<u>**Using the Unit Circle**</u>

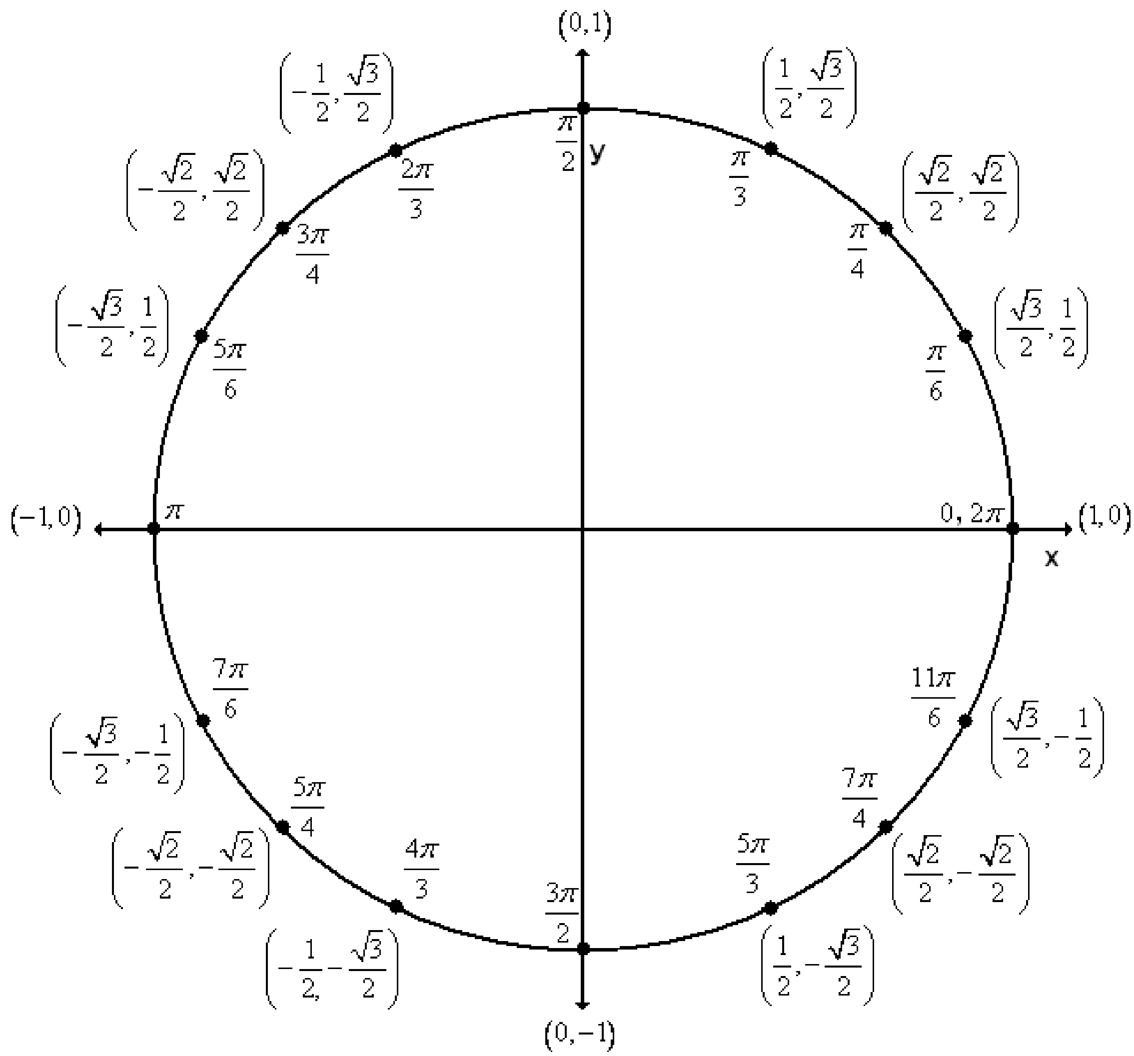
The hypotenuse of the unit circle has a length of one unit. Therefore, whenever any angle needs to be evaluated using any of the trigonometric functions, the following will be used.

$$\sin\theta = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Think of the sine of an angle being the y value of the coordinate, the cosine of an angle as being the x value of the coordinate, and the tangent of an angle being x over y. Then the reciprocals will be taken for the second set of functions.



When reading through the following examples, refer to the unit circle on the previous page.

Examples

Find the exact value of the six trigonometric functions for $\frac{4\pi}{3}$.

