Completing the Square

When solving a quadratic equation by completing the square, the goal is to create a perfect square binomial on the left side of the equal sign. A perfect square binomial resembles the following.

x^2+6x+9

You can see that x^2+6x+9 ***is really*** $(x+3)^2$***, therefore, it is a perfect square; just as 4 is the perfect square of 2.***

****The most important rule when completing the square is "You can only complete the square when the leading coefficient is one." If the leading coefficient is any other number, you will need to multiply the entire equation by its' reciprocal. This will yield a leading coefficient of one. Observe the following example.***

$x^2+8x-4=0$	***Begin with the quadratic equation in standard form*** $ax^2+bx+c=0$
$x^2+8x \quad =4$	***Begin by adding 4 to both sides of the equation***
$\left(\frac{b}{2}\right)^2=\left(\frac{8}{2}\right)^2=(4)^2=16$	***Now we need to create a perfect square binomial. We need to find the missing number after the 8x, so we need to evaluate*** $\left(\frac{b}{2}\right)^2$***.***
$x^2+8x+16=4+16$	***Once we evaluate b over 2 squared, we add the result to both sides of the equation.***
$(x+4)^2=20$	***Now, on the left side of the equal sign is the perfect square in factored form. When you evaluate b over 2, that number, in this case 4, is what goes in the factor.***
$\sqrt{(x+4)^2}=\pm\sqrt{20}$	***Now take the square root of both sides of the equation. Do not forget to use*** $\pm$ ***on the right side.***
$(x+4)=\pm 2\sqrt{5}$	***Simplify the radical if possible.***
$x=-4\pm 2\sqrt{5}$	***Now subtract 4 to both sides. Since we do not have two separate rational solutions, the answers will be written as algebraic expressions. For now, the solution may be left like this, however in the future, it will be necessary to use each separately.***

Here is a more complicated example.

$$2x^2 - 7x + 3 = 0$$

Here is the quadratic equation in standard form $ax^2 + bx + c = 0$

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Begin by multiplying the entire equation by ½.

$$x^2 - \frac{7}{2}x \quad = -\frac{3}{2}$$

Subtract $\frac{3}{2}$ ***to both sides.***

$$\left(\frac{b}{2}\right)^2 = \left(-\frac{7}{2} \div 2\right)^2 = \left(-\frac{7}{2} \cdot \frac{1}{2}\right)^2 = \left(-\frac{7}{4}\right)^2 = \frac{49}{16}$$

Now we need to create a perfect square binomial. We need to find the missing number, so we need to evaluate $\left(\frac{b}{2}\right)^2$***.***

$$x^2 - \frac{7}{2}x + \frac{49}{16} = -\frac{3}{2} + \frac{49}{16}$$

Once we evaluate b over 2 squared, we add the result to both sides of the equation.

$$\left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$$

Now, on the left side of the equal sign is the perfect square in factored form. When b over 2 is evaluated, the result is -7/4. That is the number that goes in the binomial on the left.

$$\sqrt{\left(x - \frac{7}{4}\right)^2} = \pm\sqrt{\frac{25}{16}}$$

Now take the square root of both sides of the equation.
Do not forget to use $\pm$ ***on the right side.***

$$x - \frac{7}{4} = \pm\frac{5}{4}$$

Simplify the radical if possible.

$$x = \frac{7}{4} \pm \frac{5}{4}$$

$$x = \frac{7}{4} + \frac{5}{4} \quad and \quad x = \frac{7}{4} - \frac{5}{4}$$

$$x = \frac{12}{4} \qquad x = \frac{2}{4}$$

$$x = 3 \quad and \quad x = \frac{1}{2}$$

Now add 7/4 to both sides. And simplify the solutions if possible.

In this example, once everything is divided by 2, the leading coefficient is one. This means the "completing the square" method can now be used. This is a complicated problem because the b term is a fraction. Which means all those rules regarding fractions will be coming into play. Be mindful of such problems in the future.

Being able to complete the square is VITAL as it will be used many times in the future!!!