Lesson 5: Vectors and Projectile Motion

5.1 Introduction: Vectors vs. Scalars
(a) Read page 69 of the supplemental Conceptual Physics text. Name at least 3 vector quantities and at least 3 scalar quantities.

5.2 Experiment: Describing Motion at an Angle
(a) Diagonal motions can be described by using right triangles. To examine this, work in teams of three or four to accomplish the next task. Use the floor tiles as gridlines. Pick a corner for everyone to start at. One student walk sideways 4 squares from that corner, the second student walk 3 squares forward, and the third student walk diagonally to a point where he can view both students directly along “gridlines.” (A fourth student can stay at the starting point for reference)

(b) Now measure the distances from each student to the starting point. Write those 3 distances here.
(diagonal part) $R_{\text{measured}} =$

(sideways part) $R_x \text{ measured} =$

(forward part) $R_y \text{ measured} =$

(c) Since this forms a right triangle, the Pythagorean Theorem can be used to relate these lengths. Use this mathematical relationship and the two shorter distances to calculate the diagonal distance.

(d) How close did it come?

*Any motion can be broken into two movements like this. A diagonal move is just like a “sideways” and “forward” move. This is true of anything that involves direction, not just displacements. These two parts of the motion are called “components.” Mathematically, the x and y components of a vector are defined as:
(the sideways one) $R_x = R \cos \theta$ and (the forward one) $R_y = R \sin \theta$

(e) Use a protractor to measure the angle between the “4 square” student and the “diagonal” student. (It should be around 37°) Record your measured angle.

measured angle: _____________°
*We can use what we know mathematically about right triangles to calculate the exact angle using sine, cosine, and tangent functions. If we use the two smaller sides (the two components of the vector), we can use the tangent function to find the ratio of the sides. Using the inverse tangent function on your calculator, you can solve for the angle with more precision.

(f) Use your calculator to solve for the angle by plugging it into the arctan or tan\(^{-1}\) on your calculator. Record your answer and show your calculation.

(g) How does this answer compare to the angle you measured with the protractor?

5.3 Experiment: Using Vectors and Their Components

(a) Choose an “origin” at a corner of the squares on the floor. Now have one student walk a few paces in any direction. Measure the distance and direction (be specific: include an angle and reference the angle to something) and record the values you measure. Mark the spot with a small piece of masking tape.

Distance moved = ________________

at an angle of ____________________°

measured from:

(b) Now move again from the origin to the spot you marked, but with these two guidelines: (2) You can only move sideways and forward at right angles. (2) You can only turn ONCE. Have each group member try it.
How far did you have to go each time and in which directions?

1\textsuperscript{st} motion:

Direction of 1\textsuperscript{st} motion:

2\textsuperscript{nd} motion:

Direction of 2\textsuperscript{nd} motion:

(c) Use your two distances from part (b) to calculate the angle with \(\tan^{-1}\) on your calculator.
(d) Is this the angle you expected?

Explain.

*This method isn’t just for displacements. It works for \textit{anything} that involves direction (in other words, any “vector quantity”): velocities, accelerations, forces, momenta, electric fields, etc.

5.4 Demonstrations: Independence of Vertical and Horizontal Motion for a Projectile

(a) Examine the examples given on the handout. For each, discuss with each other and write your prediction for each.
(b) Explain the reason for each of your predictions.
(c) Now observe the demonstrations and describe what happens for each.

\textbf{Bullet and block}

A large very powerful gun shoots a bullet that easily goes through a block. In the process the block is knocked off the post and falls the ground. Which hits the ground first, the bullet or the block? Why? Explain.
A) PREDICTION:

B) Explanation for prediction:

C) What actually happens:

**Pop cart**

It is a cart that rolls and has a vertical tube that shoots a small ball straight into the air when triggered. When pushed across a level track, a small vertical bar will trigger the ball to shoot into the air without slowing down the cart. Where will the ball land after it as been ejected from the cart? Why? Explain.

A) PREDICTION:

B) Explanation for prediction:

C) What actually happens:

**River versus stream and boat**

How long does it take to get across river? Why? Explain.

A) PREDICTION:
B) Explanation for prediction:

C) What actually happens:

**Bomber drops a bomb**

Which picture of the bombers dropping their bombs is correct? Why? Explain.

A) PREDICTION:

B) Explanation for prediction:

C) What actually happens:

**Where does the bomb land**

When the bomb hits the ground where will the bomber be? Why? Explain.
A) PREDICTION:

B) Explanation for prediction:

C) What actually happens:

**Monkey hunter 1**
Should the monkey hold on or drop from the tree to escape the hunter? Why? Explain.

A) PREDICTION:

B) Explanation for prediction:

C) What actually happens:

**Monkey hunter 2**
Should the monkey hold on or drop from the tree to escape the hunter? Why? Explain.

A) PREDICTION:
B) Explanation for prediction:

C) What actually happens:

### Toss a ball on a train

The train is moving at a constant speed of 150 m/s (336 mph) how does a ball behave when it is tossed into the air? Why? Explain.

