Trigonometric Equations

Many of the skills used for solving algebraic equations will be used to solve trigonometric equations. Trigonometric equations are solved using inverse operations. The ultimate objective of solving trigonometric equations is to find the angle that makes the statement true.

Examples

Solve the following equations in the interval $(0, 2\pi]$.

$$2\sin\theta + 1 = 0$$

$$2\sin\theta + 1 = 0$$
$$2\sin\theta = -1$$
$$\sin\theta = -\frac{1}{2}$$

Begin solving a trigonometric equation by isolating the trigonometric function involved.

$$\theta = \frac{7\pi}{6},\ \frac{11\pi}{6}$$

At this point, find all angles in the interval $(0, 2\pi]$ which make the equation true.

Solve the following equations in the interval $(0, 2\pi]$.

$$\sin^2\theta - 1 = 0$$

When dealing with trigonometric equations, $\sin^2\theta$ is the same type of thing as x^2

In other words, $(\sin 30°)^2$ is written as $\sin^2 30°$.

$$\sin^2\theta - 1 = 0$$
$$\sin^2\theta = 1$$
$$\sqrt{\sin^2\theta} = \pm\sqrt{1}$$
$$\sin\theta = \pm 1$$

Solve the equation by isolating the trigonometric function, then taking the square root of both sides. Do not forget to use $\pm$ when finding the solution.

$$\theta = \frac{\pi}{2},\ \frac{3\pi}{2}$$

Here are the angles in the interval $(0, 2\pi]$ that satisfy the equation.

Every algebraic equation that was solved previously will play into your ability to solve some of the trigonometric equations you may face.

For example, the trigonometric equation $\csc^2\theta + 3\csc\theta + 2 = 0$ is very similar to the quadratic equation $x^2 + 3x + 2 = 0$. This trigonometric equation would be solved in the same manner as the algebraic. Factor the equation out to $(\csc\theta + 1)(\csc\theta + 2) = 0$. Then, proceed to set each factor equal to zero and solve.

Here are a couple of examples of trigonometric equations involving the reciprocal functions.

Solve the following equations in the interval $(0, 2\pi]$. $\sec\theta + 2 = 0$	
$\sec\theta + 2 = 0$ $\sec\theta = -2$ $\frac{1}{\cos\theta} = -2$ $\cos\theta = -\frac{1}{2}$	***Begin by isolating the trigonometric function. Once that is done, raise both sides of the equation to the negative first power essentially taking the reciprocal of both sides. This yields one of the basic three trigonometric functions.***
$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$	***The angles in the interval*** $(0, 2\pi]$ ***that satisfy the equation are here.***
Solve the following equations in the interval $(0, 2\pi]$. $3\csc^2\theta + 6 = 10$	
$3\csc^2\theta + 6 = 10$ $3\csc^2\theta = 4$ $\csc^2\theta = \frac{4}{3}$ $\sqrt{\csc^2\theta} = \pm\sqrt{\frac{4}{3}}$ $\csc\theta = \pm\frac{2}{\sqrt{3}}$ $\frac{1}{\sin\theta} = \pm\frac{2}{\sqrt{3}}$ $\sin\theta = \pm\frac{\sqrt{3}}{2}$	***Once again, the trigonometric function is isolated.*** ***Taking the square root of both sides always results in*** $\pm$ ***answers.*** ***Since the cosecant function is really the reciprocal of the sine function, both sides are flipped over.***
$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$	***Here are the angles in the interval*** $(0, 2\pi]$ ***that satisfy the equation.***