The Remainder Theorem

When trying to find all zeros of a complex polynomial function, use the rational zero test to find all possible rational zeros. Each possible rational zero should then be tested using synthetic division. If one of these numbers work, there will be no remainder to the division problem. For every potential zero that works, there may be others that do not. Are these just useless? The answer is no. Every time synthetic division is attempted, we are actually evaluating the value of the function at the given x coordinate. When there is no remainder left, a zero of the function has just been found. This zero is an x intercept for the graph of the function. If the remainder is any other number, a set of coordinates on the graph has just been found. These coordinates would aid in graphing the function.

Let $P_{(x)}$ be a polynomial of positive degree n. Then for any number c,

$$P_{(x)} = Q_{(x)} \cdot (x - c) + P_{(c)},$$

Where $Q_{(x)}$ is a polynomial of degree n-1.

This simply means that if a polynomial $P_{(x)}$ is divided by $(x - c)$ using synthetic division, the resultant remainder is $P_{(c)}$.

When trying to find the zeros of the function $f_{(x)} = 2x^4 + 7x^3 - 4x^2 - 27x - 18$, first find all possible rational zeros. Then evaluate each one. Here is one particular example.

$$
\begin{array}{r|rrrrr}
-2 & 2 & 7 & -4 & -27 & -18 \\
   & 0 & -4 & -6 & 20 & 14 \\
\hline
   & 2 & 3 & -10 & -7 & -4
\end{array}
$$

In this example, (-2) is evaluated using synthetic division to see if it was a zero of the function. It turns out that (-2) is not a zero of the function, because there is a remainder of (-4).

Therefore, Using the Remainder Theorem, it can be stated that $f_{(-2)} = -4$.

You already saw that dividing by (-2) yields a result of (-4), giving us the statement $f_{(-2)} = -4$. This can be proven algebraically as follows.

$$f_{(-2)} = 2(-2)^4 + 7(-2)^3 - 4(-2)^2 - 27(-2) - 18$$

$$f_{(-2)} = 32 - 56 - 16 + 54 - 18$$

$$f_{(-2)} = -4$$