Foundation to Solving Trigonometric Equations

It is also possible to use the unit circle going backwards. The previous exercises require a student to evaluate the trigonometric function of an angle using the unit circle. The samples below require the student to work backwards. Given the value of the trigonometric function of an angle θ, refer to the unit circle, and find the angle θ that makes the statement true. Given a statement such as $\sin\theta = \frac{1}{2}$, we will work backwards to try to determine the angle θ that would make the statement true.

<u>For Example</u>

Find all values of θ in the interval $\left(0, 2\pi\right]$ that make the statement $\cos\theta = \frac{1}{2}$ true.

Referring to the unit circle, look for x coordinates of $\frac{1}{2}$. This happens in two places, $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

As a result, $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

Find all values of θ in the interval $\left(0, 2\pi\right]$ that make the statement $\tan\theta = -\sqrt{3}$ true.

The only coordinate that has $\sqrt{3}$ in it is $\frac{\sqrt{3}}{2}$. That rules out any and all of the 45° angles or multiples thereof.

It would therefore follow, that the $\sqrt{3}$ is a result of either $\left(\frac{\sqrt{3}}{2} \div \frac{1}{2}\right)$ or $\left(\frac{1}{2} \div \frac{\sqrt{3}}{2}\right)$.

Working backwards will reveal what the result is.

$$\left(\frac{\sqrt{3}}{2} \div \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{2} \cdot \frac{2}{1}\right) = \sqrt{3} \qquad \left(\frac{1}{2} \div \frac{\sqrt{3}}{2}\right) = \left(\frac{1}{2} \div \frac{2}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

According to the work above a y value of $\frac{\sqrt{3}}{2}$ divided by an x value of $\frac{1}{2}$ would yield a result of $\sqrt{3}$.

This occurs at $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ and $\frac{5\pi}{3}$.

Since tangent is y divided by x, and in this case, the tangent of θ is negative, it would stand to reason that one of the coordinates used will be a negative, while the other is a positive. As discussed earlier, the tangent of θ will be negative in quadrants II and IV.

The solution to this problem is: $\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$