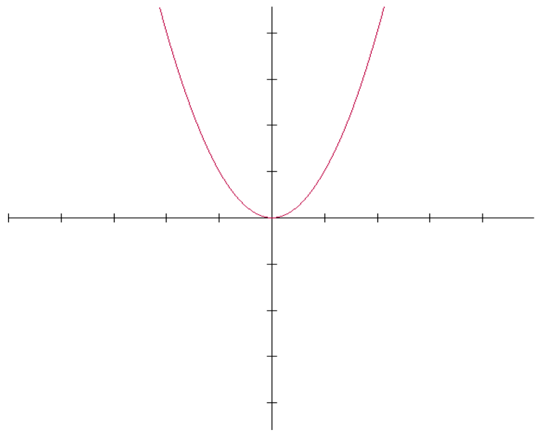
Quadratic Functions

The translation of a function is simply the shifting of a function. In this section, for the most part, we will be graphing various functions by means of shifting the parent function. We will go over the parent function for a variety of algebraic functions in this section. It is much easier to see the effects different constants have on a particular function if we use the parent function. We will begin with quadratics. Observe the following regarding a quadratic function in standard form.

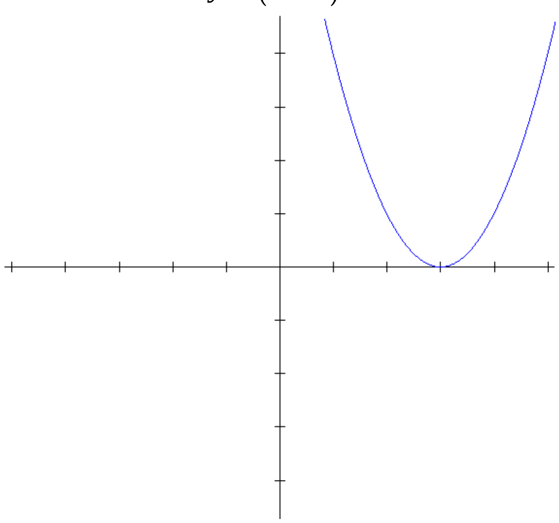
$$y = a(x-h)^2 + k$$

$$y = x^2$$



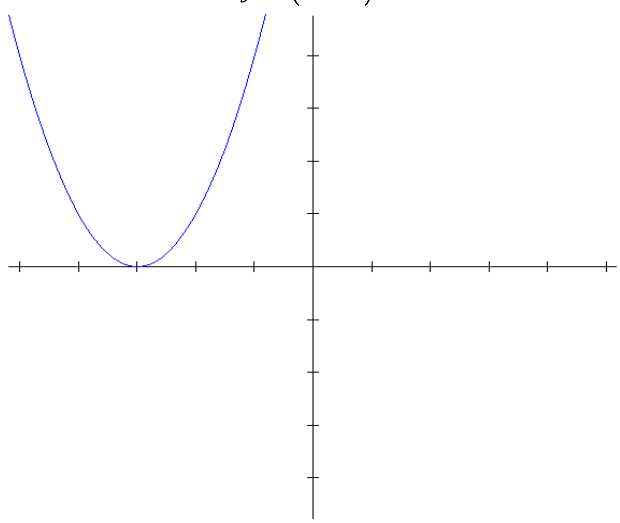
Notice that in the equation above, the h and k values are zero, while the value of a is one. This gives you the parent function for all quadratics. Everything else is merely a manipulation of the parent function.

$$y = (x-3)^2$$



The graph of the function shifts right 3.

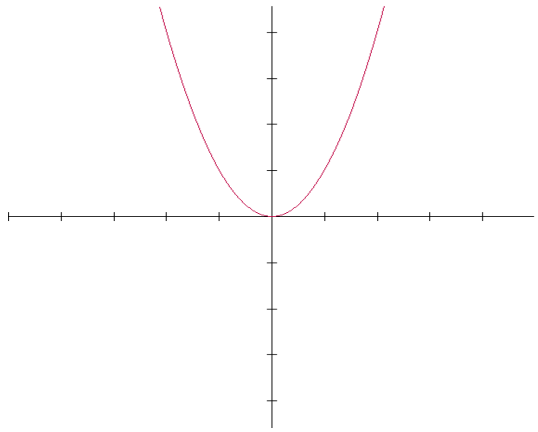
$$y = (x+3)^2$$



The graph of the function shifts left 3.

