Maximum and Minimum Values

In some problems you will be asked about the maximum or minimum <u>value</u> of a function. Whenever you are asked about the value of a function, we are always referring to the y value. In the case of a parabola, the maximum or minimum value will be the y value of the vertex. The x value of the vertex is where you get that maximum or minimum. For example, let's look at a business; where the profit margin is given by the formula $P = -(x-32)^2 + 8000$***, where x is the number of units sold, and P is the profit. According to the equation for the graph of the function, the parabola will open down, and the vertex will be located at*** $(32, 8000)$***. This means the maximum profit is $8000. The business will achieve maximum profitability if 32 units are sold. So the x value is where the max occurred, and the y value is simply what the maximum or minimum is depending on the case and function.***

Therefore, it can be concluded…

If the parabola opens down, meaning $a < 0$***, the function will have a maximum.***
If the parabola opens up, meaning $a > 0$***, the function will have a minimum.***

In either case, the maximum or minimum <u>value</u> of the function is the y value of the vertex.

When asked to find the maximum or minimum value of a function, or where this occurs, it is not necessary to actually write the function in standard form to find the vertex. As previously stated, working from the general form of a quadratic function, $y = ax^2 + bx + c$***, the vertex can be found. Evaluating for*** $-\frac{b}{2a}$***, will reveal the x value of the vertex. This tells you where the maximum or minimum value occurs. Keeping our previous example in mind, you could simply be asked how many units need to be sold to maximize profits. If you are being asked were the maximum or minimum occur,*** $-\frac{b}{2a}$ ***will give you the solution to the problem. If you are asked to take it a step further and find the maximum or minimum value, plug*** $-\frac{b}{2a}$***, in this case 32, back into the problem to find the y value of the vertex. Why is this important? Consider the following example.***

Find the maximum value of the function $y = -0.25x^2 + 5.62x + 2$.
You can see putting this in standard form will be relatively complicated. Even more so if the leading coefficient is a decimal value not easily converted to a fraction. Following this procedure, the maximum value of the function is found.

$$-\frac{b}{2a} = -\frac{(5.62)}{2(-0.25)} = 11.24$$

now substitute 11.24 *for* x,

$$y = -0.25(11.24)^2 + 5.62(11.24) + 2$$

$$y = 33.5844$$

The maximum value of this function is 33.5844