Logarithmic Functions

$$f_{(x)} = a\log_n(bx+c)+d \qquad f_{(x)} = a\ln(bx+c)+d$$

Logarithmic functions will be graphed in the same manner as radical functions. It is first necessary to find the domain of the logarithmic function. The range of a logarithmic function is all real numbers, so only the domain needs to be found. To find the domain of a logarithmic function evaluate $bx+c>0$. Remember, this is not ≥, because you cannot take the log of zero. Once the domain is found, it will tell in which direction the function is moving. This inequality will also help find the vertical asymptote for the function.

If the x inside the log does not have a negative coefficient, the curve will be on the right side of the vertical asymptote. If the coefficient in front of x is 1, begin with the key point of (1,0). From that point on, treat the function just like an exponential function. Adding or subtracting to either the x or y values to find the new key point making the graph shift.

If the x inside the log has a negative coefficient, the curve will be on the left side of the vertical asymptote. If the coefficient in front of x is -1, begin with the key point of (-1,0) and shift from there.

***Once again, just like exponential growth and decay functions, watch the value of "a", as it affects the scale of the function. If the value of "a" is some number other that 1 or -1, find the key point algebraically before you translate the function.**

As the function shifts, it will be helpful to draw a broken line for both the horizontal and vertical asymptotes. It is OK to cross the horizontal asymptote, as you will find the key point always rests on it. The vertical asymptote, however, may never be crossed.

$$f_{(x)} = a\log_n(bx+c)+d \qquad f_{(x)} = a\ln(bx+c)+d$$

Solving for $bx+c=0$, will yield the equation for the vertical asymptote. The equation for the horizontal asymptote is $y=d$.

$$f_{(x)} = \log_3(x-4)+2$$

Finding the domain.

$$x-4>0$$
$$x>4$$

Notice the similarity in the procedures.

Finding the vertical asymptote.

$$x-4=0$$
$$x=4$$

Finding the horizontal asymptote.

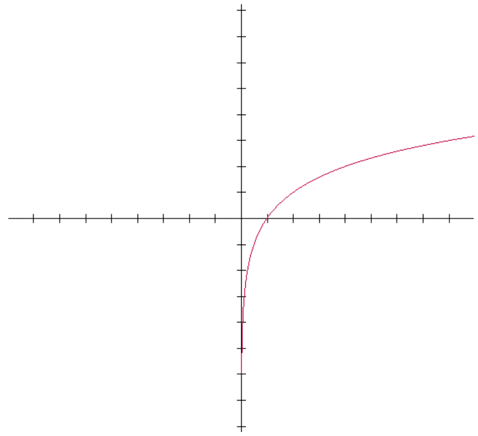
$$y=2$$

There is no real work involved with finding the horizontal asymptote. Identify the vertical shift. This is the equation of the horizontal asymptote.

***If the variable x inside the log has a coefficient other than 1 or -1, the key point will be different. The key point must then be found algebraically. To find the x value of the key point solve for $bx+c=1$. Substitute that solution back into the problem to find the y value.**

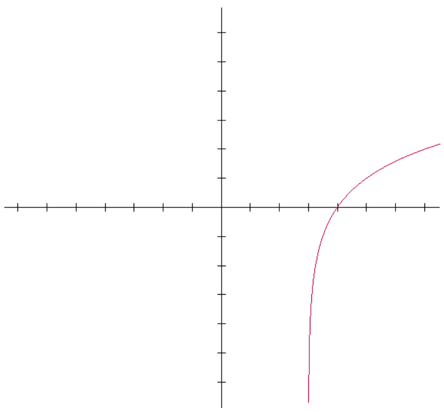
$$f_{(x)} = a\log_n(bx+c)+d \qquad f_{(x)} = a\ln(bx+c)+d$$

$$f_{(x)} = \log_2 x$$



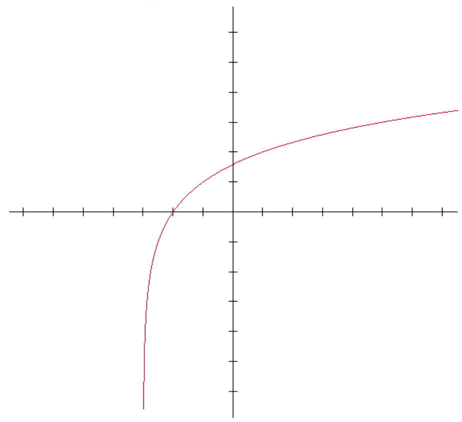
The parent function has the key point at (1, 0)

$$f_{(x)} = \log_2(x-3)$$



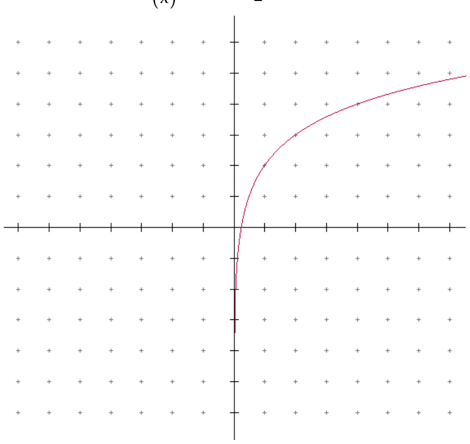
The graph of this function shifts right 3. Notice the key point moved to the right 3 places to (4,0).

$$f_{(x)} = \log_2(x+3)$$



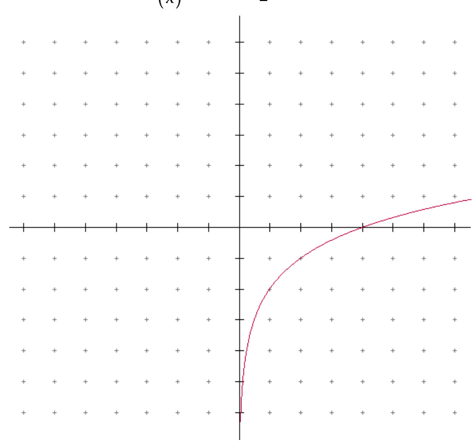
The graph of this function shifts to the left 3. The new key point is (-2,0).

$$f_{(x)} = \log_2 x+2$$



This function shifts up 2. Add 2 to the y value of the key point, and it is now at (1,2).

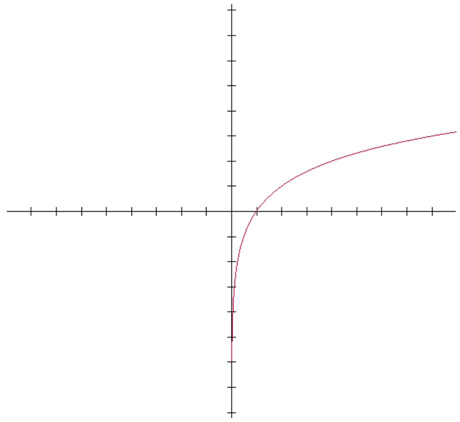
$$f_{(x)} = \log_2 x-2$$



This function shifts down 2. Subtracting 2 from the y value of the key point results in (1,-2).

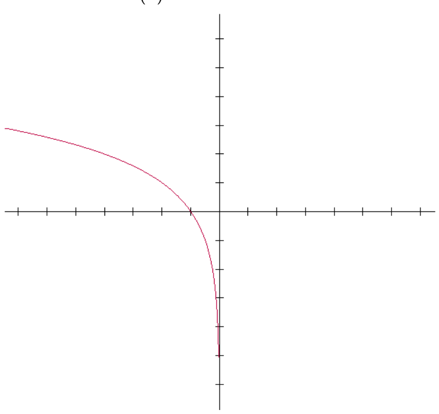
$$f_{(x)} = a\log_n(bx+c)+d \qquad f_{(x)} = a\ln(bx+c)+d$$

