Lesson 10: Circular Motion

10.1 Experiment: Making the Rounds

How does an object have to be pushed to make it go in a circle? Is it one of the natural ways things move? The Moon goes approximately in a circle, after all. Is that just how it naturally goes? We will be able to answer these questions by the end of this lesson.

(a) In this experiment, you will be using a meter stick to tap a ball into a circle. Keep tapping it continuously with the meter stick so that it goes at a constant speed in a circle around you with you at its center. **PREDICT:** Which direction do you think you’ll have to tap it for it to go around you in the circle? Include a top-view diagram of your prediction. Show the circle and the force vector (arrow) in the direction of your taps.

(b) Now try it: once you get the ball rolling, use your meter stick to continuously tap the ball so that it will roll around you in a circle. Remember to keep tapping! To see what works and what doesn’t, try to keep your taps the same each trial: for example, just tap sideways the whole time and if that doesn’t work, try a different thing. (Don’t just push it around randomly!) Be sure to give everyone a chance to try it out.

(c) What direction did you have to tap it to keep it going around you in a circle? Include a top-view diagram.

(d) What direction did it roll when you stopped tapping? Draw this in on a top-view diagram.

(e) What direction did you push it before tapping to get it moving circularly in the first place?

*In order to move something around in a circle, there has to be an unbalanced net force applied toward the center of the circle. Not surprisingly, this force is called the “center-seeking” force, or the **centripetal force** \( F_c \). Because this net force is directly proportional to the acceleration, it follows that the corresponding “center-seeking” acceleration, or **centripetal acceleration** \( a_c \), also points inward to the center. The centripetal acceleration can be described by this equation: \( a_c = \frac{v^2}{r} \) where \( v \) is the tangential speed and \( r \) is the radius. The velocity at any given moment points tangent to the circle because of the object’s inertia.*
10.2  Exercise: Circular Motion

One day, a chino cowgirl notices a chino cowboy riding by on his horse. Not wanting this opportunity to pass her by, she takes out her lasso and catches him in it so he won’t get away.

(a) Which direction does the cowgirl have to pull to keep the cowboy in a circle at a constant speed around her?

(b) How can she speed up or slow down the horse with her rope or is she only able to pull inward?

(c) What is the direction of the motion of the velocity of his horse at any given moment?

(d) Since force and acceleration are proportional, the acceleration points the same direction as the force.

(e) PREDICT: If a steel ball on the table is set in motion inside a ring with a gap in it, which way will the ball go when it reaches the ring? Draw your prediction here.

(f) What actually happens?

(g) PREDICT: If you spin a top and drop water on it as it spins, how will the drops fly off? Draw your prediction here.

(h) What actually happened?

(i) What do these say about the direction of the velocity of an object moving with uniform circular motion?
10.3  **Experiment: Spinning practice**

(a) What direction do you have to push to keep an object spinning in a circle?

(b) Use the bent hanger adjusted so the end of the hook points toward the center of rotation for this experiment. Now balance a penny on the tip of the hanger. Give it a few short swings to get it up to speed, then spin it over the top! What happens to the penny?

(c) What direction does the normal force from the tip of the hanger push to keep the penny in a circle?

10.4  **Exercise: Velocity in a Circle**

(a) When you ride a carousel (on a broken horse that doesn’t go up and down), let’s say you move around a circle of radius “r” at a constant speed. What mathematical equation can you use to find out how much distance over the ground you go in each revolution by using the radius “r”?

(b) If we name the period of time for one rotation around a circle to be “The Period” (T – capital T because it’s special), we can find the velocity using radius ”r” and time “T”. What’s the equation that we can use?

(c) To check, let’s make it a carousel of radius 3 meters that revolves once every 3.14 seconds. How fast are you going as you ride? It should be 6 m/s. Show the calculations with everything all plugged in.
10.5 Experiment: Force, Velocity, and Radius in Circular Motion

What is the mathematical relationship that describes uniform circular motion? These experiments will help us find out.

(a) The pendulum bob used in Lesson 8 will be your rotating mass. Thread the string through the tubing and hang masses from the bottom of the loop to create tension in the string, which will be the centripetal force keeping the pendulum bob going in a circle. Use a bit of tape or paper clip to mark the string so you can keep the radius of the pendulum bob’s path fairly constant by keeping the mark at the bottom of the tube.

Measure the mass of the rotating pendulum bob and fill it into the table below.

