y = a\sec(bx + c) + d$$

Amplitude **Period** **Phase Shift** **Initial Interval**

Find each indicated value.

A) $y = 3\sin 4\pi x - 2$

Amp:

Per:

P.S.:

I.I.:

B) $y = -2\cos\left(\pi - \dfrac{x}{2}\right)$

Amp:

Per:

P.S.:

I.I.:

C) $y = -3\csc(\pi - 3x) + 6$

Amp:

Per:

P.S.:

I.I.:

D) $y = \dfrac{1}{4}\sec\left(\dfrac{2x}{3} + 2\pi\right)$

Amp:

Per:

P.S.:

I.I.: