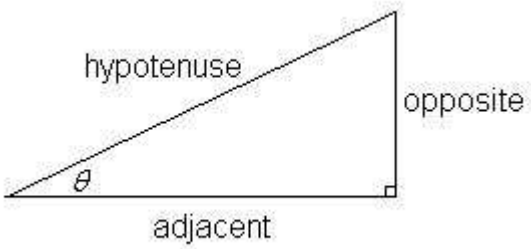
Basic Trigonometric Functions

There are six basic trigonometric functions used in trigonometry. Below are the names of the six functions and their three letter abbreviation.

Sine (sin)	**Cosecant (csc)**
Cosine(cos)	**Secant (sec)**
Tangent (tan)	**Cotangent (cot)**



These six trigonometric functions are used to evaluate acute angles in a right triangle. The ratio of the lengths of two sides of a right triangle will be used to evaluate a given angle θ. We will go back to something introduced in geometry for this.

Soh-Cah-Toa

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

The second set of trigonometric functions, are reciprocal functions.

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Therefore, it follows that...

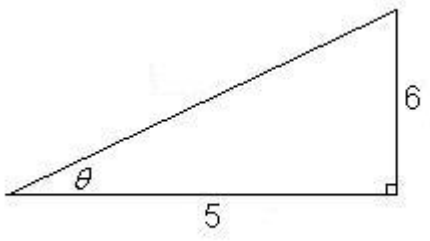
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Exercises that require finding the exact value of the six trigonometric functions follow on the next few pages. Sometimes, only two of the three sides of a triangle will be given requiring the student to find the third.

When given the lengths of two sides of a right triangle, how can the length of the third side be found?

Example

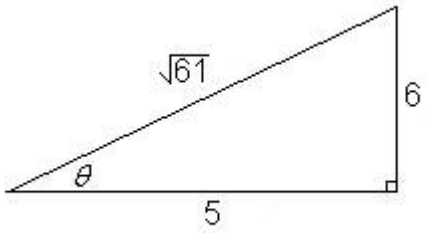
Find the exact values of the six trigonometric functions of θ.



In this example, the length of the side adjacent to the angle θ measures 5 units, while the opposite side measures 6 units. In order to find the missing side, the Pythagorean Theorem must be used.

$$a^2 + b^2 = c^2$$
$$(5)^2 + (6)^2 = c^2$$
$$25 + 36 = c^2$$
$$61 = c^2$$
$$\sqrt{61} = c$$

Using the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{61}$ units.



Once the value of the hypotenuse is found, we can find the exact value of the six trigonometric functions of the angle θ.

$$\sin\theta = \frac{6}{\sqrt{61}} = \frac{6\sqrt{61}}{61} \qquad \csc\theta = \frac{\sqrt{61}}{6}$$
$$\cos\theta = \frac{5}{\sqrt{61}} = \frac{5\sqrt{61}}{61} \qquad \sec\theta = \frac{\sqrt{61}}{5}$$
$$\tan\theta = \frac{6}{5} \qquad \cot\theta = \frac{5}{6}$$

Evaluate the first three functions using Soh-Cah-Toa.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Evaluate second set of functions by finding the reciprocals of the first three.
Do not forget to rationalize any denominators if needed.

$$\sin\theta = \frac{6\sqrt{61}}{61} \qquad \csc\theta = \frac{\sqrt{61}}{6}$$
$$\cos\theta = \frac{5\sqrt{61}}{61} \qquad \sec\theta = \frac{\sqrt{61}}{5}$$
$$\tan\theta = \frac{6}{5} \qquad \cot\theta = \frac{5}{6}$$

Here are the exact values of the six trigonometric functions of the angle θ. Radicals are left in the solutions because we need the <u>exact</u> values, not estimates.