Exponential Functions

This information was covered in a previous section of the workbook, but it won't hurt to go over it again.

Standard exponential function

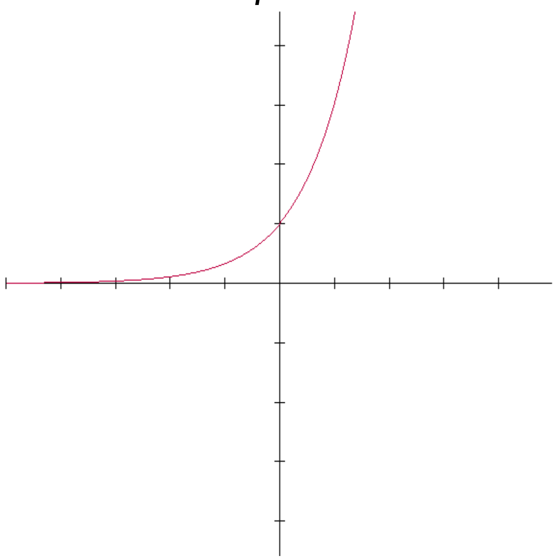
$$f_{(x)} = ca^{x-h} + k$$

The c term is a constant that can make to graph reflect about a horizontal axis or change to scale of the graph of the function proportionately. If c is a positive value, then you will have a standard looking growth or decay curve. If c is negative, the growth or decay curve will flip upside down. We will get into the effects different values of h and k have on this function shortly. What we will concentrate on here is identifying an exponential function as being growth or decay, and finding the range, domain and key point of the function.

Exponential Growth

$$f_{(x)} = ca^{x-h} + k$$

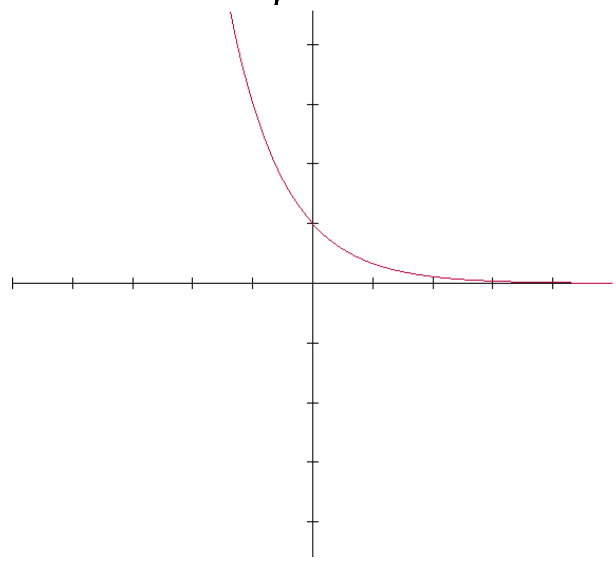
If a>1



Exponential Decay

$$f_{(x)} = ca^{x-h} + k$$

If 0<a<1



Obviously if $a = 1$***, we are raising 1 to various powers, and we wind up getting a horizontal line because no matter what you do, raising one to any power still yields a result of one. Pay special attention to the exponential decay function. The statement*** $0 < a < 1$ ***is saying that the value of*** a ***is a fraction whose value is between zero and one. Do not make the mistake of just looking for a fraction to determine whether or not the function is decay. Make sure the value of the fraction is between zero and one.***

The values for variables $h,\ k,\ and\ \ c$ ***act to make the graph shift left/right, up/down, change the scale or will reflect the function about a horizontal axis.***

Notice the key point for each of these functions is the point $(0,1)$. This information is vital. This key point will shift depending on the values of h, k and c. To find the x value of the key point, evaluate $x-h=0$. In other words, find the value of x that would create a problem such as 3 to the zero power. This number is the x value of the key point. To find the y value, substitute the x value back in. Refer to the following example.