<table>
<thead>
<tr>
<th>Situation</th>
<th>Radius (m)</th>
<th>Time for 10 revolutions (s)</th>
<th>Time for 1 revolution (s)</th>
<th>( V_{\text{rotating mass}} ) ( \frac{2\pi r}{T_{1\text{rev}}} ) (m/s)</th>
<th>( m_{\text{hanging}} ) (kg)</th>
<th>( F_{\text{tension}} = m_{\text{h}} g ) ( \left[ m_{\text{h}} \times 9.8 \text{m/s}^2 \right] ) (m/s(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td></td>
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(b) Use 0.2m as your radius of rotation and 100g as the hanging mass to fill in the values in the table.

*It takes more force to spin a more massive thing around because of its increased inertia. The equation that describes this looks like this:

\[
F_c = \frac{mv^2}{r}
\]

(c) Now calculate the centripetal force using the equation above and fill it into the table below.

<table>
<thead>
<tr>
<th>Situation</th>
<th>( F_{\text{tension}} = m_{\text{h}} g ) ( \left[ m_{\text{h}} \times 9.8 \text{m/s}^2 \right] ) (m/s(^2))</th>
<th>( m_{\text{rotating}} \frac{V^2}{r} ) (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td></td>
<td></td>
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</table>

(d) How does the force of tension holding the pendulum bob in the circle compare to the calculation of \( mv^2/r \)?
10.6  Exercise: Free Body Diagrams for objects undergoing Uniform Circular Motion

(a) For each of the following cases, (1) draw 2 free body diagrams: one side view and one top view so you can see the circle there. Then (2) identify the centripetal force in each case: $F_c = \underline{\hspace{2cm}}$

a. Ball spun around head by a string in horizontal circle

b. UFO of Doooooom carnival ride (like a spinning barrel)

c. Car driving around circular track
d. Car driving around track with banked (tilted) curve

e. Ferris wheel (no need for a top view this time)
   i. Top of motion
   ii. Bottom of motion
   iii. Halfway between top and bottom
10.7  **Exercise: Gravitational Attractions**

(a) In a zoo at sea level, a statue of Ellie the elephant weighs 29,310N. The statue of Ellie is taken to be shown at a zoo at the top of a nearby mountain. When it is weighed there, however, the statue weighs 29,250N!

   a. Come up with ideas as to why this might be. (Assume both scales work equally well.)

b. If the force of gravity acting on the statue is the force of the Earth on the statue, what is the reaction force?

   The reaction to the gravitational force of Earth on the statue can be written as

   the force of ______________________ on ______________________.

   c. To clarify, there is a force acting on the Earth. What kind of force is this?

   d. Does Ellie the elephant have her own gravity?

   If so, what does Ellie’s gravity pull on?

   e. Is the gravitational attraction between Ellie the elephant and Earth larger or smaller when Ellie is raised up higher (farther from the center of Earth)?

   * All objects with mass have a gravitational attraction to all other objects with mass. They pull on each other just as Newton’s 3rd Law of Action and Reaction predicts, with equal sized forces acting in opposite directions. When masses are farther apart, they pull less. In fact, the equation that describes this force is

   $$ F = G \frac{m_1 m_2}{d^2} $$
$m$ stands for masses, $d$ stands for distance between centers of the masses, $F$ is force, and $G$ is a constant.

f. The Earth has a radius of around $6 \times 10^6$ m to its center. When standing on the surface of the planet at sea level, how far apart are the centers of Ellie and Earth? (Hint: Is Ellie tall enough to include her height?)

g. Earth has around $6 \times 10^{24}$ kg of mass. Ellie has 2637 kg of mass. Using this information and the equation shown above, calculate the value of $G$. (Hint: Which value of force should be used with the radius of the Earth?)

*You should find that $G$ is a very small number. Experimental results show that $G = 0.0000000000667$ Nm$^2$/kg$^2$ (more easily written as $6.67 \times 10^{-11}$ Nm$^2$/kg$^2$). Gravity between masses doesn’t pull very hard unless at least one of the objects has a lot of mass, like a planet (Earth, for example) or a star (the Sun, for example) or other large astronomical objects.

h. Using the value of gravitational force at the top of the mountain, 29,250 N (Earth and Ellie are farther apart) along with any necessary information given above, calculate the distance between Earth and Ellie. (It should be a bit more than $6 \times 10^6$ m)

i. Calculate the height of the mountain. (Hint: How much farther is the mountaintop than sea level where the radius of Earth was measured?) The answer should be around 6600 m.