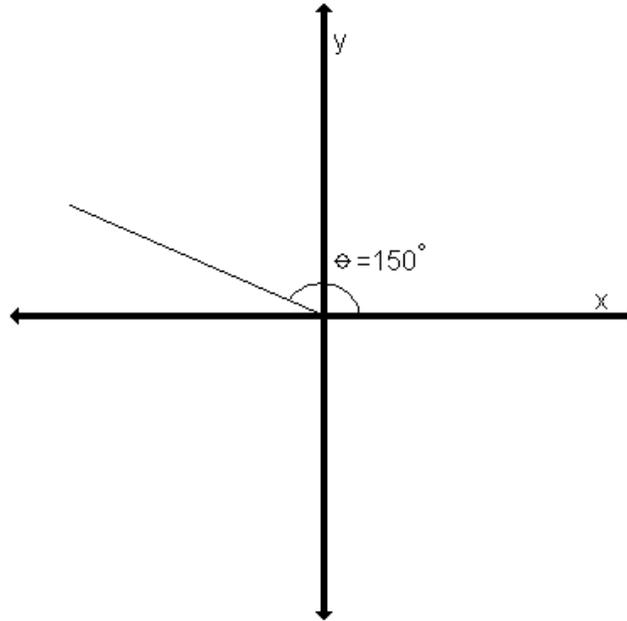


Reference Angles

The angles we will evaluate in trigonometry will always rest between the terminal side of that angle, and the x axis. Below is a graphical representation of a 150° angle.

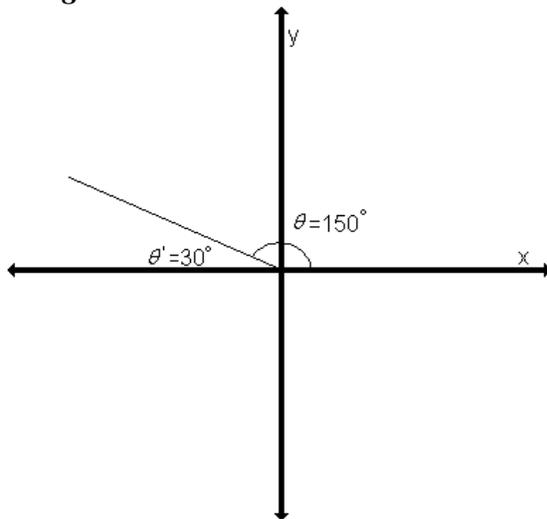


In the illustration above, the terminal side of a 150° angle, in standard position, resides in quadrant II. In order to evaluate the six trigonometric functions of an angle θ , we are required to use a reference angle.

A reference angle is the acute angle θ' (read as theta prime) formed by the terminal side of the angle θ , and the x axis.

REFERENCE ANGLES ARE ALWAYS DRAWN IN RELATION TO THE X AXIS.

Therefore, to evaluate the six trigonometric functions of a 150° angle in standard position, a 30° angle will be used.



Since the horizontal in quadrant II represents 180° , evaluate $180^\circ - 150^\circ$ to find θ' , which is this case is 30° . This is not the same method that will be used for every angle.

If for example, we need to find a reference angle of a 200° angle, evaluate $200^\circ - 180^\circ = 20^\circ$.

Here are some guidelines for finding reference angles. The method used to find a reference angle depends on the quadrant in which the terminal side of the angle resides.

- If the terminal side of an angle θ rests in quadrant I, $\theta' = \theta$.
- If the terminal side of an angle θ rests in quadrant II, $\theta' = 180^\circ - \theta$ or $\theta' = \pi - \theta$.
- If the terminal side of an angle θ rests in quadrant III, $\theta' = \theta - 180^\circ$ or $\theta' = \theta - \pi$.
- If the terminal side of an angle θ rests in quadrant IV, $\theta' = 360^\circ - \theta$ or $\theta' = 2\pi - \theta$.

Example 1

The angle $\theta = -150^\circ$. Find the reference angle θ' .

Begin by finding the positive coterminal angle to a -150° angle.

$$-150^\circ + 360^\circ = 210^\circ$$

The terminal side of a 210° angle resides in quadrant III. Therefore, to find the reference angle use $\theta' = \theta - 180^\circ$.

$$210^\circ - 180^\circ = 30^\circ$$

$$\theta' = 30^\circ$$

Example 2

The angle $\theta = 2.5$. Find the reference angle θ' .

In this case, there is no degree symbol. This means the measure of angle θ is 2.5 radians.

First use the formula to convert radians to degrees.

$$2.5 \cdot \frac{180^\circ}{\pi} \approx 143.239^\circ$$

Since the question was asked in terms of radians, the answer must be given in the same way. Converting the angle measure to degrees allows us to get a clear picture of where the terminal side of the angle will lie. In this case, the terminal side of an angle that measures 2.5 radians lies in quadrant II.

To find the reference angle, evaluate $\theta' = \pi - \theta$.

$$\theta' = \pi - 2.5$$

$$\theta' \approx 0.642 \text{ radians}$$

As illustrated in example 1 on the previous page, it is sometimes necessary to find a coterminal angle first. If θ is negative, first find the coterminal angle, then use that to find the reference angle. If the measure of the original angle is given in degrees, its reference angle must also be in degrees. If the measure of the original angle is given in radians, then the reference angle found must also be in radians. Exact solutions should be found whenever possible. In example 2 on the previous page, it was impossible to give an exact solution, because the measure of angle θ did not include π . Therefore, a decimal approximation had to be made.