$f_{(x)}=2^{x-3}+5$

to find the key point evaluate $x-3=0$

$x-3=0$

$x=3$ *this is the x value of the key point*

now substitute 3 *back into the problem for* x

$f_{(3)}=2^{3-3}+5$

$f_{(3)}=2^{0}+5$

$f_{(3)}=1+5$

$f_{(3)}=6$

so the key point is $(3,6)$

$$f_{(x)}=ca^{x-h}+k$$

As we work to translate these functions, use $(0,1)$, as the default key point to any exponential growth or decay curve that is above the horizontal asymptote where the value of "c" is 1. In other words, if the value of "c" is positive one, use the point $(0,1)$ to assist you in shifting the function. If the graph of the function is below the horizontal asymptote, and the "c" value is -1, you will use $(0,-1)$ as the key point. If the value of "c" is <u>any</u> other number, you must find the key point algebraically. Consider the example above.

$f_{(x)}=2^{x-3}+5$

The first thing I did was notice that the graph of this function shifts right 3 and up 5. Pay special attention to the value of k. That tells you where your new horizontal asymptote is going to be, in this case, at $y=5$.

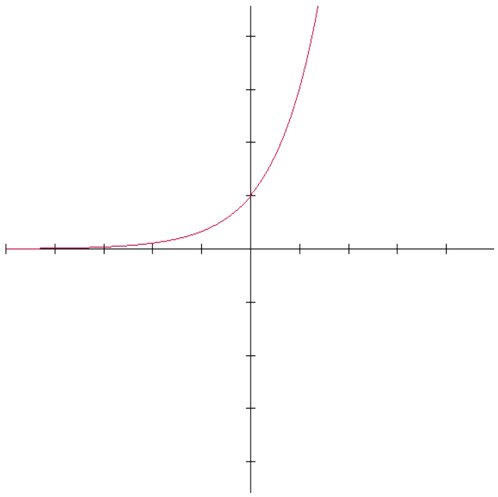
Since the value of c is positive one, begin at the key point $(0,1)$. Since the graph will shift right 3 and up 5, simply add 3 to the x value, and 5 to the y value of the key point. This produces a new key point of $(3,6)$. Observe how this information matches the work above. The key point in these functions acts as the vertex in a parabola. It gives you a point of reference with which to shift the function. Make sure the correct key point is used from the beginning, either $(0,1)$ or $(0,-1)$. <u>Remember</u>, if the "c" term is a number other that 1 or -1, the key point is actually multiplied by that number. For example, the function $f_{(x)}=3(2)^{x}$ has a key point of $(0,3)$, not $(0,1)$.

Here we will look at how to graph these functions by means of translation.

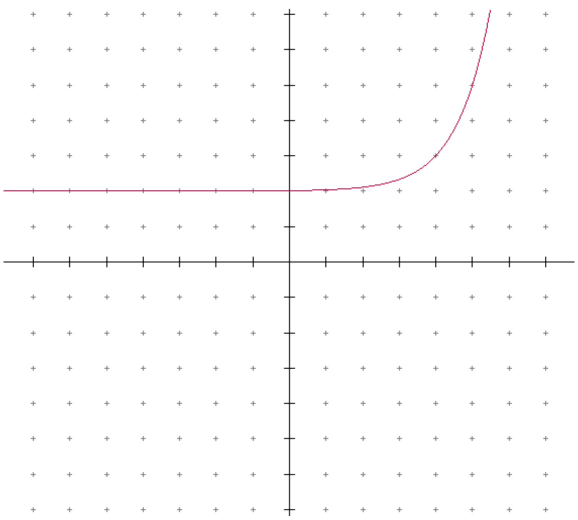
<u>Exponential Growth</u>

$$f_{(x)} = ca^{x-h} + k$$

$$f_{(x)} = 3^x$$

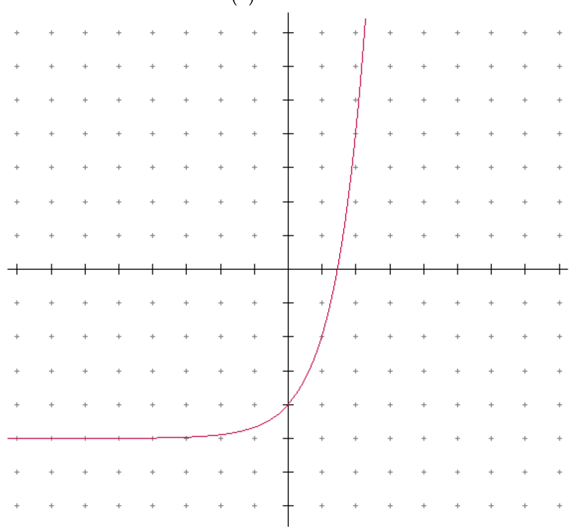


$$f_{(x)} = 3^{x-4} + 2$$



Notice that the horizontal asymptote is at y=2. Since the graph of this function is going to be above the x axis, begin with the key point (0,1). This function shifts right 4 and up 2. The dots have been left on the graph so it would be easier to see. Adding 4 to the x value of the key point, and 2 to the y value, the new key point is at (4,3). Just remember where to begin, and do not cross the horizontal asymptote.