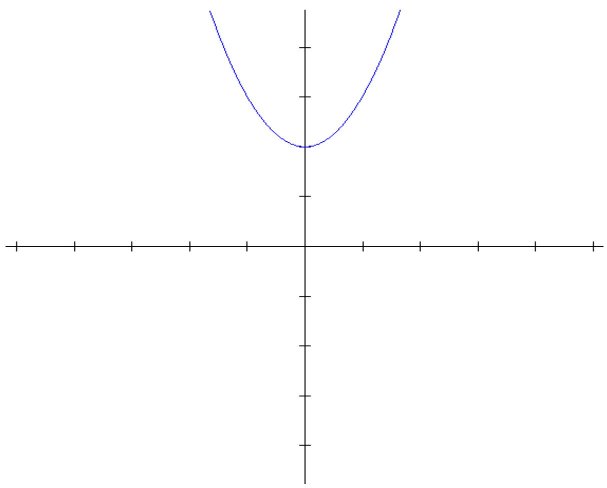
The number inside the parenthesis makes the graph shift to the left or right. Remember P.L.N.R., Positive Left Negative Right, tells about the horizontal shift needed to graph the function.
If the function above is $f_{(x)}$, the functions below would be $f_{(x-3)}$ and $f_{(x+3)}$ respectively. This is important to know, because in the future, you will be required to graph functions based solely on the picture provided. No equation will be given. You must rely solely on your knowledge of translating graphs.

$y = x^2$



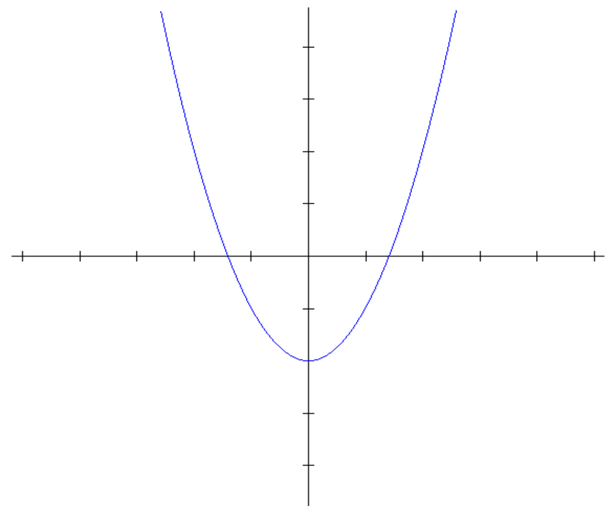
Once again, the parent function is illustrated above, and translations of it below.

$y = x^2 + 2$



The graph of the function shifts up 2.

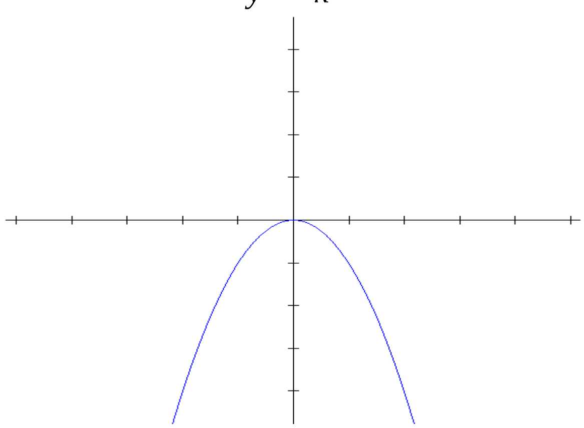
$y = x^2 - 2$



The graph of the function shifts down 2.

In these examples, the k ***value is what is changing. The value of*** k ***dictates a vertical shift of the function. In this case, consider the parent function as being*** $f_{(x)}$***. Given no information regarding the specific equation of the function, the equations for these two translations of*** $f_{(x)}$ ***are*** $f_{(x)} + 2$***, and*** $f_{(x)} - 2$.

