Find the equation of a parabola that passes through (1,0), (2,-1) and (3,0).

$$(1,0),\ (2,-1),\ (3,0)$$

Given

$$y = ax^2 + bx + c$$

Use the general form of a parabola

$$\begin{aligned} 0 &= a + b + c \\ -1 &= 4a + 2b + c \\ 0 &= 9a + 3b + c \end{aligned}$$

Substitute the values of x and y into the general form of a parabola for each set of coordinates setting up 3 equations.

$$\begin{array}{ll} Eq_1 & a + b + c = 0 \\ Eq_2 & 4a + 2b + c = -1 \\ Eq_3 & 9a + 3b + c = 0 \end{array}$$

You have now created a system of equations in 3 variables.

$$\begin{array}{ll} Eq_1 & a + b + c = 0 \\ Eq_2 & 4a + 2b + c = -1 \end{array} \qquad \begin{array}{ll} Eq_1 & a + b + c = 0 \\ Eq_3 & 9a + 3b + c = 0 \end{array}$$

One of these equations will be used twice. It doesn't matter which. In this case, equation 1 will be used twice.

$$\begin{array}{ll} -Eq_1 & -a - b - c = 0 \\ +Eq_2 & 4a + 2b + c = -1 \\ \hline Eq_4 & 3a + b = -1 \end{array} \qquad \begin{array}{ll} -Eq_1 & -a - b - c = 0 \\ +Eq_3 & 9a + 3b + c = 0 \\ \hline Eq_5 & 8a + 2b = 0 \end{array}$$

Multiplying equation 1 by -1 and combining the equations yielded two new equations. You <u>must</u> get rid of the same variable each time.

$$\begin{array}{ll} Eq_4 & 3a + b = -1 \\ Eq_5 & 8a + 2b = 0 \end{array}$$

Now we have a system of equations in two variables.

$$\begin{array}{ll} -2Eq_4 & -6a - 2b = 2 \\ +\ Eq_5 & 8a + 2b = 0 \\ \hline & 2a = 2 \\ & a = 1 \end{array}$$

Multiply equation 4 by -2, and add the result to equation 5. This yields a numerical value for the variable a.

$a = 1$

Substitute into equation 5

$$\begin{aligned} 8(1) + 2b &= 0 \\ 2b &= -8 \\ b &= -4 \end{aligned}$$

$a = 1 \quad b = -4$

Substitute into equation 1

$$\begin{aligned} (1) + (-4) + c &= 0 \\ -3 + c &= 0 \\ c &= 3 \end{aligned}$$

Once the value of the first variable is found, substitute that number, in this case 1, into either equation 4 or 5 and solve for the remaining variable. Now that the value of two of the variables is known, go back to equation 1, substitute and find the value of the third variable.

$$a = 1 \quad b = -4 \quad c = 3$$

The value of all three variables has now been found.

$$y = x^2 - 4x + 3$$

Substitute these values into the general form of a parabola, and the equation of the parabola that passes through (1,0), (2,-1) and (3,0) is found.