$$f_{(x)} = \log_2 x$$



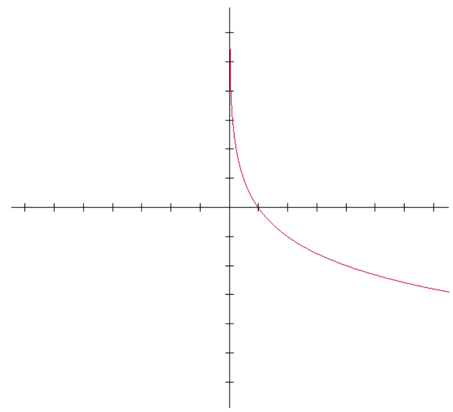
The parent function has the key point at (1, 0)

$$f_{(x)} = \log_2(-x)$$



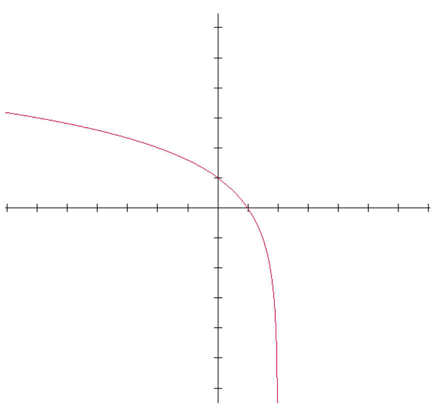
The graph of this function reflects about the vertical asymptote. Key point is now (-1,0).

$$f_{(x)} = -\log_2 x$$



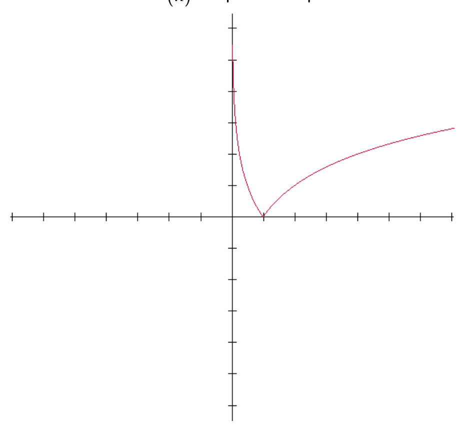
The graph of this function is reflected about the horizontal asymptote. Key point is still at (1,0).

$$f_{(x)} = \log_2(2-x)$$



Since the coefficient of x is -1, this graph will be on the left side of the vertical asymptote. Begin with the key point (-1,0), and shift right 2 because it is positive. Add 2 to the x value of the key point. The new key point is (1,0).

$$f_{(x)} = \left|\log_2 x\right|$$

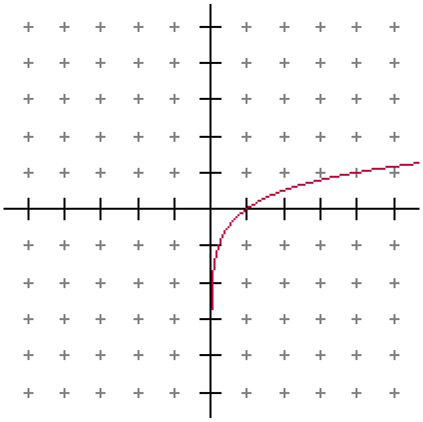


Notice the negative portion of the graph reflected above the x axis.

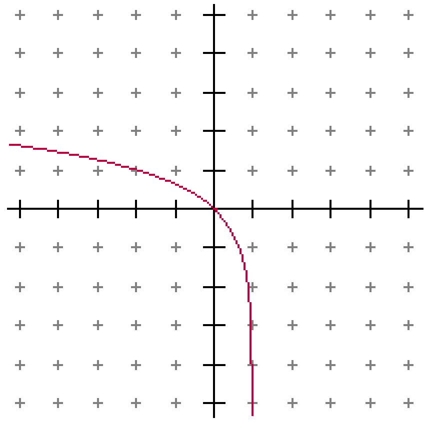
Match the appropriate graph with its equation below. Explain why each of your solutions is

true.