$y = -x^2$



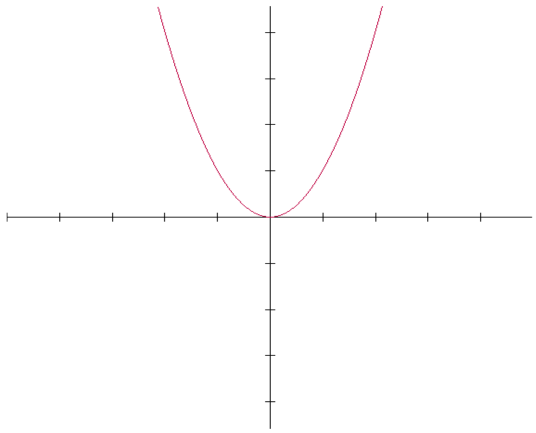
Now, on the left we have the opposite of the parent function. In this particular example, the value of a***, in the standard form is -1. A negative reflects the graph of the function about the horizontal axis. Once again, if the parent function given is referred to as*** $f_{(x)}$***, this function is*** $-f_{(x)}$.

We have seen how to graph a function by shifting the parent function. You may have noticed that we graphed $-f_{(x)}$***, but not*** $f_{(-x)}$***. The reason we did not see*** $f_{(-x)}$***, is because this is the graph of an even function. That means that if a –x were plugged in to the function, it would make no difference. The outcome would be the same. However, if we are dealing with a different type of function, one that was not even,*** $f_{(-x)}$ ***would cause the graph of the function to reflect about a vertical axis. In other words, if*** $-f_{(x)}$ ***makes a graph flip upside down,*** $f_{(-x)}$ ***would make the graph flip from right to left, or left to right, whatever may be the case.***

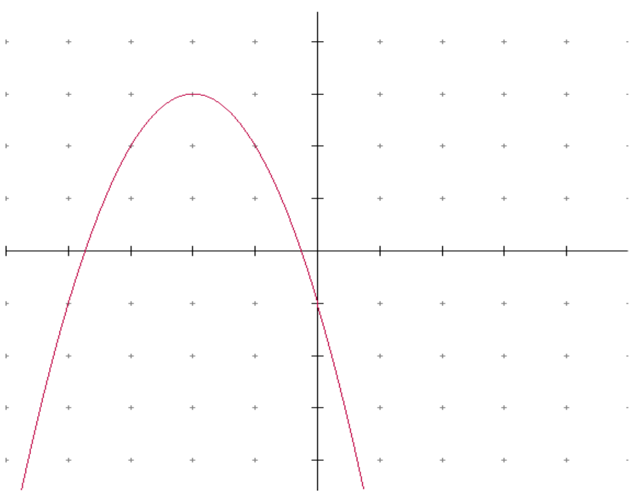
Lets see how differing values of a***,*** h ***and*** k ***will cause various shifts of the function.***

$$y = a(x-h)^2 + k$$

Once again, take note of the parent function $y = x^2$



$$y = -(x+2)^2 + 3$$

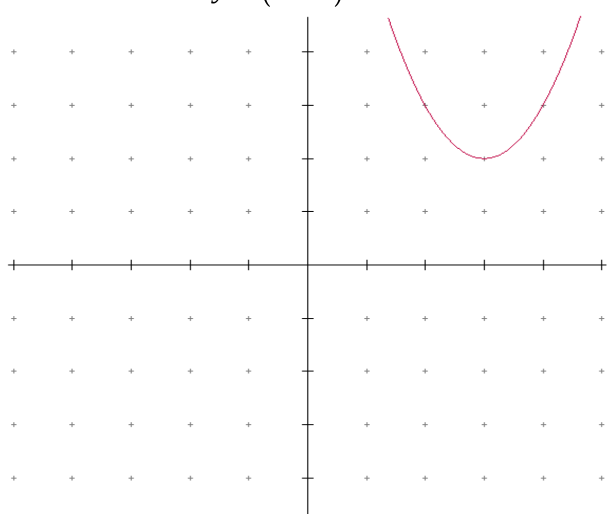


This graph opens down, and shifts left 2, up 3.

