<u>The Standard Form of a Parabola</u>

In this class we will be graphing quadratic functions using the standard form of a parabola.

$$y = a(x-h)^2 + k$$

This is the standard form of a parabola, where a, h and k represent real number constants.

In Algebra I, students are taught to graph a parabola in the form $y = ax^2 + bx + c$***, by first finding the vertex. This meant the student had to first evaluate*** $-\frac{b}{2a}$ ***to find the x value of the vertex. The student would then proceed to evaluate*** $f_{\left(-\frac{b}{2a}\right)}$ ***to find the y value of the vertex. In other words, first find the x value of the vertex, then plug it back into the problem to find the y value. From here the student would set up a table to find any additional coordinates. The results would then be graphed. From this point forward, you will be given the function in the form*** $y = ax^2 + bx + c$***, and rewrite it in the standard form,*** $y = a(x-h)^2 + k$***. Once in this form, it is easier to identify the vertex, x-intercepts and any other information you need to graph the function.***

Once the function is in standard form, the vertex is given by (h,k)***. The*** a ***value determines if the function opens up or down, and gives some indication as to how wide or narrow the graph will be. The*** h ***value dictates how the function shifts horizontally. Remember, in the standard form of a parabola you see*** $-h$***, but in the vertex you see*** h***. This means the x value of the vertex is the opposite of what you see when it is standard form. Remember P.L.N.R., Positive Left Negative Right, and it will help you with the shift. The*** k ***value of the function is the y value of the vertex, thereby making the graph of the function shift vertically.***

As stated earlier, the process of completing the square is vital in mathematics. This is the method used to put the equation of a parabola in standard form. As you complete the square of the function $y = ax^2 + bx + c$***, the dependant variable,*** y***, <u>must</u> be kept in the problem. The*** y ***variable will keep track of the*** a ***term for you since the completing the square method cannot be used if the leading coefficient is any number other that one. The*** y ***variable holds on to the number for you, then gives it back at the end. The following example shows this happening.***

Example

$$y = -2x^2 + 8x - 14$$

Remember to keep y in the problem.

$$\frac{y}{-2} = \frac{-2x^2}{-2} + \frac{8x}{-2} - \frac{14}{-2}$$

To put into standard form, we will need to complete the square, which means the coefficient for the x^2 term must be one, so divide the whole equation by -2.

$$\frac{y}{-2} = x^2 - 4x + 7$$

Division by -2 yields this equation

$$\frac{y}{-2} - 7 \quad = x^2 - 4x$$

Subtract 7 to both sides of the equation

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

To find the missing term evaluate $\left(\frac{b}{2}\right)^2$ to create a perfect square binomial on the right.

$$\frac{y}{-2} - 7 + 4 = x^2 - 4x + 4$$

Add the result, 4, to both sides of the equation

$$\frac{y}{-2} - 3 = (x-2)^2$$

Simplify

$$\frac{y}{-2} = (x-2)^2 + 3$$

Now we need to begin isolating the y, so add 3 to both sides of the equation

$$-2\left(\frac{y}{-2}\right) = -2\left[(x-2)^2 + 3\right]$$

To isolate the y, multiply the entire equation by -2.

$$y = -2(x-2)^2 - 6$$

Notice the y was just holding on to -2 throughout the entire process. The equation for this parabola is now in standard form.

As previously stated, this skill is vital. In the future, it will be necessary to complete the square twice within the same problem. This is the procedure used when putting the equation of a circle, ellipse, or hyperbola into standard form. In each case, you will be required to complete the square for two different variables in order to write the equations in standard form.