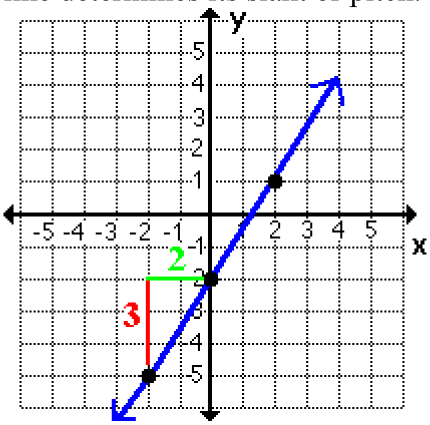
THE SLOPE OFA LINE

As we begin study of linear functions, we must be aware of the slope of a line. The slope of a line determines its slant or pitch.



As you can see from the picture on the left, in order to graph this line, I had to go up 3 and right 2. This determines the slant of the line, or slope as we call it. One way to think of slope is as **Rise Over Run**. In this case, we rise 3 and run 2 to the right.

FINDING THE SLOPE OF A LINE

There is a mathematical formula that allows you to find the slope of a line. Given two points, (x_1, y_1) and (x_2, y_2), we can find the slope of any line. The sub 1 and sub 2 are used only to distinguish between the first point given, and the second point given. Once we have these points, the slope of the line will be given by the following formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where m is the slope of the line.

Here are some examples on finding the slope of a line given two points.

<table>
<tr><td colspan="2">Example 1
Find the slope of the line that contains the points $(-3,-5)$ and $(7,3)$.</td><td></td><td colspan="2">Example 2
Find the slope of the line that contains the points $(2,8)$ and $(12,-2)$.</td></tr>
<tr><td>$x_1 \ \ y_1 \quad x_2 \ y_2$
$(-3,-5),(7,\ 3)$
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$m = \frac{3-(-5)}{7-(-3)}$
$m = \frac{3+5}{7+3}$
$m = \frac{8}{10}$
$m = \frac{4}{5}$</td><td>To begin with, label the coordinates as $x_1\ y_1$ and $x_2\ y_2$. This will ensure that the numbers are placed correctly using the formula. They are written in this order simply because of the first point and second point. Recall from your previous math classes, that minus a negative changes to plus positive.

Once the ratio has been found, reduce if possible.

This is the slope of the line.</td><td></td><td>$x_1\ y_1 \quad x_2\ y_2$
$(2,\ 8),(12,-2)$
$m = \frac{y_2 - y_1}{x_2 - x_1}$
$m = \frac{-2-8}{12-2}$
$m = \frac{-10}{10}$
$m = -\frac{10}{10}$
$m = -1$</td><td>Once the coordinates are labeled, place the into the formula for slope.

This problem is straight forward, just be careful with the negative sign.

One of the properties of fractions is that a negative can be moved from the numerator to the front of the fraction.

Reduce the fraction</td></tr>
</table>

<table>
<tr><td colspan="2">For examples 3 and 4, we will look at coordinates that contain fractions.</td></tr>
<tr><td>Example 3
Find the slope of the line that contains the points $\left(\frac{3}{2},4\right)$ and $\left(\frac{25}{3},10\right)$.</td><td></td></tr>
<tr><td>$\begin{matrix} x_1 & y_1 & x_2 & y_2 \end{matrix}$
$\left(\frac{3}{2},4\right),\left(\frac{25}{3},\ 10\right)$

$m=\frac{y_2-y_1}{x_2-x_1}$

$m=\frac{10-4}{\frac{25}{3}-\frac{3}{2}}$

$m=6\div\left(\frac{25}{3}-\frac{3}{2}\right)$

$m=6\div\left(\frac{25}{3}\left(\frac{2}{2}\right)-\frac{3}{2}\left(\frac{3}{3}\right)\right)$

$m=6\div\left(\frac{50}{6}-\frac{9}{6}\right)$

$m=6\div\left(\frac{41}{6}\right)$

$m=\frac{6}{1}\cdot\frac{6}{41}$

$m=\frac{36}{41}$</td><td>In this problem we are dealing with fractions. The coordinates are first labeled as appropriate.

In this step, the coordinates have been substituted into the formula for the slope of a line.