If this graph is a translation of the function $f_{(x)}$

It would by written as $-f_{(x+2)} + 3$.

$$y = (x-3)^2 + 2$$



This graph opens up and shifts right 3, and up 2

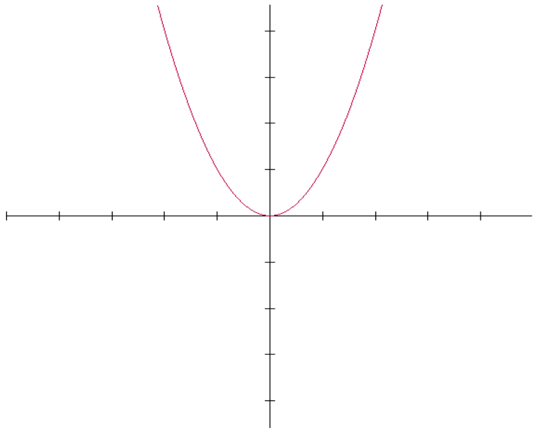
If this graph is a translation of the function $f_{(x)}$

It would by written as $f_{(x-3)} + 2$.

We will be graphing functions using only $f_{(x)}$ ***in the "translations of functions" section.***

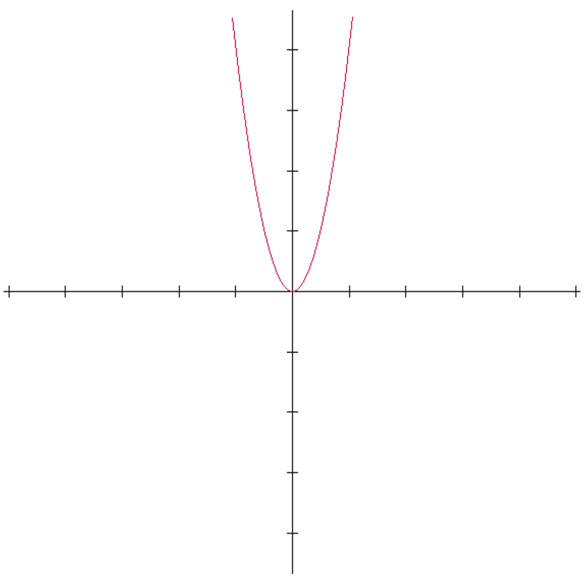
$$y = a(x-h)^2 + k$$

$$y = x^2$$



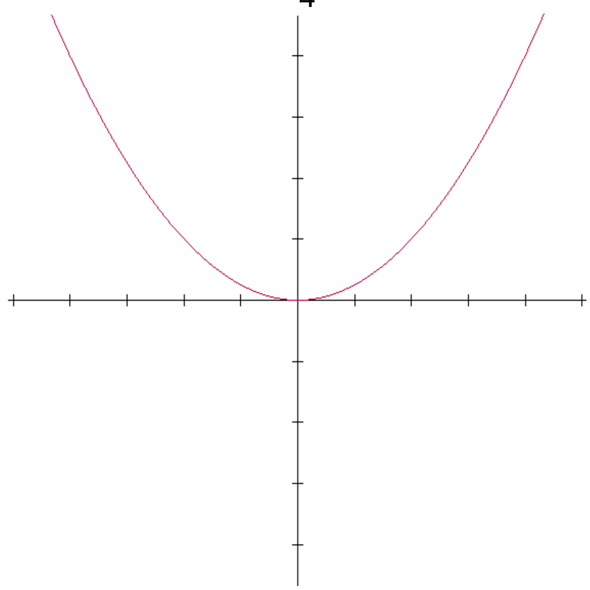
Here we will see how the value of a for the quadratic function in standard form affects the graph of the function. To illustrate this, we will look at the graph of a parabola that has its vertex on the origin.

$$y = 4x^2$$



This graph seems very narrow, but what is actually happening, is the value of the function is increasing very rapidly. The y values are increasing at 4 times their normal rate. The rapid increase causes the graph to appear narrow.

$$y = \frac{1}{4}x^2$$



This graph is wider than the parent function. In this case, the y values of the function are increasing at ¼ their normal rate, causing a more gradual increase.

As you can see, if the value of the leading coefficient is a whole number, the y values of the graph will increase rapidly causing a narrow and steeper curve. In contrast, if the leading coefficient is a fraction, the y values of the function will increase mildly, causing a more gradual curve.

