

MATRICES AND SYSTEMS OF EQUATIONS

THE ORDER OF A MATRIX

$$\begin{array}{c} \text{Columns} \\ \downarrow \\ \text{Rows} \rightarrow \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \end{array}$$

Order
Rows x Columns

Determine the order of the following matrices.

$$\begin{array}{ccc} 1 & 2 & 3 \\ \begin{bmatrix} 3 & 2 & -4 \\ 5 & 1 & 6 \end{bmatrix} & \begin{bmatrix} 1 \\ -6 \\ 5 \\ 7 \end{bmatrix} & [6] \end{array}$$
$$\begin{array}{ccc} 4 & & \\ \begin{bmatrix} 2 & 8 \\ -1 & 0 \\ -4 & 1 \end{bmatrix} & & \end{array}$$

WRITING AN AUGMENTED MATRIX

Given the system

$$\begin{cases} 3x - 2y = 8 \\ 4x + y = 6 \end{cases}$$

The Augmented Matrix is

The coefficient matrix is

SOLVING A SYSTEM USING AUGMENTED MATRICES

Before we solve systems using augmented matrices, let's practice a bit.

A. Write the augmented matrix that represents the system.

$$\begin{cases} 7x - 3y + 6z = 8 \\ 2x \quad + 4z = 1 \\ -2y + z = 9 \end{cases}$$

B. Write the system of equations represented by the matrix below.

$$\left[\begin{array}{ccc|c} 2 & 3 & -4 & 1 \\ 0 & 8 & 6 & 0 \\ 3 & 0 & 0 & 4 \end{array} \right]$$

C. Write the system of equations represented by the matrix below.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{array} \right]$$

When solving the system using an augmented matrix: $\begin{cases} 5x - 5y = -5 \\ -2x - 3y = 7 \end{cases}$

First Create an augmented matrix for the system

$$\left[\begin{array}{cc|c} 5 & -5 & -5 \\ -2 & -3 & 7 \end{array} \right]$$

Our Goal is to make this look like

$$\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}$$

Or if given a 3 x 3 we want:

$$\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

Why?

To accomplish this, we use elementary row operations.

Identify the elementary row operations being performed to obtain the new row equivalent matrix.

Original Matrix

$$\begin{bmatrix} -1 & -2 & 3 & -2 \\ 2 & -5 & 1 & -7 \\ 5 & 4 & -7 & 6 \end{bmatrix}$$

New Row Equivalent Matrix

$$\begin{bmatrix} -1 & -2 & 3 & -2 \\ 0 & \square & 7 & -11 \\ 0 & -6 & \square & -4 \end{bmatrix}$$

On the next page, we will solve the following

Solve the system using an augmented matrix:

$$\begin{cases} -x + y - z = -14 \\ 2x - y + z = 21 \\ 3x + 2y + z = 19 \end{cases}$$

$$\begin{bmatrix} -1 & 1 & -1 \div -14 \\ 2 & -1 & 1 \div 21 \\ 3 & 2 & 1 \div 19 \end{bmatrix}$$

How do we make this 1 using only multiplication or division?