A compound equation can be difficult to work with. In this case, rewrite the problem.

In order to add fractions, we must have a common denominator. In this case, the common denominator will be 6. Multiply the first fraction by 2 over 2, and the second by 3 over 3.

When dividing fractions, multiply by the reciprocal.

Since the resultant fraction cannot be reduced, we have found the slope of the line.</td></tr>
<tr><td></td><td></td></tr>
</table>

Example 4 Find the slope of the line that contains the points $\left(-\frac{1}{2},\frac{7}{3}\right)$ and $\left(\frac{13}{4},-\frac{5}{2}\right)$.	
$\overset{x_1}{\left(-\frac{1}{2}\right.},\overset{y_1}{\left.\frac{7}{3}\right)},\overset{x_2}{\left(\frac{13}{4}\right.},\overset{y_2}{\left.-\frac{5}{2}\right)}$ $m=\frac{y_2-y_1}{x_2-x_1}$ $m=\frac{-\frac{5}{2}-\frac{7}{3}}{\frac{13}{4}-\left(-\frac{1}{2}\right)}$ $m=\frac{-\frac{5}{2}-\frac{7}{3}}{\frac{13}{4}+\frac{1}{2}}$ $m=\left(-\frac{5}{2}-\frac{7}{3}\right)\div\left(\frac{13}{4}+\frac{1}{2}\right)$ $m=\left(-\frac{5}{2}\left(\frac{3}{3}\right)-\frac{7}{3}\left(\frac{2}{2}\right)\right)\div\left(\frac{13}{4}+\frac{1}{2}\left(\frac{2}{2}\right)\right)$ $m=\left(-\frac{15}{6}-\frac{14}{6}\right)\div\left(\frac{13}{4}+\frac{2}{4}\right)$ $m=-\frac{19}{6}\div\frac{15}{4}$ $m=-\frac{19}{6}\cdot\frac{4}{15}$ $m=-\frac{76}{90}$ $m=-\frac{38}{45}$	*In this problem we are dealing with fractions. The coordinates are first labeled as appropriate.* *In this step, the coordinates have been substituted into the formula for the slope of a line.* *A compound equation can be difficult to work with. In this case, rewrite the problem.* *In order to add fractions, we must have a common denominator. In this case, the common denominator will be 6. Multiply the first fraction by 2 over 2, and the second by 3 over 3.* *When dividing fractions, multiply by the reciprocal.* *Since the resultant fraction cannot be reduced, we have found the slope of the line.*

SPECIAL CASES

The following are special cases one may run into when finding the slope of a line.

<table>
<tr><td colspan="2">Example 5
Find the slope of the line that contains the points $(7,8)$ and $(7,3)$.</td><td></td><td colspan="2">Example 6
Find the slope of the line that contains the points $(2,7)$ and $(12,7)$.</td></tr>
<tr><td>$x_1\ y_1\ \ x_2\ y_2$
$(7,8),(7,\ 3)$
$m=\frac{y_2-y_1}{x_2-x_1}$
$m=\frac{3-8}{7-7}$
$m=\frac{-5}{0}$
$m=undefined$</td><td>Once the coordinates are labeled as $x_1\ y_1$ and $x_2\ y_2$. They can be placed correctly using the formula. Before we begin, notice that the x values are the same.

As the problem is set up and simplified, the denominator becomes 0.

Since we can never have zero in a denominator, our slope in this problem is undefined.

If we were to plot these two points, we would find that they line up vertically. That means this is a vertical line.

Since the answer to this question is undefined, we can conclude that the slope of a vertical line is undefined.</td><td></td><td>$x_1\ y_1\ \ \ x_2\ y_2$
$(2,\ 7),(12,7)$
$m=\frac{y_2-y_1}{x_2-x_1}$
$m=\frac{7-7}{12-2}$
$m=\frac{0}{10}$
$m=0$
or
$m = no\ slope$</td><td>Once the coordinates are labeled, place the into the formula for slope.

In this problem, the numerator becomes zero. Zero divided by any number is still zero, therefore, our slope is zero.

If we were to plot these two points they would line up horizontally.

Since these points line up horizontally and using the formula yields a result of zero, we conclude that the slope of a horizontal line is 0 or no slope.</td></tr>
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