Describe the movement of each of the following quadratic functions. Describe how each opens and if there is any horizontal or vertical movement. Be sure to state how many spaces it moves, for example: ***This graph opens down, and shifts left 2, up 3.***

A) $y = -3(x-4)^2 + 2$

B) $y = 2(x+3)^2 - 8$

C) $y = \frac{1}{2}(x-3)^2$

D) $y = \left(x + \frac{1}{2}\right)^2 - \frac{2}{3}$

E) $y = -(x+5)^2 + 6$

F) $y = 7(x-3)^2 + 1$

G) $y = -\frac{1}{5}(x-7)^2 + 4$

H) $y = 3(x+6)^2 + 8$

I) $y = -4(x-3)^2 - 2$

J) $y = x^2 - 3$

K) $y = -\frac{1}{5}(x+14)^2$

L) $y = -2x^2 + 8$

As you describe the graphs of the quadratic functions above, you wrote that it shifts to the left or right, and up or down. What is actually shifting?

Write the equation for a quadratic function in $y = a(x-h)^2 + k$ form that opens down, shifts left 3 and up 7.

Write the equation for a quadratic function in $y = a(x-h)^2 + k$ form that opens up, shifts right 4 and down 2.

Write the equation for a quadratic function in $y = a(x-h)^2 + k$ form that opens up, and only shifts down 4.

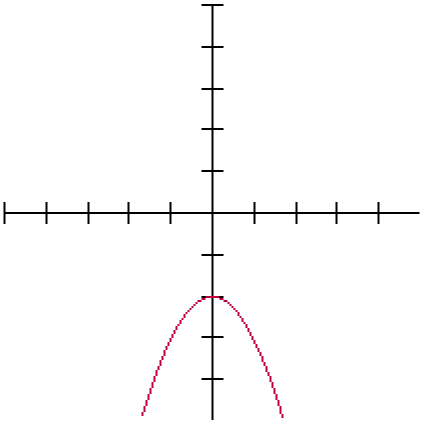
Write the equation for a quadratic function in $y = a(x-h)^2 + k$ form that opens down and shifts to the left 8 spaces.

Write the equation for a quadratic function in $y = a(x-h)^2 + k$ form that opens down and shifts up 7.

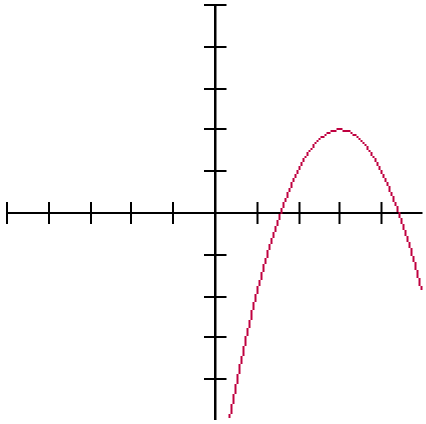
Is a quadratic function a one-to-one function? Why or why not? What does this tell you about the inverse of a quadratic function?

Match the appropriate graph with its equation below. Explain why each of your solutions is true.

