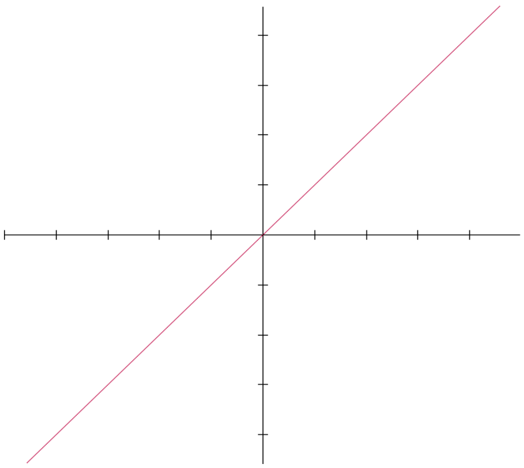
<u>Piece-Wise Functions</u>

The best way to describe a piece-wise function is to look at a simple example. Consider the absolute value function.

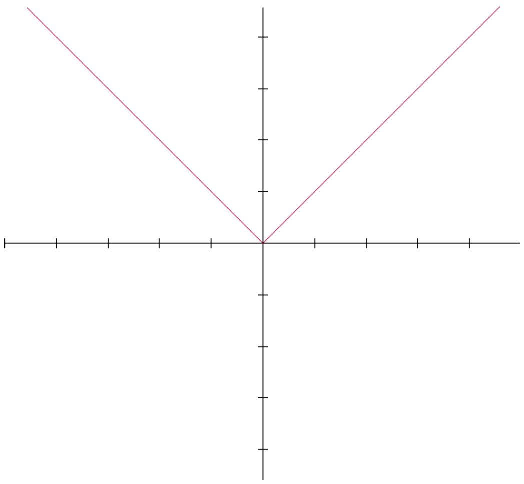
$y = x$



This is the graph of the function $y = x$. In this case, the x and y values of coordinates are identical. For example, (-3,-3). You can see the x and y values are the same.

Now, lets take a look at what happens when we want the absolute value of x.

$y = |x|$



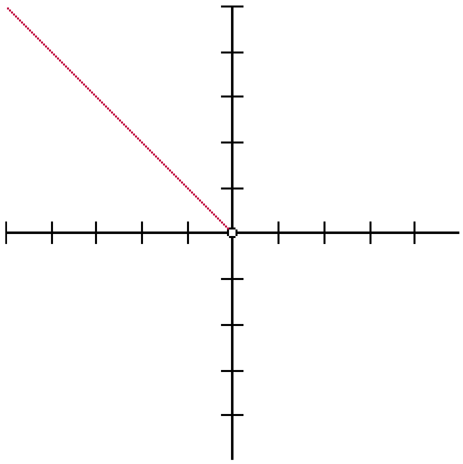
If the graph of $y = x$ above is $f_{(x)}$, the function to the left is $|f_{(x)}|$. We know that the absolute value of a number cannot be negative. If we take the absolute value of $f_{(x)}$, it would cause the left portion of the original graph to reflect above the x axis. This results in all y values of the function being positive. This is where the graph of the absolute value of x comes from.

It is possible to get this same graph if the linear function $y = x$ is graphed in the interval $[0, \infty)$, and the function $y = -x$ in the interval $(-\infty, 0)$. What happens here is a specific section of two different graphs is drawn. If these two sections are placed together on the same plane, it would result in the graph of $y = |x|$. Such an equation would appear as follows.

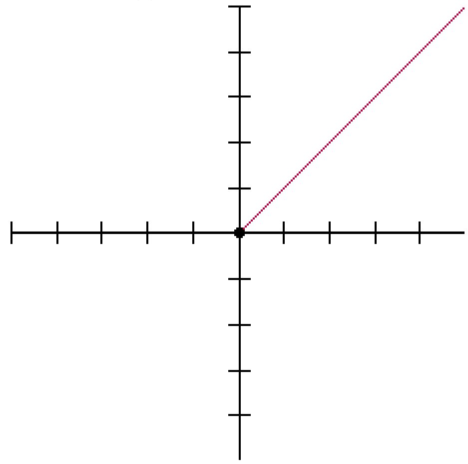
$$f_{(x)} = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$f_{(x)} = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$f_{(x)} = -x, \quad x < 0$

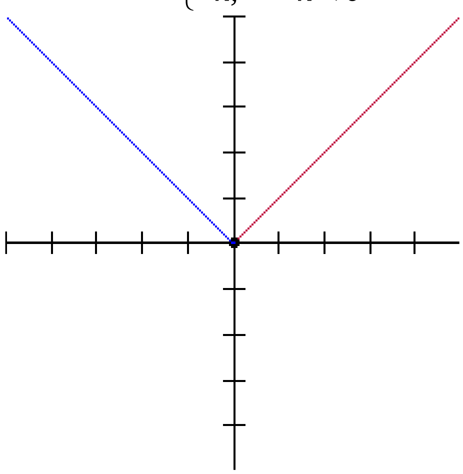


$f_{(x)} = x, \quad x \geq 0$



Notice the graph of the function on the left has a hole when x = 0. Since the function does not exist when x is zero, because of the domain of the function, an open circle must be used at that point. A pronounced dot is placed on the function to the right, showing the graph of the function in the interval $x \geq 0$***. This is how you plot a point when you have the greater than or equal to sign in the domain of the function. If both of these functions are placed on the same plane, the result would be as follows.***

$$f_{(x)} = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

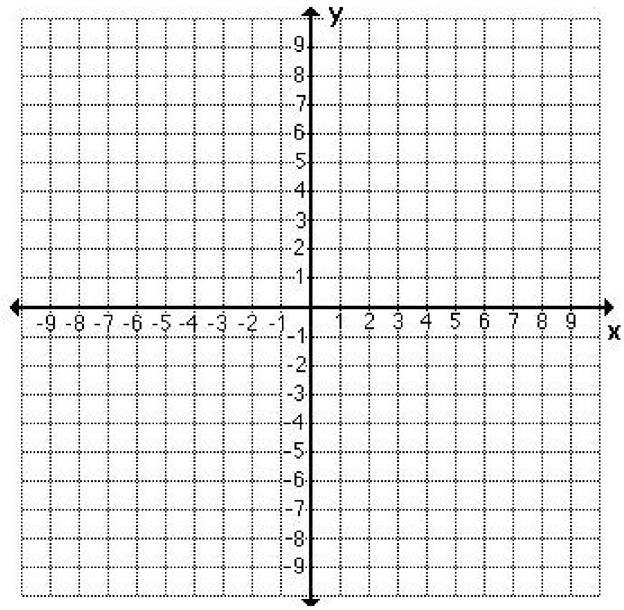


Since the open circle and solid dot are right on top of each other, the dot would "fill in" the hole on the first curve. This results in a continuous function. If the hole were still there, the function would have a removable discontinuity. If there is a complete break in the curve, from one portion of the graph to the next, it would be a discontinuous function.

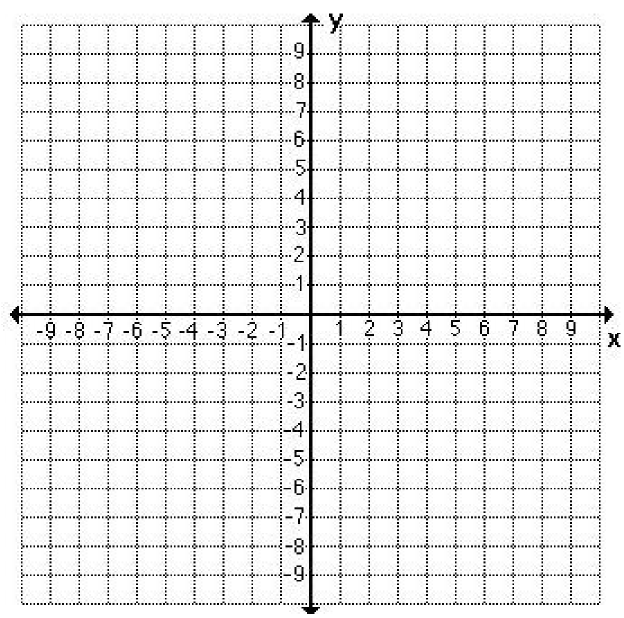
The simplest way to graph a piece-wise function is to graph the entire function and erase the portions that are not needed. This will be done for each part of the overall graph until a complete picture has been created. Very simply, a piece-wise function is just as it sounds, pieces of different functions put together to create one graph. Some examples of piecewise functions can be found in the **"Translations of Functions"** ***section of this chapter.***

Graph each of the following piece-wise functions.

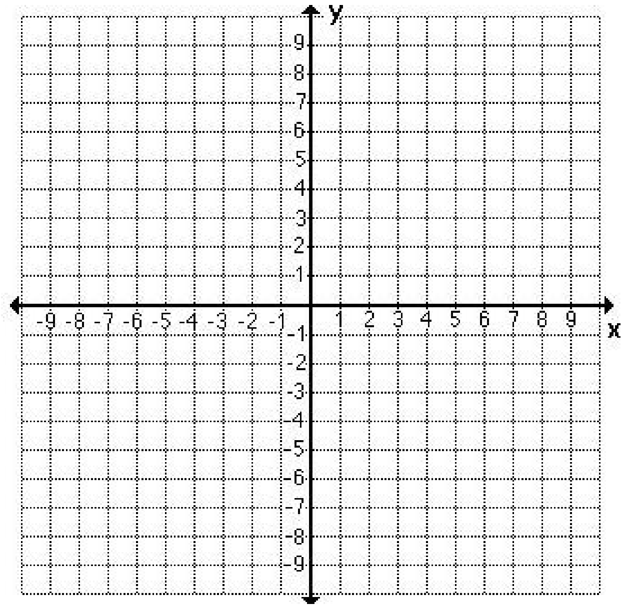
A) $f_{(x)} = \begin{cases} x+5, & x \le -3 \\ 2, & -3 < x < 1 \\ x+1, & x \ge 1 \end{cases}$



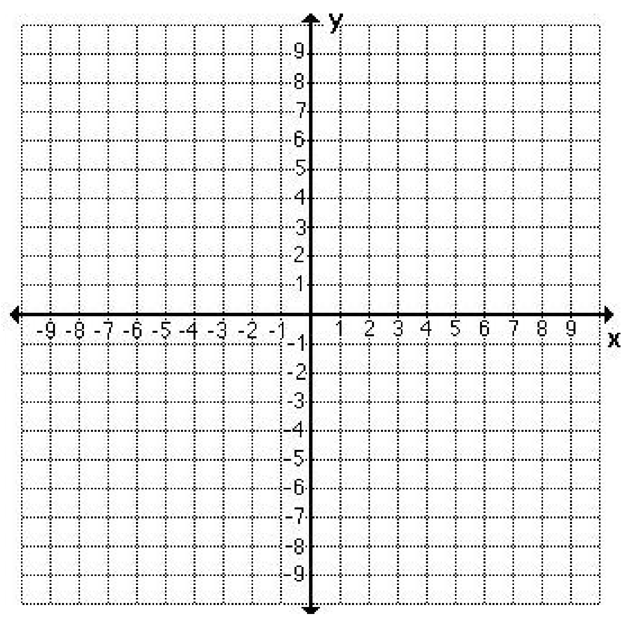
B) $f_{(x)} = \begin{cases} -(x-2)^2, & x < 2 \\ (x-2)^2, & x \ge 2 \end{cases}$



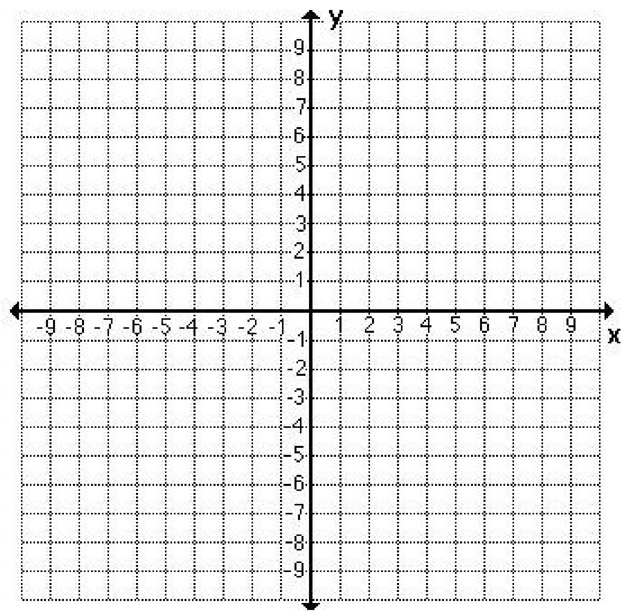
C) $f_{(x)} = \begin{cases} x+2, & x \le -4 \\ -2, & -4 < x < 3 \\ (x-3)^2 - 3, & x \ge 3 \end{cases}$