$$f_{(x)} = 3^x - 5$$



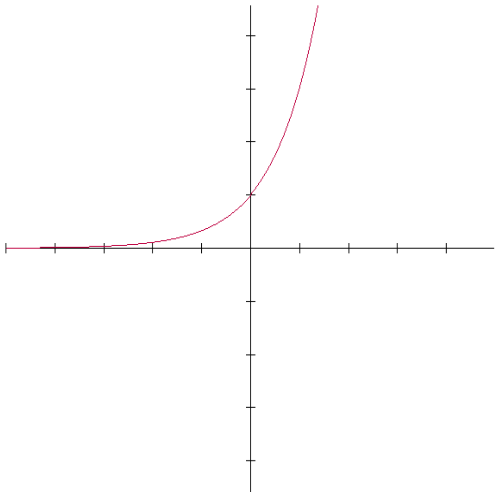
In this function, the value of k is -5. This tells you the new horizontal asymptote will be at y = -5. Since the value of the constant "c" is a positive one, begin with the key point (0,1). This function will only shift down 5 spaces. Therefore, subtract 5 from the y value of the key point which is 1. This results in: (1 – 5 = -4) therefore, the new key point is at (0,-4).

Graphing exponential functions by translation is relatively simple. The most difficult part will be finding the x and y intercepts as the x-intercept will involve the use of logarithms.

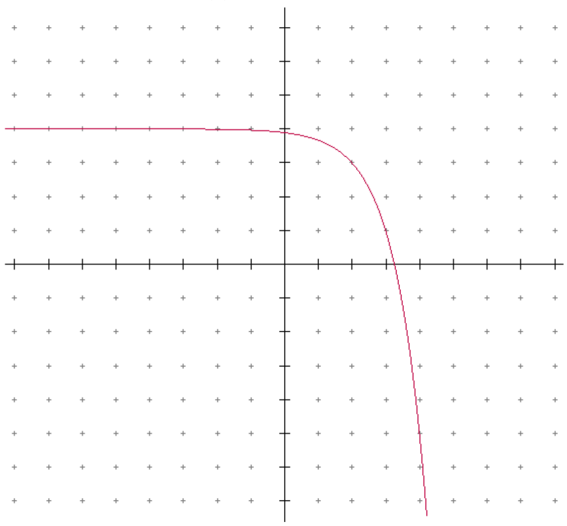
Exponential Growth

$$f_{(x)} = ca^{x-h} + k$$

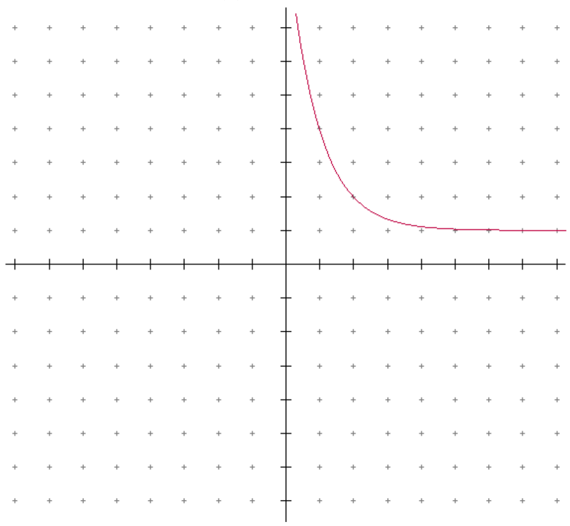
$$f_{(x)} = 3^x$$



$$f_{(x)} = -3^{x-2} + 4$$



$$f_{(x)} = 3^{2-x} + 1$$



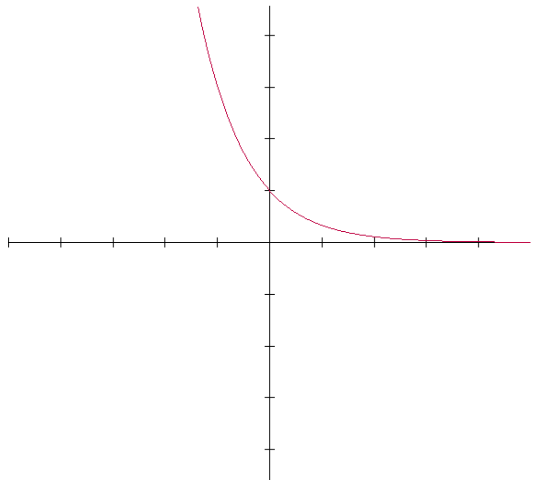
Since the value of "c" in the equation of this function is -1, we must begin with the key point of (0,-1). This is the key point, because that value of "c" caused the graph to reflect about the horizontal asymptote. The entire function will shift up 4, so the new horizontal asymptote is y = 4. The curve is going to shift right 2 and up 4. By adding 2 to the x value of the key point, and 4 to the y value, the new point can be found at (2,3). Notice the graph runs right though that point.

Here we have something that looks like decay. The value of "a" in this function is greater that one, so it should be growth. What really happened here, is the laws of exponents went to work. 3^{2-x} ***Is the same thing as*** $3^{-(x-2)}$. ***The power of a power rule says this can be seen as*** $\left(3^{-1}\right)^{x-2}$. ***This simplifies to*** $\left(\frac{1}{3}\right)^{x-2}$ ***, a decay curve. OK, so we begin with a decay curve that has a key point of (0,1). Add 2 to the x value, and 1 to the y value of the key point, and the new key point is (2,2), with a horizontal asymptote of y = 1.***

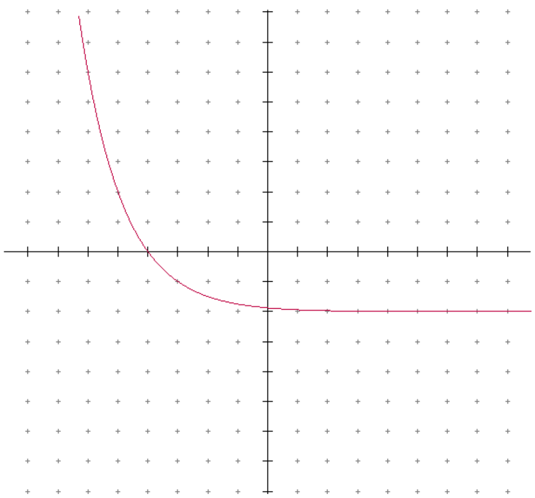
Exponential Decay

$$f_{(x)} = ca^{x-h} + k$$

$$f_{(x)} = \left(\frac{1}{2}\right)^{x}$$

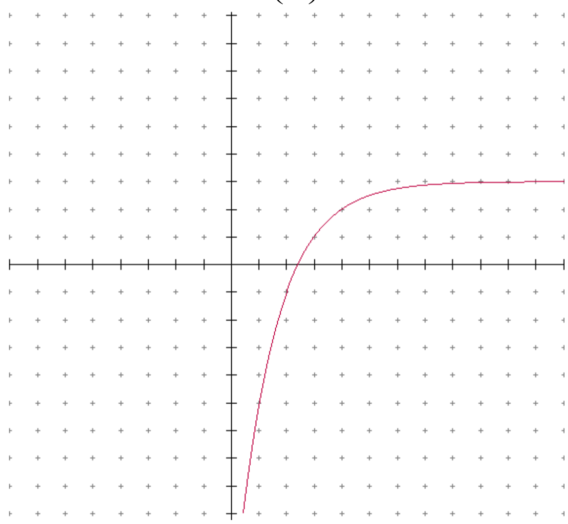


$$f_{(x)} = \left(\frac{1}{2}\right)^{x+3} - 2$$



This is an exponential decay curve. The original graph will lie above the x axis. Therefore, begin with the key point (0,1). The key point will shift to the left 3, and down 2, so subtract 3 from the x value and 2 from the y value of the key point, and the coordinates of the new point will be at (-3,-1). The graph of this function shifts down 2, so the horizontal asymptote of this function is y = -2.