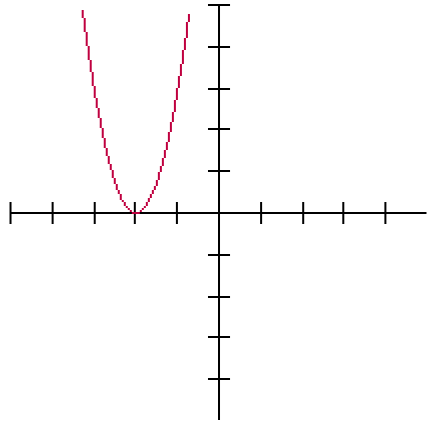
A



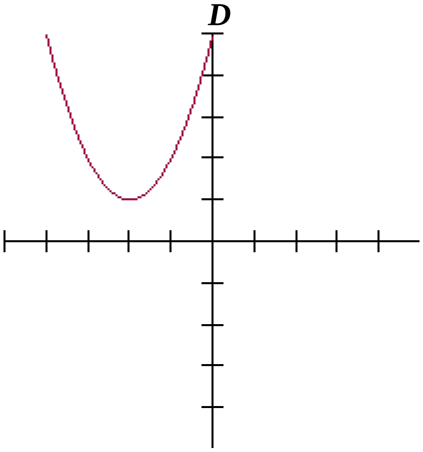
B



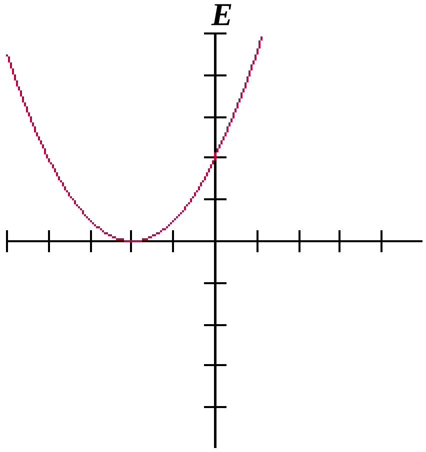
C



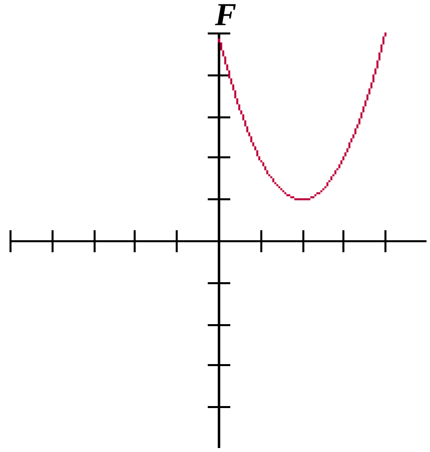
D



E



F



1) $f_{(x)}=\frac{1}{2}(x+2)^2$

2) $f_{(x)}=-x^2-2$

3) $f_{(x)}=(x+2)^2+1$

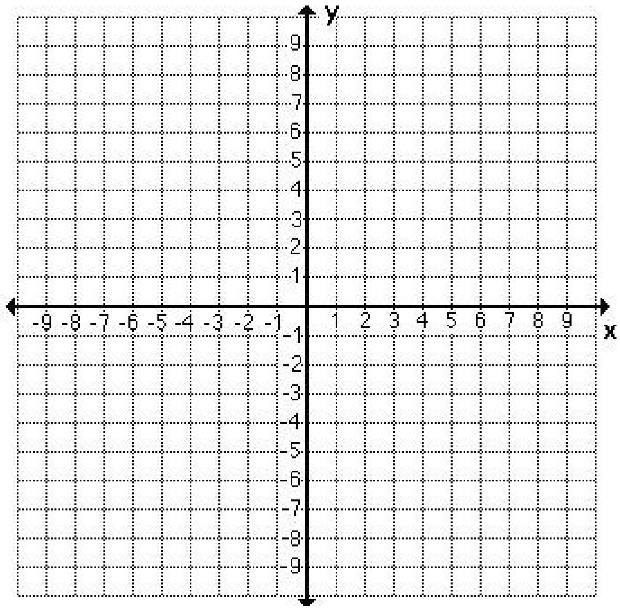
4) $f_{(x)}=(x-2)^2+1$

5) $f_{(x)}=3(x+2)^2$

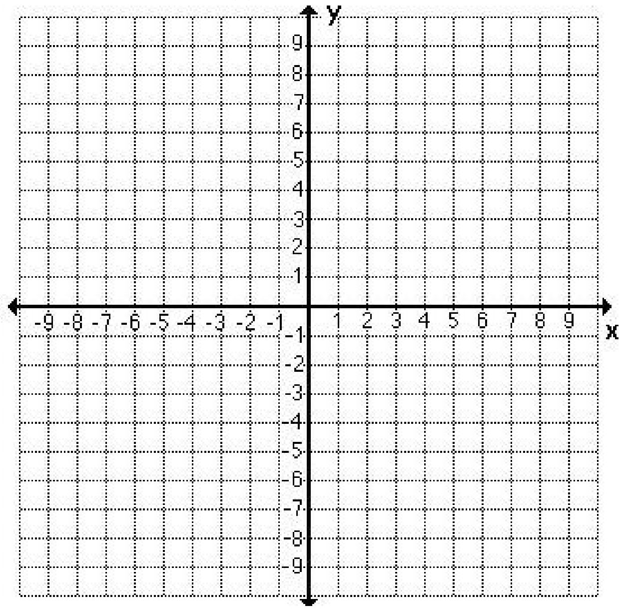
6) $f_{(x)}=-(x-3)^2+2$

Graph each of the following functions. You may need to use an axis of symmetry to graph some of these. Label the vertex, y-intercept, and all x-intercepts.