![Diagram of a ball being tossed on a moving train]

A) PREDICTION:

B) Explanation for prediction:

C) What actually happens:

### Connected Rocket A and Rocket B

Could Rocket B slow down or speed up Rocket A? Why? Explain.

![Diagram of two connected rockets]

A) PREDICTION:
5.5 Experiment: Analyzing the Path of a Projectile

Use the data provided for a projectile’s position in flight to plot a graph of its vertical and horizontal positions at equal intervals of time for the analysis in this section.

(a) What is the shape of the curved path the ball takes?

(b) Measure the differences between the **horizontal** positions of the projectile in equal intervals of time.

(c) Use these values to create a velocity vs. time graph of the **horizontal** motion of the projectile. Attach graph.

(d) How does the velocity change? How would you describe this motion?

(e) Now measure the differences between the **vertical** positions of the projectile in equal intervals of time.

(f) Use these values to create a velocity vs. time graph of the **vertical** motion of the projectile.

(g) How does the velocity change this time? How would you describe this motion?

(h) Now use your velocity values to create an acceleration graph of the vertical and horizontal motions.

(i) What is the horizontal acceleration of the projectile according to your graph?

(j) What is the vertical acceleration of the projectile according to your graph? Calculate its value here:
5.6 Exercise: Calculating 2-Dimensional Kinematics Projectile Motion Problems

(a) Suppose that a ball is thrown at a speed of 10 m/s horizontally off a 50 meter tall cliff. In order to find out its furthest horizontal distance and final velocity, we can use what we discovered in the previous experiment. First, sketch the situation here:

(b) How quickly is the ball moving on each axis at the beginning of the toss? To find out, we need to break the initial velocity into its components. What are the horizontal and vertical components of the launch?

\[ v_y = \_\_\_\_\_\_\_\_\_ m/s \quad \quad \quad \quad \quad \quad v_x = \_\_\_\_\_\_\_\_\_ m/s \]

(c) How much time will the ball spend in the air until it lands? Will it take more, less, or the same amount of time as it would take if it were dropped from the same height? (“Dropped” means “released from rest”)

(d) Let’s begin by analyzing the vertical component of the motion: Write out all five variables for constant acceleration first, writing in the values of the three you know. That leaves two variables, so you can put in a question mark for the one you’re trying to solve for – in this case, the time.

(e) Write in the value of the vertical component of the initial velocity \(v_y\). What is the vertical acceleration of the ball as it falls? Write in that value as well.
(f) How far does the ball fall?

Will that be a positive or negative value of displacement?

(Write the displacement into the list of givens a few steps back also.)

(g) Now use the three values we just determined to solve for the fall time. Show your calculations.

Total time before hitting the floor = _________________ s

(h) Next, we’ll find the horizontal distance traveled.
   What is the horizontal motion of a projectile like? How does it change in time? Is it constant?

(i) We found the horizontal component of the velocity earlier. We will use this crucial information to solve for the distance traveled in the next few steps. What equation relates velocity to displacement and time for constant speed motion?

(j) Since it takes the same time for the vertical and horizontal components of the motion to happen, we will use the time we solved for with the vertical values to solve for the horizontal motion.
   Plug that time value into your equation for velocity, displacement, and time to solve for the horizontal distance traveled.

(k) Last, we will solve for the final velocity.
   Is the ball falling downward or rising as it lands?
Use the givens in the vertical analysis to solve for the final velocity on the y axis:

Does this velocity represent the entire final speed? Is it moving along the x-axis still at the end or is it going straight down? (Remember what the x-axis motion is like)

What is the final velocity on the x-axis? \( V_{x\text{ final}} = \) \underline{____________________} m/s

To find the actual final speed, we must combine both final velocity vector components and solve for the overall value by making a right triangle with the final x-component as the horizontal side of the triangle and the y-component as the vertical side of the triangle. The hypotenuse is the resultant speed (the actual speed it’s moving diagonally as it lands)
Perform the necessary calculations to solve for the final speed: (Hint: it will involve the Pythagorean Theorem so draw a triangle here)

\[ V_{\text{final}} = \underline{____________________} \text{ m/s} \]

The final angle can be found by using the \( \tan^{-1} \) function as shown below:

\[ \theta = \tan^{-1}(v_y/v_x) = \underline{__________}^\circ \]

(be sure your calculator is in degree mode so that you get the answer in degrees)

(m) Use the equation shown above to solve for the direction (angle) and write the full answer below in the blanks:

\[ \theta = \underline{____________________}^\circ \]

\[ V_{\text{final}} = \underline{____________________} \text{ m/s at } \underline{________________________}^\circ \text{ below the horizontal} \]