

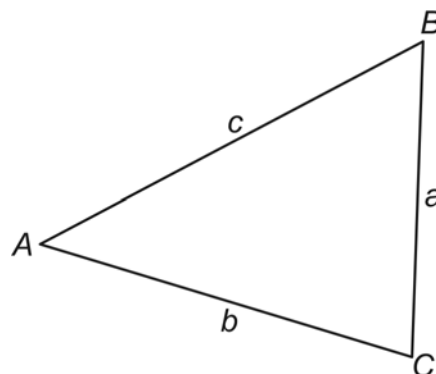
# The Law of Sines

NAME \_\_\_\_\_

Right triangle trigonometry can be used to solve problems involving right triangles. However, many interesting problems involve non-right triangles. In this lesson, you will use right triangle trigonometry to develop the *Law of Sines*. The law of sines is important because it can be used to solve problems involving non-right triangles as well as right triangles.

Consider oblique  $\triangle ABC$  shown to the right.

1. Sketch an altitude from vertex B.
2. Label the altitude  $k$ .
3. The altitude creates two right triangles inside  $\triangle ABC$ . Notice that  $\angle A$  is contained in one of the right triangles, and  $\angle C$  is contained in the other. Using right triangle trigonometry, write two equations, one involving  $\sin A$ , and one involving  $\sin C$ .



$$\sin A = \text{_____} \qquad \sin C = \text{_____}$$

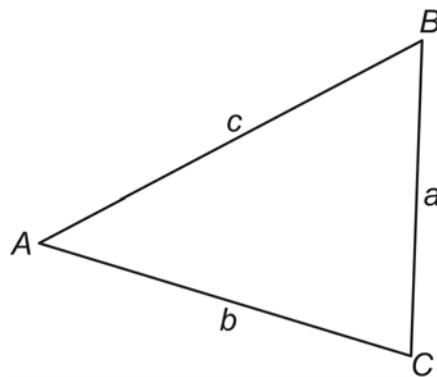
4. Notice that each of the equations in Question 3 involves  $k$ . (Why does this happen?) Solve each equation for  $k$ .
5. Since both equations in Question 4 are equal to  $k$ , they can be set equal to each other. (Why is this possible?) Set the equations equal to each other to form a new equation.
6. Notice that the equation in Question 5 no longer involves  $k$ . (Why not?) Write an equation equivalent to the equation in Question 5, regrouping  $a$  with  $\sin A$  and  $c$  with  $\sin C$ .

Again, consider oblique  $\triangle ABC$ .

7. This time, sketch an altitude from vertex  $C$ .

8. Label the altitude  $k$ .

9. The altitude creates two right triangles inside  $\triangle ABC$ . Notice that  $\angle A$  is contained in one of the right triangles and  $\angle B$  is contained in the other. Using right triangle trigonometry, write two equations, one involving  $\sin A$  and one involving  $\sin B$ .



$$\sin A = \text{—————}$$

$$\sin B = \text{—————}$$

10. Notice that each of the equations in Question 9 involves  $k$ . (Why does this happen?) Solve each equation for  $k$ .

11. Since both equations in Question 10 are equal to  $k$ , they can be set equal to each other. (Why is this possible?) Set the equations equal to each other to form a new equation.

12. Notice that the equation in Question 11 no longer involves  $k$ . (Why not?) Write an equation equivalent to the equation in Question 11, regrouping  $a$  with  $\sin A$  and  $b$  with  $\sin B$ .

13. Use the equations in Question 6 and Question 12 to write a third equation involving  $b$ ,  $c$ ,  $\sin B$ , and  $\sin C$ .

Together, the equations in Questions 6, 12, and 13 form the *Law of Sines*. The law of sines is important, because it can be used to solve problems involving both right and non-right triangles, because it involves only the sides and angles of a triangle.