$$f_{(x)} = -\left(\frac{1}{2}\right)^{x-4} + 3$$



This is an exponential decay function that is reflected and lies below the horizontal asymptote. The initial key point here is (0,-1). Since this graph will shift right 4 and up 3, add 4 to the x value of the key point and 3 to the y value. This yields a result of (4,2). Since the entire graph shifted upwards 3 spaces, the horizontal asymptote is y = 3. Once again, when graphing, do not cross the horizontal asymptote.

The translations of these functions are very similar to that of other functions we have seen. A point of reference with which to shift is all that is needed. Most important is to make sure to always use the appropriate key point to start with. <u>Draw the horizontal asymptote first</u>, that way the graph of the function does not accidentally cross it.

$$f_{(x)} = ca^{x-h} + k$$

The domain of any exponential function is $(-\infty, \infty)$. The values of c and k terms will determine the range of the function. Since the horizontal asymptote of an exponential function is given by y=k, the value of k will determine where the horizontal asymptote of the function lies, whereas the value of c will determine if the function is above or below that asymptote. Be careful not to use brackets when describing the range of an exponential function. The horizontal asymptote <u>must not be touched</u>, so only parenthesis may be used to describe the range in interval notation.

Find the range and domain of each of the following exponential functions.

A) $f_{(x)} = 2^{x+6} - 4$

B) $f_{(x)} = -\left(\frac{1}{2}\right)^{x-1} + 3$

C) $f_{(x)} = 2(3)^{x+1} - 5$

D) $f_{(x)} = 5^{-x} - 3$

E) $f_{(x)} = -2(5)^{x+2} - 3$

F) $f_{(x)} = e^{x+2} - 3$

G) $f_{(x)} = \left(\frac{5}{4}\right)^{x-8} + 2$

H) $f_{(x)} = -2^{x-3} - 7$

I) $f_{(x)} = -4^{3-x} + 2$

J) $f_{(x)} = 2\left(\frac{1}{3}\right)^{x-5} + 1$

K) $f_{(x)} = -6^{x-7} - 1$

L) $f_{(x)} = -e^{x-2} + 3$

Here is an example of finding the x and y intercept of an exponential function.

$$f_{(x)} = 3^{x+2} - 4$$

Finding the x intercept.

Begin by substituting 0 for $f_{(x)}$

$$0 = 3^{x+2} - 4$$

$$4 = 3^{x+2}$$

$$\log 4 = \log 3^{x+2}$$

$$\log 4 = (x+2)\log 3$$

$$\log 4 = x\log 3 + 2\log 3$$

$$\log 4 - 2\log 3 = x\log 3$$

Now divide both sides by log 3.

$$\frac{\log 4 - 2\log 3}{\log 3} = \frac{x\log 3}{\log 3}$$

$$x = \frac{\log 4 - 2\log 3}{\log 3}$$

$$x \approx -0.7381$$

Finding the y intercept.

Begin by substituting 0 for x.

$$f_{(x)} = 3^{0+2} - 4$$

$$f_{(x)} = 3^{2} - 4$$

$$f_{(x)} = 9 - 4$$

$$f_{(x)} = 5$$

As you can see, this function has an x intercept of approximately (-0.74,0), and a y intercept of (0,5).

Find the key point to each of the following functions.