$\sin\frac{4\pi}{3}=-\frac{\sqrt{3}}{2}$ $\qquad$ $\csc\frac{4\pi}{3}=-\frac{2}{\sqrt{3}}=-\frac{2\sqrt{3}}{3}$

$\cos\frac{4\pi}{3}=-\frac{1}{2}$ $\qquad$ $\sec\frac{4\pi}{3}=-2$

$\tan\frac{4\pi}{3}=\left(-\frac{\sqrt{3}}{2}\div-\frac{1}{2}\right)=\left(-\frac{\sqrt{3}}{2}\cdot-2\right)=\sqrt{3}$ $\qquad$ $\cot\frac{4\pi}{3}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

$\sin\frac{4\pi}{3}=-\frac{\sqrt{3}}{2}$ $\qquad$ $\csc\frac{4\pi}{3}=-\frac{2\sqrt{3}}{3}$

$\cos\frac{4\pi}{3}=-\frac{1}{2}$ $\qquad$ $\sec\frac{4\pi}{3}=-2$

$\tan\frac{4\pi}{3}=\sqrt{3}$ $\qquad$ $\cot\frac{4\pi}{3}=\frac{\sqrt{3}}{3}$

Locate the coordinates at $\frac{4\pi}{3}$. The y value at $\frac{4\pi}{3}$ is $-\frac{\sqrt{3}}{2}$. The x value at $\frac{4\pi}{3}$ is $-\frac{1}{2}$. Therefore, sin $\frac{4\pi}{3}=-\frac{\sqrt{3}}{2}$, and cos $\frac{4\pi}{3}=-\frac{1}{2}$. Tangent is y over x, so the quotient of the two is found. The remaining three are evaluated using the reciprocal. All denominators <u>must</u> be rationalized. The exact value of the function means do not use decimal approximations..

Find the exact value of the six trigonometric functions for $-\frac{11\pi}{6}$.

$\sin-\frac{11\pi}{6}=\frac{1}{2}$ $\qquad$ $\csc-\frac{11\pi}{6}=2$

$\cos-\frac{11\pi}{6}=\frac{\sqrt{3}}{2}$ $\qquad$ $\sec-\frac{11\pi}{6}=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$

$\tan-\frac{11\pi}{6}=\left(\frac{1}{2}\div\frac{\sqrt{3}}{2}\right)=\left(\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\right)=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$ $\qquad$ $\cot-\frac{11\pi}{6}=\sqrt{3}$

$\sin-\frac{11\pi}{6}=\frac{1}{2}$ $\qquad$ $\csc-\frac{11\pi}{6}=2$

$\cos-\frac{11\pi}{6}=\frac{\sqrt{3}}{2}$ $\qquad$ $\sec-\frac{11\pi}{6}=\frac{2\sqrt{3}}{3}$

$\tan-\frac{11\pi}{6}=\frac{\sqrt{3}}{3}$ $\qquad$ $\cot-\frac{11\pi}{6}=\sqrt{3}$

In this case, $-\frac{11\pi}{6}$ is located in quadrant I. Moving in a clockwise direction, it is evident that $-\frac{11\pi}{6}$ is the same as $\frac{\pi}{6}$. This can also be found using coterminal angles. If we add 2π to $-\frac{11\pi}{6}$, the result is $\frac{\pi}{6}$. From this point, evaluate the six trigonometric functions.

Find the exact value of the six trigonometric functions for $\frac{19\pi}{6}$

$\sin\frac{19\pi}{6}=-\frac{1}{2}$ $\qquad$ $\csc\frac{19\pi}{6}=-2$

$\cos\frac{19\pi}{6}=-\frac{\sqrt{3}}{2}$ $\qquad$ $\sec\frac{19\pi}{6}=-\frac{2\sqrt{3}}{3}$

$\tan\frac{19\pi}{6}=\frac{\sqrt{3}}{3}$ $\qquad$ $\cot\frac{19\pi}{6}=\sqrt{3}$

In this example, it is obvious that $\frac{19\pi}{6}$ is greater than 2π. This is called a periodic function. This means the angle makes at least one complete revolution before coming to rest. To find the angle that must be used, in this case, subtract 2π from $\frac{19\pi}{6}$. The result of this operation is $\frac{7\pi}{6}$. Therefore, in order to find the exact value of the six trigonometric functions of $\frac{19\pi}{6}$ use the angle $\frac{7\pi}{6}$.