Coterminal Angles

Coterminal Angles *are angles who share the same initial side and terminal sides. Finding coterminal angles is as simple as adding or subtracting* $360°$ *or* 2π *to each angle, depending on whether the given angle is in degrees or* *radians.* *There are an infinite number of coterminal angles that can be found. Following this procedure, all coterminal angles can be found. This is the basis for solving trigonometric equations which will be done in the future.*

Radians are often used in trigonometry to represent angle measures. Radian measures are very common in calculus, so it is important to have an understanding of what a radian is.

Definition of a Radian

A radian is the measure of a central angle θ **that intercepts an arc s equal in length to the radius r of the circle. There are** 2π **, or approximately 6.28318, radians in a complete circle. Thus, one radian is about 57.296 angular degrees.**
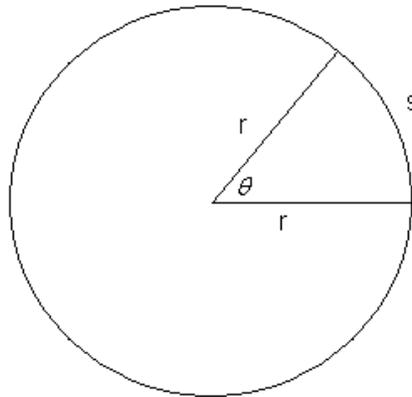
In other words, if we were to take the length of the radius of a circle, and lay in on the edge of a circle, that length would be one radian.

The number π *is often used when describing radian measure. The approximate value of* π *is 3.14159... A plane, in trigonometry, can not only be divided into quadrants using degree measures, but radian as well. Observe the following moving in a counterclockwise direction.*
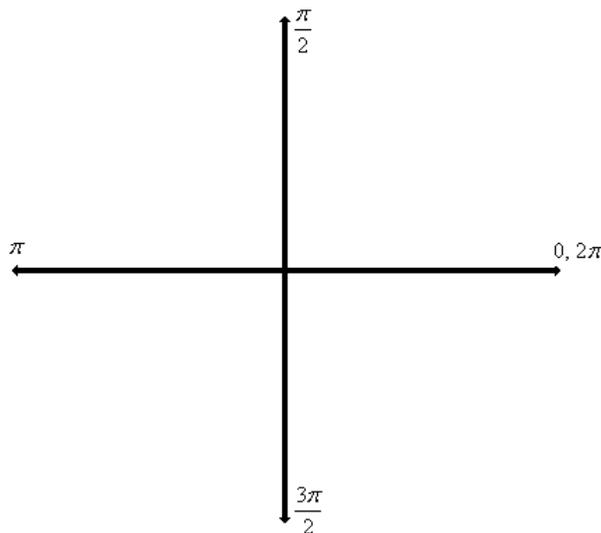
When studying trigonometry, angles are usually measured in radians.

In relation to degrees, 180° is π radians. This means 2π radians is 360°. Since the approximate value of π is 3.14159..., it follows that 360° is approximately 6.28318...radians. <u>*When evaluating angles in trigonometry or calculus, always be aware of whether the question is given in terms of degrees or radians. If no degree symbol is given, the problem is in radians.*</u>

Examples of finding coterminal angles

Find one positive angle that is coterminal to 50°.

Since the terminal side of a 50° angle resides in quadrant I, the terminal side of its coterminal angle must share that side. This means the new angle would make one complete revolution before having its terminal side come to rest at the same place.

Therefore, to find the coterminal angle to a 50° angle, just add 360°.

$$50° + 360° = 410°$$

Below is the graphical representation of a 50° angle. Since its coterminal angle must share the same terminal side, it is reasonable to create a new angle that makes one complete revolution and ends up in the same place.

To find the coterminal angle of a 50° angle, add 360°. It would follow that 50°+360°=410°. A 410° angle is illustrated below. From the graphical representation of the angle, we can conclude that these two angle do indeed share the same terminal side, meaning they are coterminal.
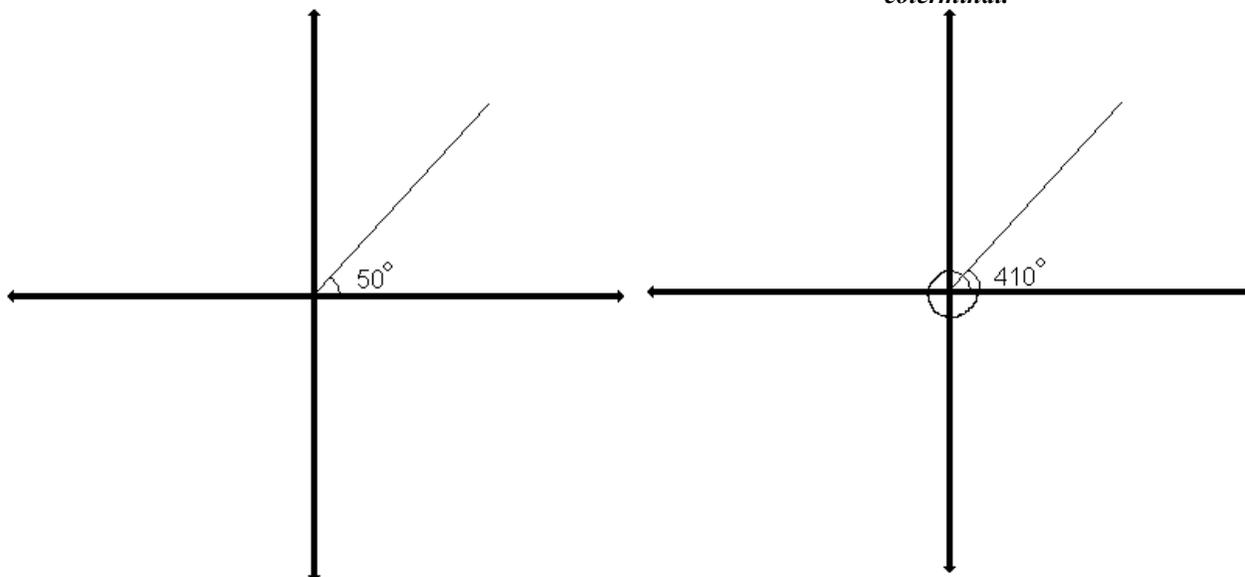
Find one positive angle that is coterminal to 110°.

$$110° + 360° = 470°$$

Find two positive angles that are coterminal to -30°.

$$-30° + 360° = 330°$$
$$330° + 360° = 690°$$
In this case, the two positive coterminal angles to -30° are 330° and 690°.

If more than one positive coterminal angle needs to be found, simply add another 360°. This would essentially make the new angle complete two full revolutions before its terminal side comes to rest.

Find one negative angle that is coterminal to 30°.

A negative angle moves in a clockwise direction. In this case, to find the negative coterminal angle, subtract 360° from 30°.

$$30° - 360° = -330°$$

Below is a 30° angle in standard position. This angle opens in a counterclockwise direction. *Here is a -330° angle. As the angle opens clockwise, it shares the same terminal side as the 30° angle.*
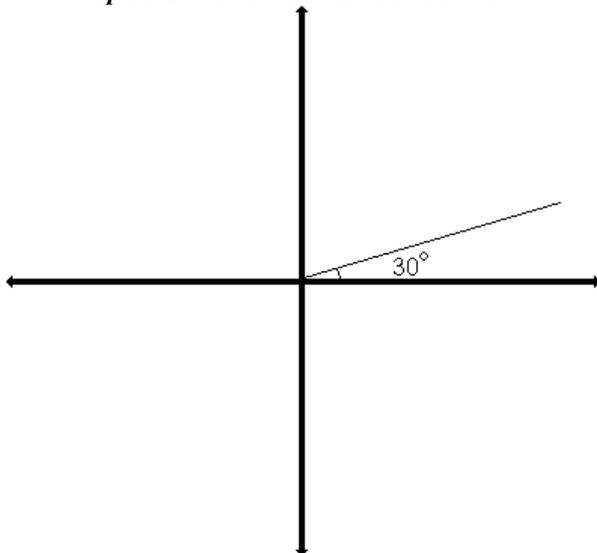
30°   -330°
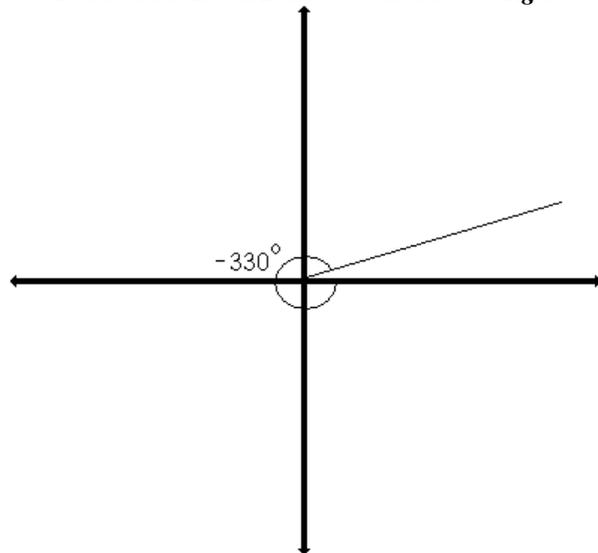
Find one negative angle that is coterminal to 150°.

$$150° - 360° = -210°$$

Find one negative angle that is coterminal to 415°.

$$415° - 360° = 55°$$

Although 55° is a coterminal angle to 415°, this is not a solution to the problem. The problem specifically asked for a negative angle, so the process needs to take place one more time.

$$55° - 360° = -305°$$

These were all examples of finding coterminal angles. If the initial angle is given in the form or radians, add or subtract 2π instead of 360°.

Find a positive and negative angle that is coterminal to an angle that is $\dfrac{\pi}{6}$ radians.

$$\frac{\pi}{6} + 2\pi$$

$$\frac{\pi}{6} + \frac{12\pi}{6}$$

$$\frac{13\pi}{6}$$

Adding 2π to the original angle yields the positive coterminal angle.

$$\frac{\pi}{6} - 2\pi$$

$$\frac{\pi}{6} - \frac{12\pi}{6}$$

$$-\frac{11\pi}{6}$$

By subtracting 2π from the original angle, the negative coterminal angle has been found.

Find two positive angles that are coterminal to an angle that is $\dfrac{11\pi}{2}$ radians.

$$\frac{11\pi}{2} - 2\pi$$

$$\frac{11\pi}{2} - \frac{4\pi}{2}$$

$$\frac{7\pi}{2}$$

$$\frac{7\pi}{2} - 2\pi$$

$$\frac{7\pi}{2} - \frac{4\pi}{2}$$

$$\frac{3\pi}{2}$$

Since $\dfrac{11\pi}{2}$ is more than one complete revolution, 2π was subtracted from the initial angle yielding a coterminal angle of $\dfrac{7\pi}{2}$. This is still at least one full revolution, so 2π was subtracted yet again. This process resulted in the two positive coterminal angles of $\dfrac{7\pi}{2}$ and $\dfrac{3\pi}{2}$.