A) $f_{(x)} = 3^{x+4} - 2$

B) $f_{(x)} = -4^{x-2} + 1$

C) $f_{(x)} = 2^{4-x} + 5$

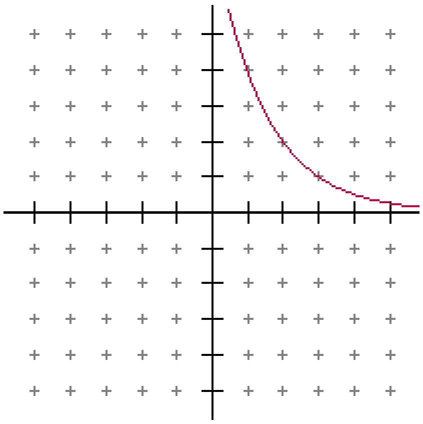
D) $f_{(x)} = 3(2)^{x+1} - 5$

E) $f_{(x)} = 2\left(\frac{1}{2}\right)^{x+4} - 3$

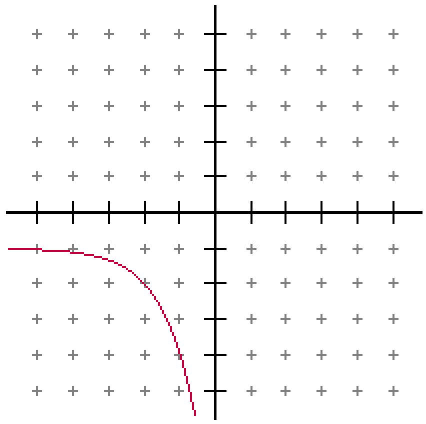
F) $f_{(x)} = -3^{x+2} - 4$

Match the appropriate graph with its equation below. Explain why each of your solutions is true.

