Absolute Value Inequalities

For absolute value inequalities, you will create a compound inequality.

$$|x-6|+3 \leq 10$$

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$$\underline{-3 \quad -3}$$

First subtract 3 from both sides of the inequality to isolate the absolute value.

$$|x-6| \leq 7$$

The absolute value has now been isolated.

At this point you will write a compound inequality. Remember to use the same sign that is used in the original problem. Once this is done, the absolute value symbols are gone. Solve just as you would a normal compound inequality.
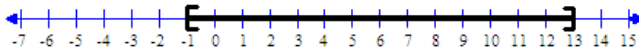
$$-7 \leq x-6 \leq 7$$

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$$+6 \qquad +6 \quad +6$$

Add six to all parts of the inequality to isolate the x.

$$-1 \leq x \leq 13$$

Here is the solution to the equation.

Here is the solution represented graphically on a number line.

$$[-1,13]$$

Here is the solution in interval notation.

Here is another example.

$$22 - |5 - x| \leq 16$$

$$22 - |5 - x| \leq 16$$
$$-22 \qquad \quad -22$$

Subtract 22 from both sides to isolate the absolute value.

$$-|5 - x| \leq -6$$

The absolute value has not quite been isolated here.

$$\frac{-|5 - x|}{-1} \geq \frac{-6}{-1}$$

Divide both sides of the inequality by -1. When this is done, the inequality sign will flip.

$$|5 - x| \geq 6$$

The absolute value has now been isolated.

$$-6 \geq 5 - x \geq 6$$

Using the same inequality symbol, create a compound inequality.

$$-6 \geq 5 - x \geq 6$$
$$-5 \quad -5 \qquad -5$$

Subtract 5 from all parts of the compound inequality.

$$-11 \geq -x \geq 1$$

Once 5 has been subtracted, we still have a negative sign in front of x.
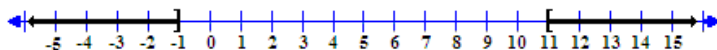
$$\frac{-11}{-1} \leq \frac{-x}{-1} \leq \frac{1}{-1}$$

Divide everything by -1 to get x by itself. It is important to remember that the signs must flip because we divided by a negative number.

$$11 \leq x \leq -1$$

The variable has been isolated, but this is not the solution. To be in standard form, the smaller number must come first.

$$-1 \geq x \geq 11$$

Rewriting the inequality place the smaller number on the left and larger on the right. Make sure when this is done, that the sign are correctly oriented. The signs would move with the numbers. As shown below.

$$11 \leq x \leq -1$$
$$-1 \geq x \geq 11$$

Here is the solution represented graphically on a number line.

$$(-\infty, -1] \cup [11, \infty)$$

Here is the solution in interval notation.