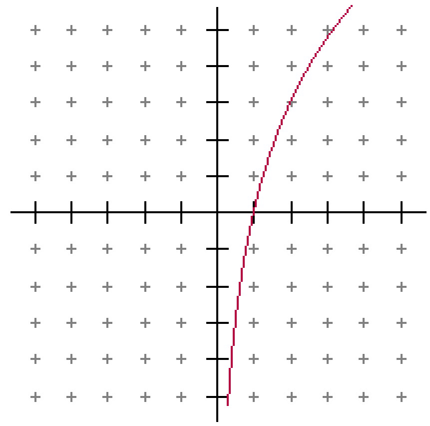
A



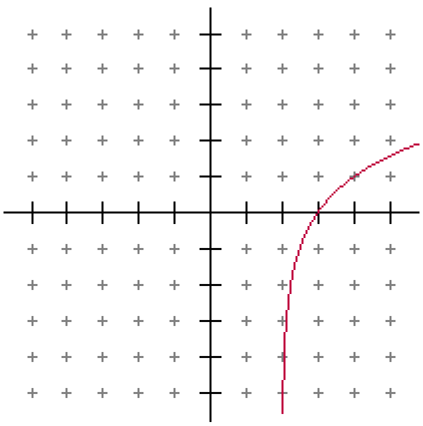
B



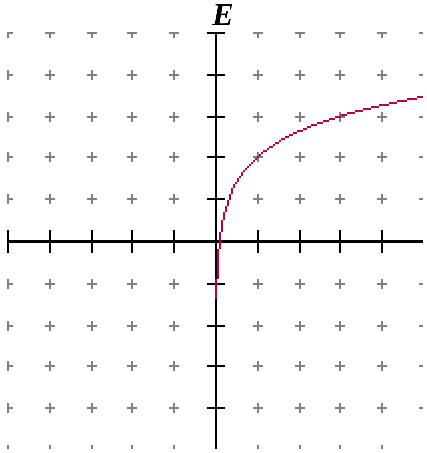
C



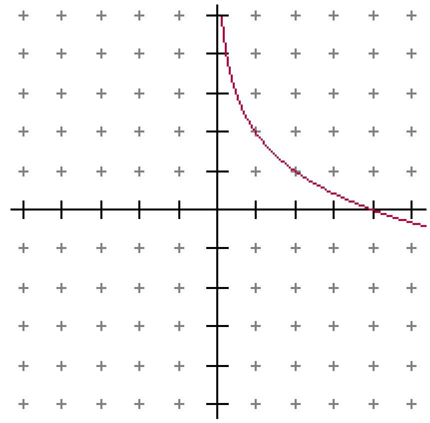
D



E



F



1) $f_{(x)} = \log_2(x-2)$

2) $f_{(x)} = \log_3(1-x)$

3) $f_{(x)} = -\log_2 x + 2$

4) $f_{(x)} = \log_3 x + 2$

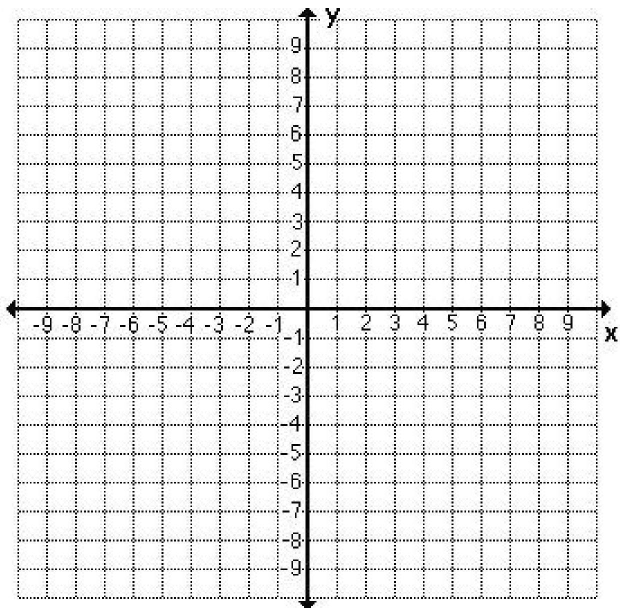
5) $f_{(x)} = \frac{1}{2}\log_2 x$

6) $f_{(x)} = 3\log_2 x$

The translation of a logarithmic function is almost identical to that of an exponential function. Just make sure to identify on which side of the vertical asymptote the graph of the function will reside. This will determine which key point to begin with. Remember to draw both asymptotes to graph the function and watch for the value of "a" which will affect key point.

Graph each of the following logarithmic functions by finding the asymptotes and labeling the key point. Be sure to find the x intercept and y intercept (if they exist).

A) $f_{(x)} = \log_3 x + 2$



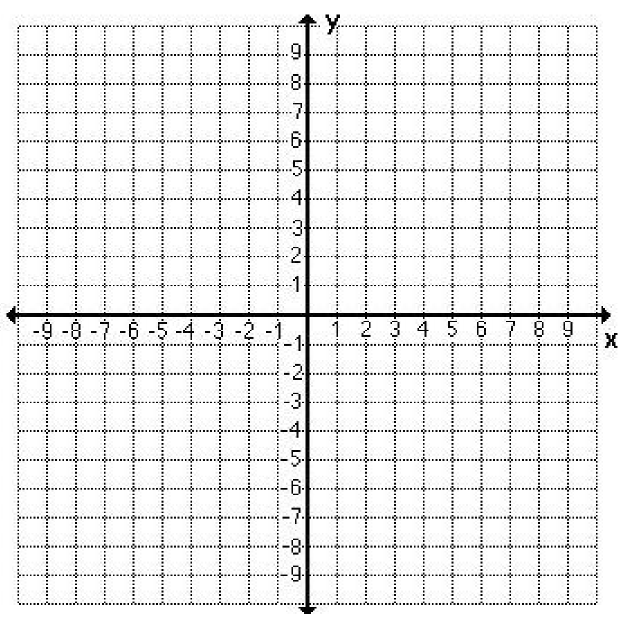
Y-intercept:

X-intercepts:

Range:

Domain:

B) $f_{(x)} = \log_2 (x + 3)$



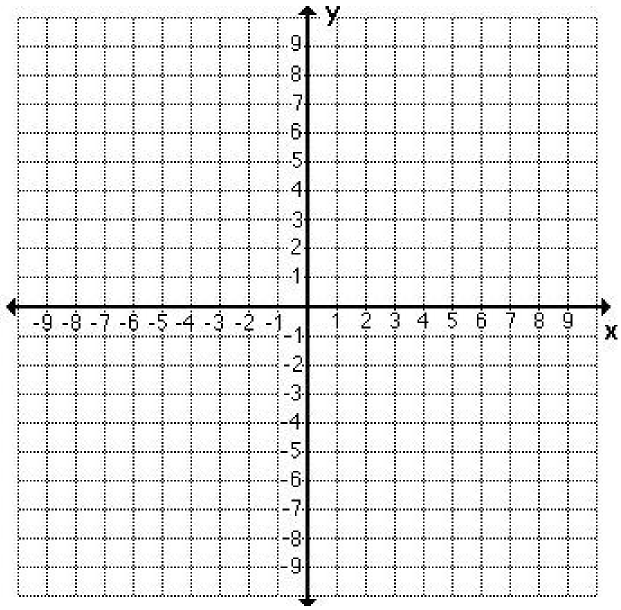
Y-intercept:

X-intercepts:

Range:

Domain:

C) $f_{(x)} = -\log_2 x + 4$



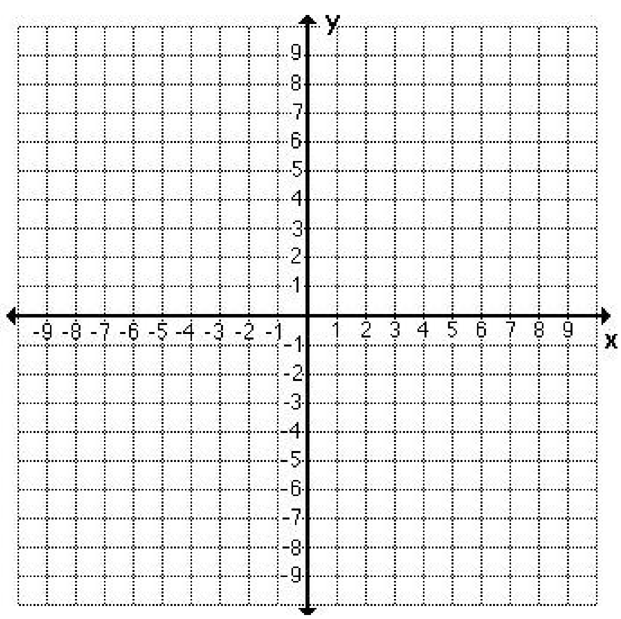
Y-intercept:

X-intercepts:

Range:

Domain:

D) $f_{(x)} = \ln(-x)$



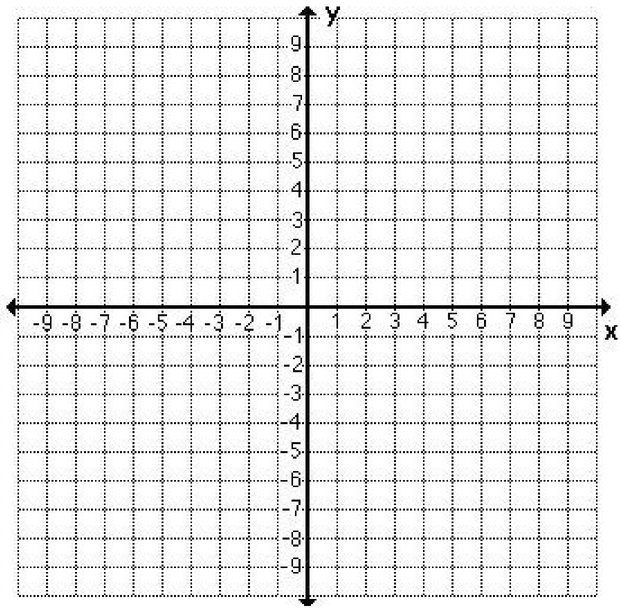
Y-intercept:

X-intercepts:

Range:

Domain:

E) $f_{(x)} = -\ln(x-2)$



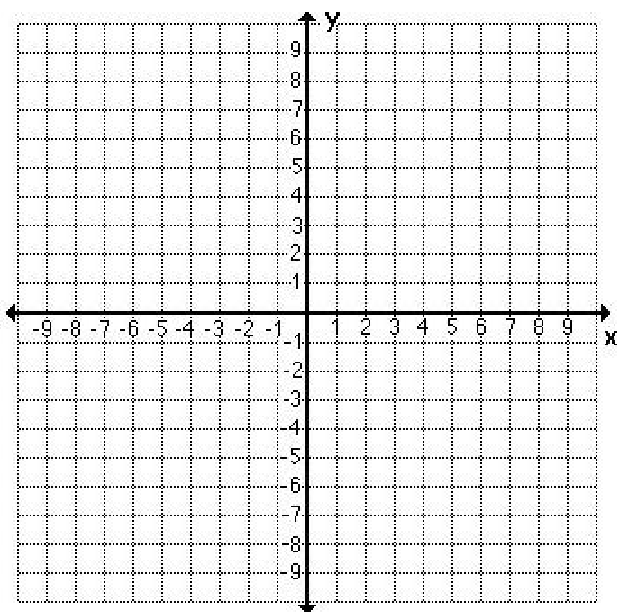
Y-intercept:

X-intercepts:

Range:

Domain:

F) $f_{(x)} = \log_2(3-x)+2$



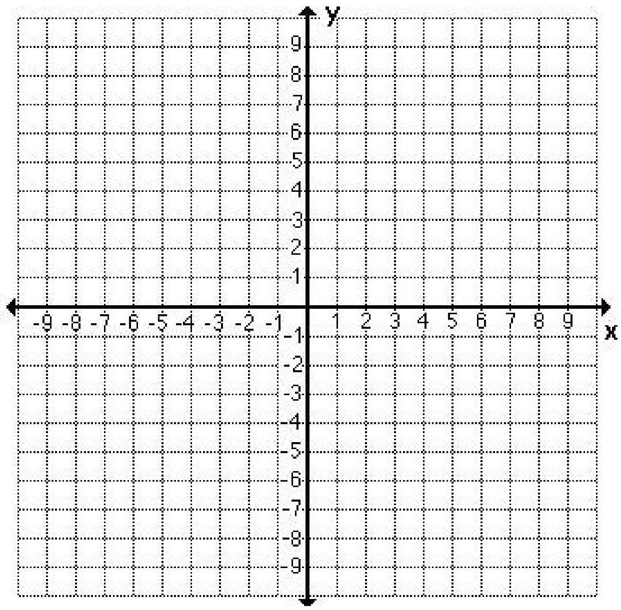
Y-intercept:

X-intercepts:

Range:

Domain:

G) $f_{(x)} = |\ln x|$



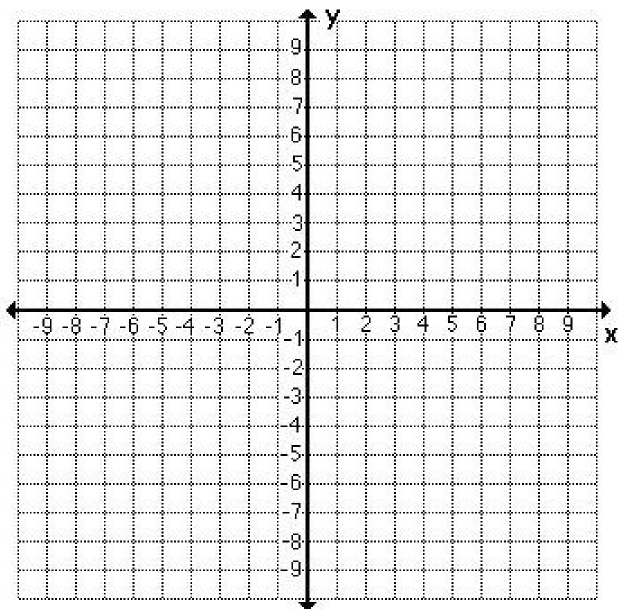
Y-intercept:

X-intercepts:

Range:

Domain:

H) $f_{(x)} = \ln|x|$



Y-intercept:

X-intercepts:

Range:

Domain:

All standard logarithmic functions (meaning a function without absolute value symbols), must have an x intercept. All standard exponential growth and decay functions must have a y intercept. Are these two statements true? Why or why not?

In order to find the domain of the logarithmic function $f_{(x)} = \log_4(x+5) - 3$**, we need to evaluate** $x+5>0$**. Why must we use this inequality?**

What is the problem with relying on a graphing calculator to graph a logarithmic function?