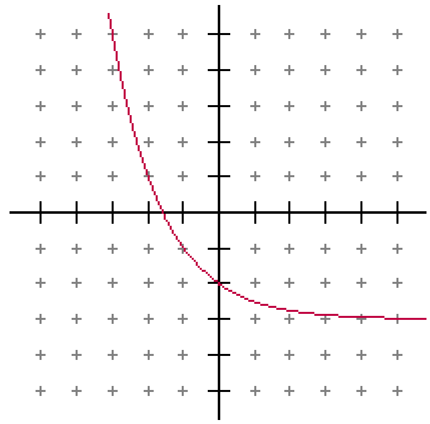
A



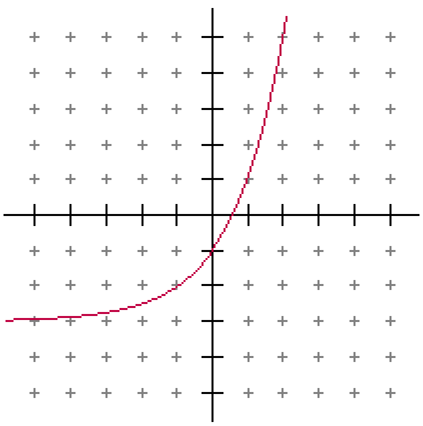
B



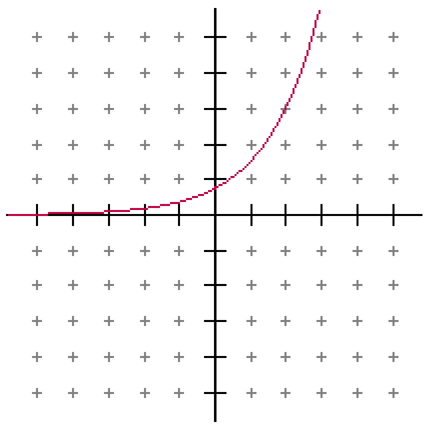
C



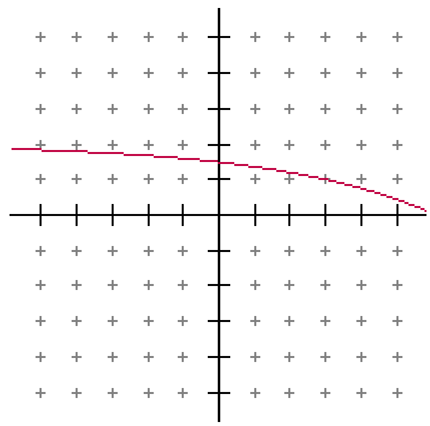
D



E



F



1) $f_{(x)} = 2^{x+1} - 3$

2) $f_{(x)} = \left(\frac{1}{2}\right)^{x} - 3$

3) $f_{(x)} = 3(2)^{x-2}$

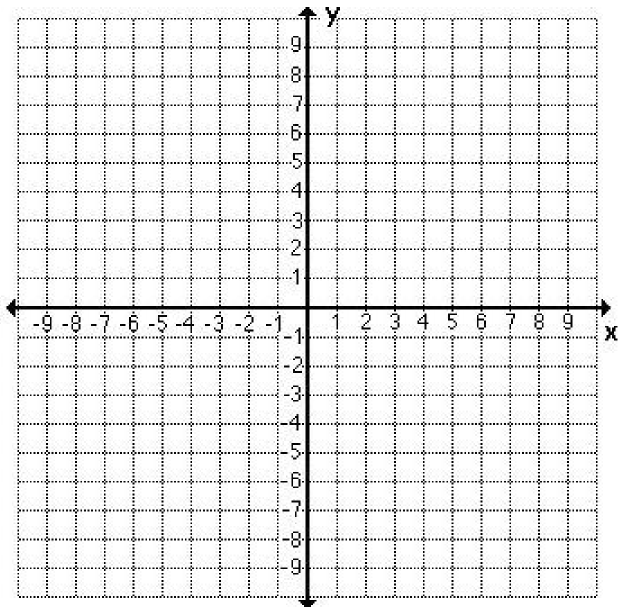
4) $f_{(x)} = -3^{x+2} - 1$

5) $f_{(x)} = -\left(\frac{5}{4}\right)^{x-3} + 2$

6) $f_{(x)} = 2^{3-x}$

Graph each of the following exponential functions. Be sure to label the key point of the function. Find the x intercept (if it exists) and y intercept of each function.

A) $f_{(x)} = 3^{x-2} - 1$



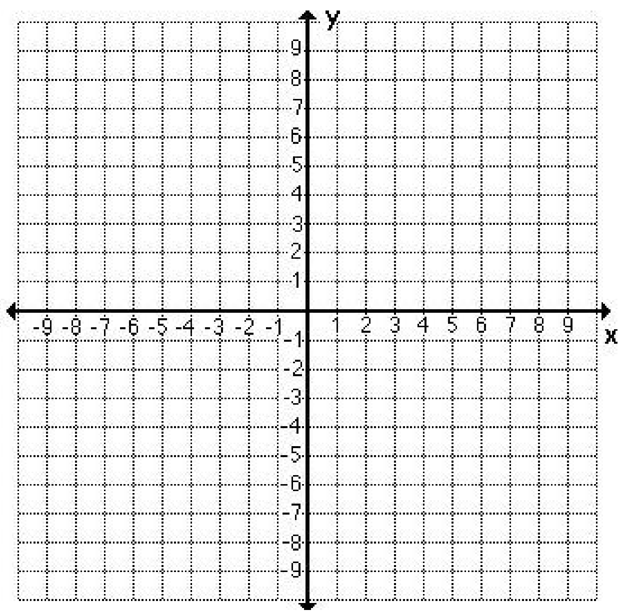
Y-intercept:

X-intercepts:

Range:

Domain:

B) $f_{(x)} = -2^{x+3} - 4$



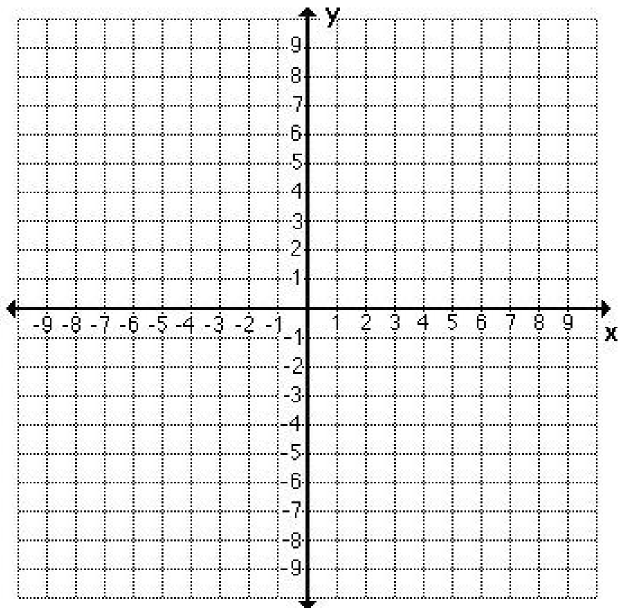
Y-intercept:

X-intercepts:

Range:

Domain:

C) $f_{(x)} = \left(\frac{1}{2}\right)^{x+2} - 3$



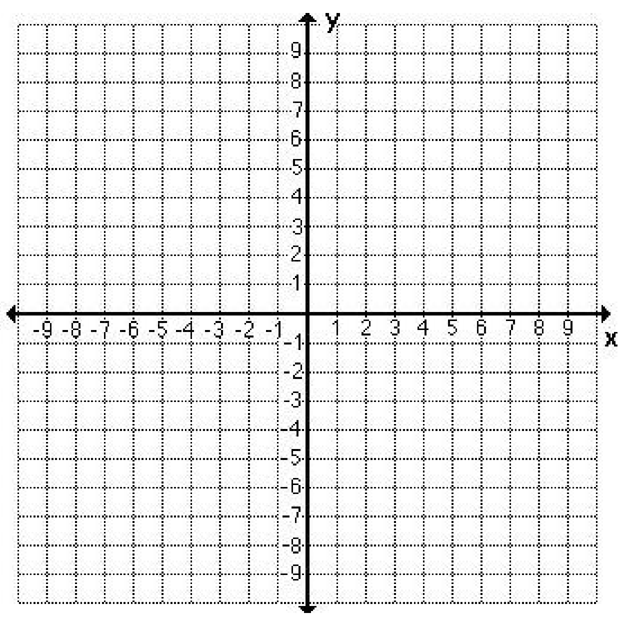
Y-intercept:

X-intercepts:

Range:

Domain:

D) $f_{(x)} = 2^{4-x}$



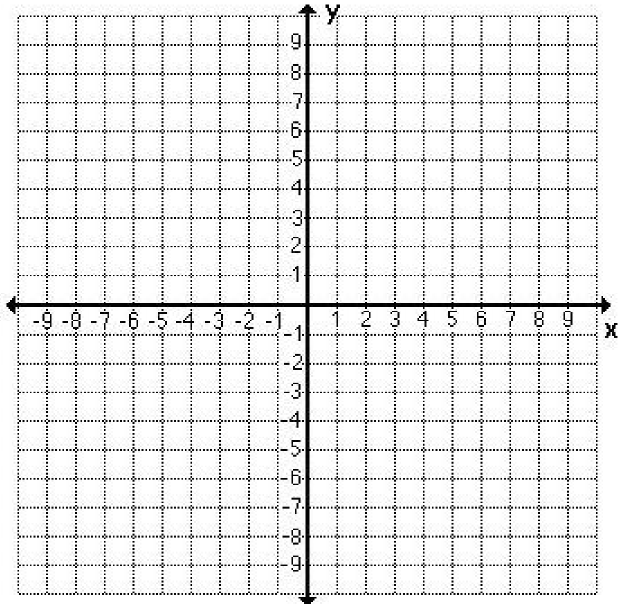
Y-intercept:

X-intercepts:

Range:

Domain:

E) $f_{(x)} = -\left(\frac{1}{3}\right)^{x-2} - 3$



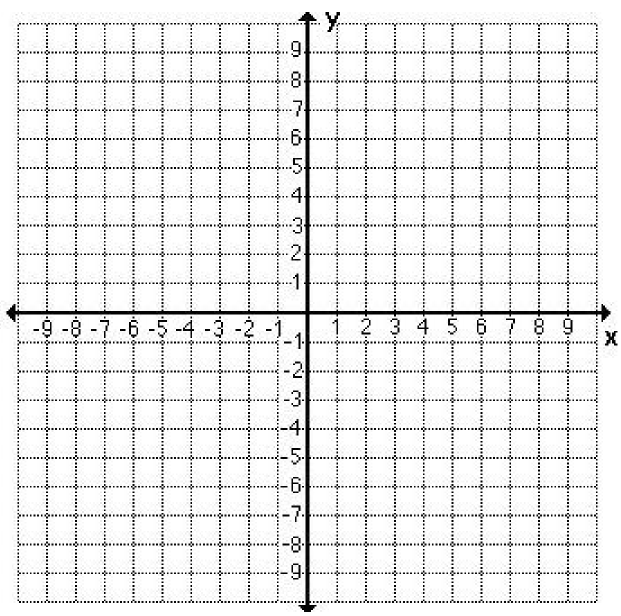
Y-intercept:

X-intercepts:

Range:

Domain:

F) $f_{(x)} = 2(3)^{x-2} + 1$



Y-intercept:

X-intercepts:

Range:

Domain: