Finding the Equation of a Line

Before we cover some examples on finding the equation of a line, we will review a few of the basics beginning with the different forms of equations that can be used to represent a line.

The Standard Form of a line: $Ax + By = C$

In order for a line to be written in standard form, two conditions must be met.

- ***Only integral coefficients may be used. (This means no fractions).***
- ***The A term be positive.***

The Point Slope Form of a line: $y - y_1 = m(x - x_1)$

Although this form of the line looks complicated, it is very easy to work with. This form of the line is typically used to write the equation of a line in standard or slope intercept form. Since one of the conditions of a line being in standard form is that fractions may not be used, we can use this formula to eliminate those fractions. In order to use this formula, we must be given the coordinates for at least one point on the line. Given the coordinates in (x, y) form, we substitute the values of x and y into the equation for x_1 and y_1. The slope of the line, m, is also substituted into the equation. Once these values have been substituted, a simple algebraic process will allow us to write the equation of a line in standard form or slope intercept form.

The Slope Intercept Form of a line: $y = mx + b$

In order to write the equation of a line in slope intercept form, we need to know the slope of the line, m, and the y-intercept. If the y intercept is given just substitute the value into the equation along with the slope and the problem is finished. If not, we will either need to find the y-intercept algebraically, or use the point slope form of a line and solve for y.

Special Cases

There are two special cases when finding the equation of a line. Those would be simple vertical or horizontal lines.

The equation for a vertical line looks like:

$x = \#$

The equation for a horizontal line looks like:

$y = \#$

Examples:

Find the equation of the vertical line that contains the point $(5, -3)$.

The equation of a vertical uses the x value so first write $x =$. We then look at coordinates and ask ourselves, "what does x equal?"

The equation of the line is $x = 5$

Find the equation of the horizontal line that contains the point $(2, -7)$.

Since the equation of a horizontal line uses the y value, we write $y =$. Now look at the coordinates and ask "what does y equal?"

The equation of the line is $y = -7$

Find the equation of the line, in both standard form and slope intercept form, that has the slope m and contains the point P.

$$P=(-3,2),\quad m=\frac{2}{5}$$

Since the slope of the line is already given in this problem we will immediately use the point slope formula to write the equations of the line.

The Point Slope Form of a line: $y-y_1=m(x-x_1)$

This formula is extremely useful if your slope is a fraction.

$$y-y_1=m(x-x_1)$$

Using the point slope formula, the slope of the line and the values of x and y should be substituted into the formula.

$$y-2=\frac{2}{5}(x-(-3))$$

Once the -3 is substituted, we are left with a statement that says minus -3. This will be changed to +3.

$$y-2=\frac{2}{5}(x+3)$$

One of the objectives of the problem is to write the equation of the line in standard form. One of the conditions for writing the equation of a line in standard form is that only integral coefficients may be used. This means no fractions. The point slope formula makes it very easy to get rid of the fractional slope.

$$5[y-2]=5\left[\frac{2}{5}(x+3)\right]$$

The denominator of the slope is 5, therefore, If I multiply the entire equation by 5 I can eliminate the fraction.

$$5y-10=2(x+3)$$

As you can see, the 5's cancel out on the right side of the equation.

$$5y-10=2x+6$$

From this point, we will write the equation in standard form and slope intercept form.

Standard Form

Recall from our classroom discussion that the standard form of the line has two conditions. The first is that only integral coefficients may be used. The second is that the A term comes first.

The Standard Form of a line: $Ax+By=C$

$$5y-10=2x+6$$

In order to write this in standard form we must use inverse operations.

$$5y-10=2x+6$$
$$-5y \qquad\qquad -5y$$

Since the coefficient of the x is already positive, subtract 5y to both sides.

$$-10=2x+6-5y$$
$$-6 \qquad -6$$

Subtracting 6 from both sides gets the variables isolated.

$$-16=2x-5y$$

Using the symmetrical property we can rewrite the equation in the proper form.

Here is the equation of the line in standard form. $2x-5y=-16$

Slope Intercept Form

When writing the equation of a line in slope intercept form simply isolate and solve for y.

The Slope Intercept Form of a line: $y=mx+b$

$$5y-10=2x+6$$

Solve for y using inverse operations.

$$5y-10=2x+6$$
$$+10 \qquad +10$$

First isolate the y by adding 10 to both sides of the equation

$$5y=2x+16$$

The y term is now isolated.

$$\frac{5y}{5}=\frac{2x}{5}+\frac{16}{5}$$

Finally to solve for y divide the entire equation by 5.

Here is the equation of the line in slope intercept form. $y=\frac{2}{5}x+\frac{16}{5}$

When asked to find the equation of a line, the following example is the most difficult type of problem. As you read through the example, notice that with the exception of finding the slope of the line, the problem is just an extension of the previous example.

Find the equation of a line, in both standard form and slope intercept form that contains the points

$$(3,-2),\ (-6,12)$$

Since the slope of the line is not given, we must begin by finding the slope of the line. To do this, us the formula for finding the slope of a line: $m=\dfrac{y_2-y_1}{x_2-x_1}$

$$(3,-2),\ (-6,12)$$

The coordinates are given to us in the form of (x_1,y_1) and (x_2,y_2) so we substitute and solve.

$$\overset{x_1}{(3,}\overset{y_1}{-2)},\ \overset{x_2}{(-6,}\overset{y_2}{12)}$$

Write a small x_1,y_1 and x_2,y_2 above the coordinates to remind yourself which values to substitute.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{12-(-2)}{-6-3}=\frac{12+2}{-6-3}=-\frac{14}{9}$$

Substituting the values into our formula, we find the slope of the line. We can now use this and one of the sets of coordinates to find the equation of the line.

As we move on to using the point slope form of a line, we need to decide which set of coordinates to use. The first thing I look for is whether or not any of these sets contain a zero. If a zero is in one of the sets of coordinates, that set should be used. If there is no zero, the next thing to look for is positive numbers. If one set of coordinates has both positive numbers, that set should be used. This will help to avoid any possible mistake with a sign. If neither of these conditions is present then it doesn't matter which set of coordinates you use.

The Point Slope Form of a line: $y-y_1=m(x-x_1)$

Now using the point slope form of a line we can finish the problem.

$$y-y_1=m(x-x_1)$$

Using the point slope formula, the slope of the line and the values of x and y should be substituted into the formula. You may use either coordinate for this step.

$$y-(-2)=-\frac{14}{9}(x-3)$$

For this particular problem, I decided to use the coordinates $(3,-2)$. As stated earlier, either set of coordinates could have been used. I chose these numbers because they were smaller values which would make the work easier.

$$y+2=-\frac{14}{9}(x-3)$$

Once the -2 is substituted, we are left with a statement that says minus -2. This will be changed to +2.

$$9[y+2]=9\left[-\frac{14}{9}(x-3)\right]$$

The denominator of the slope is 9, therefore, If I multiply the entire equation by 9 I can eliminate the fraction.

$$9y+18=-14(x-3)$$

The 9's cancel out on the right side of the equation.

$$9y+18=-14x+42$$

From this point, we will write the equation in standard form and slope intercept form.

Problem continued on the next page.

Standard Form

Recall from our classroom discussion that the standard form of the line has two conditions. The first is that only integral coefficients may be used. The second is that the A term comes first.

The Standard Form of a line: $Ax + By = C$

$9y + 18 = -14x + 42$

In order to write this in standard form we must use inverse operations.

$$\begin{aligned} 9y + 18 &= -14x + 42 \\ +14x \qquad & \qquad +14x \end{aligned}$$

Since the coefficient of the x is negative, add 14x to both sides of the equation to move the variable terms to the left side of the equal sign.

$$\begin{aligned} 14x + 9y + 18 &= \quad 42 \\ -18 & \quad -18 \end{aligned}$$

Subtracting 18 from both sides gets the variables isolated.

$14x + 9y = 24$

The problem is now in $Ax + By = C$ form and both conditions have been met.

Here is the equation of the line in standard form. $14x + 9y = 24$

Slope Intercept Form

When writing the equation of a line in slope intercept form simply isolate and solve for y.

The Slope Intercept Form of a line: $y = mx + b$

$9y + 18 = -14x + 42$

Solve for y using inverse operations.

$$\begin{aligned} 9y + 18 &= -14x + 42 \\ -18 \qquad & \qquad -18 \end{aligned}$$

First isolate the y by subtracting 18 to both sides of the equation

$9y = -14x + 24$

The y term is now isolated.

$$\frac{9y}{9} = -\frac{14x}{9} + \frac{24}{9}$$

Finally to solve for y divide the entire equation by 9.

Here is the equation of the line in slope intercept form. $y = -\frac{14}{9}x + \frac{8}{3}$