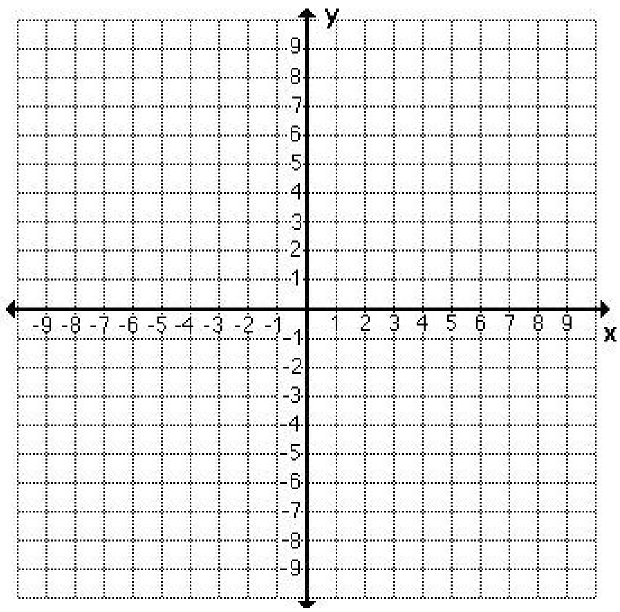
A) $f_{(x)} = (x-3)^2 + 1$



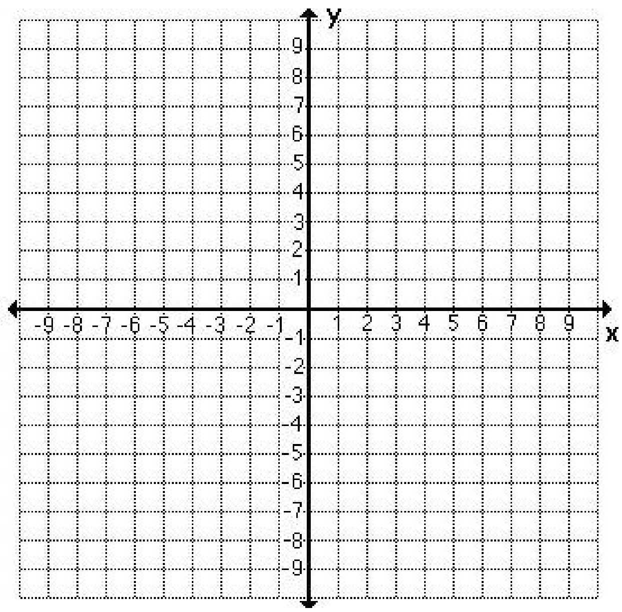
B) $f_{(x)} = -(x+4)^2 + 9$



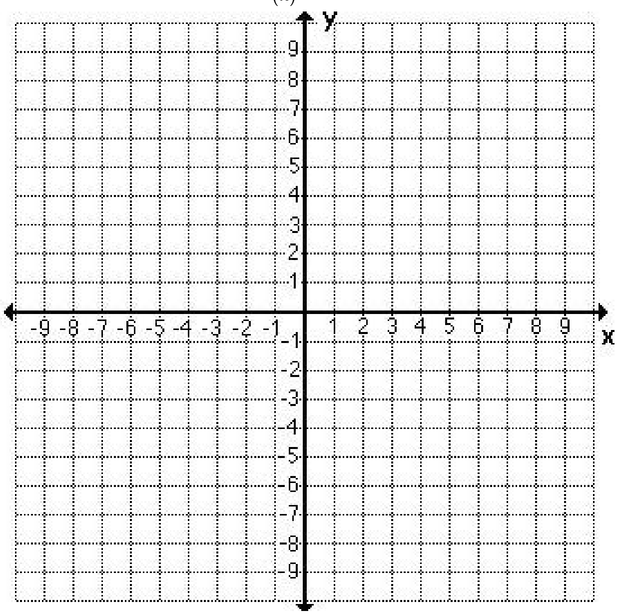
C) $f_{(x)} = \frac{1}{2}(x-5)^2$



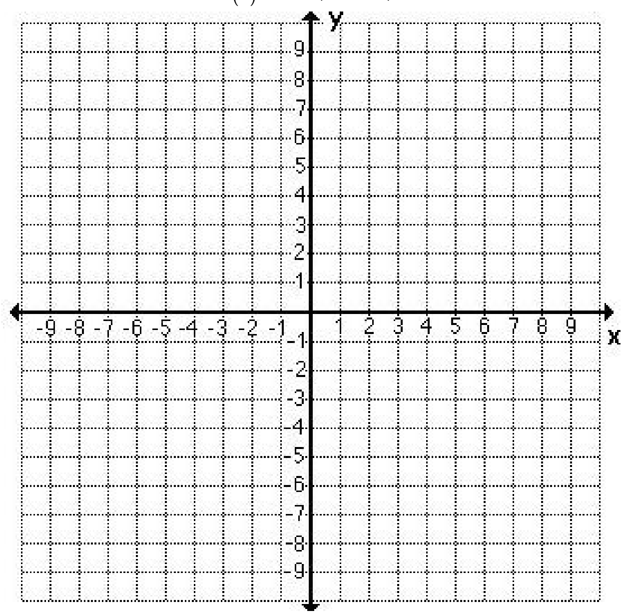
D) $f_{(x)} = -2(x+3)^2 + 5$



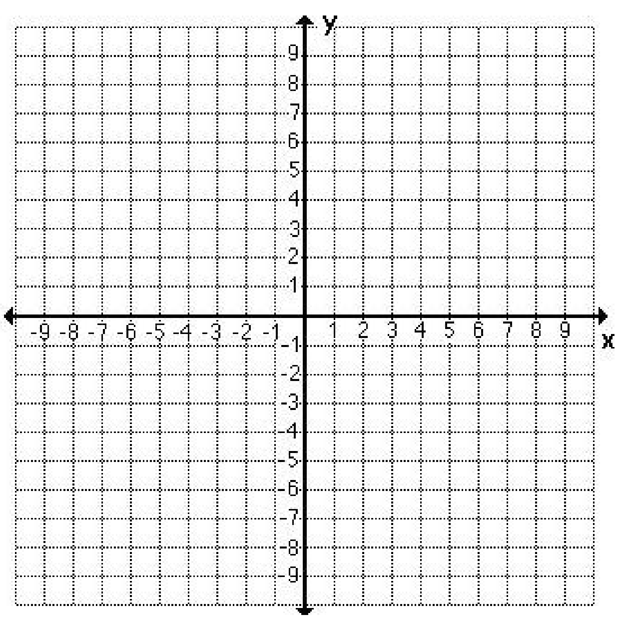
E) $f_{(x)} = x^2 + 3$



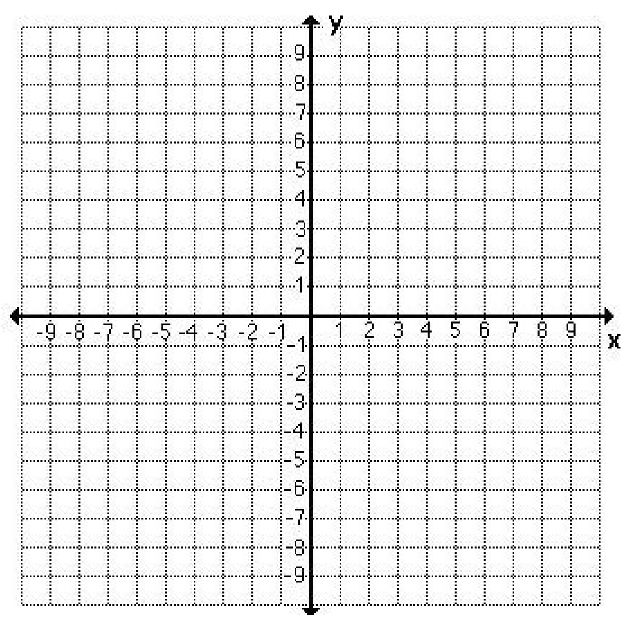
F) $f_{(x)} = -(x+3)^2 + 4$



G) $f_{(x)} = -2(x-4)^2$



H) $f_{(x)} = 2(x-1)^2 - 8$



The quadratic function given by the equation $f_{(x)} = 3(x-2)^2 + 6$ has an axis of symmetry of______________.

The quadratic function given by the equation $f_{(x)} = -3(x+6)^2 - 4$ has an axis of symmetry of______________.

The quadratic function given by the equation $f_{(x)} = a(x-h)^2 + k$ has an axis of symmetry of______________.

Considering your answers to the previous questions, we can conclude that the axis of symmetry for any quadratic function is given by the x value of the _______________.

Why does the axis of symmetry look as though we are saying x equals a number ($x = \#$)?

Why is it sometimes necessary to graph a quadratic function using the axis of symmetry?