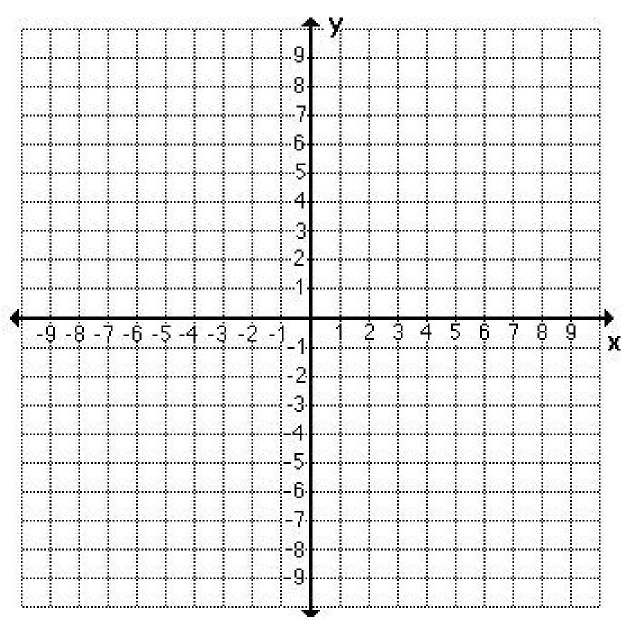
D) $f_{(x)} = \begin{cases} 3, & x < -1 \\ -x+2, & -1 \le x \le 2 \\ \frac{1}{3}(x-2)^2, & x > 2 \end{cases}$



E) $f_{(x)} = \begin{cases} \frac{3}{5}\left|x+4\right| - 2, & x < 1 \\ \sqrt{x-1} + 3, & x > 1 \end{cases}$

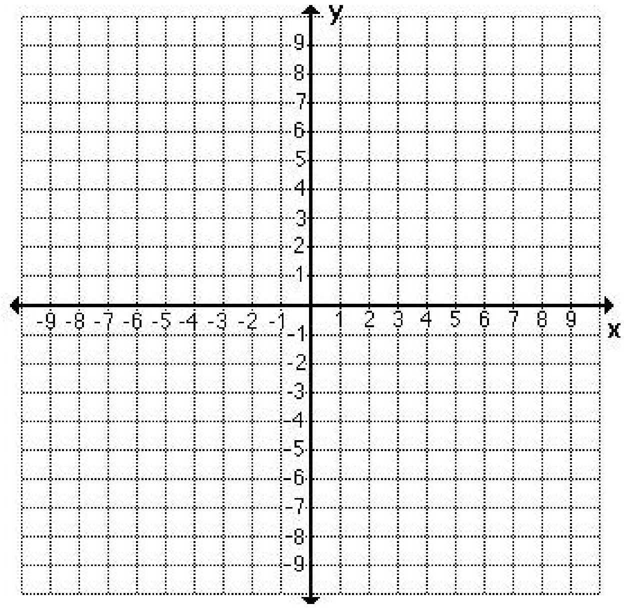


F) $f_{(x)} = \begin{cases} -\frac{2}{3}(x+4)^2 + 3, & -7 \le x < -4 \\ 3, & -4 \le x < 1 \\ -5x + 8, & 1 \le x < 2 \\ \frac{1}{3}(x-2)^2 - 2, & 2 \le x \le 5 \end{cases}$



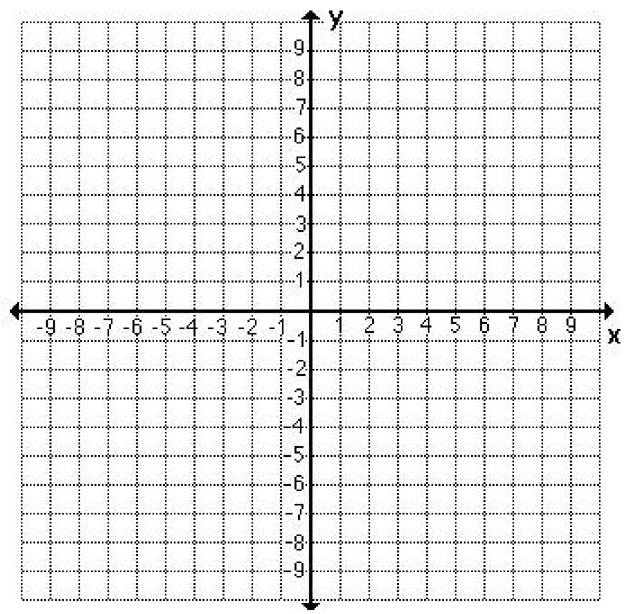
G)

$$f_{(\,)} = \begin{cases} 5, & -7 \le x \le -4 \\ -(x+3)^2 + 9, & -4 < x < 0 \\ \frac{2}{3}x + 1, & 0 \le x \le 4 \end{cases}$$



H)

$$f_{(x)} = \begin{cases} \log_2(-x), & -8 \le x < -1 \\ |x| - 1, & -1 \le x \le 1 \\ \log_3 x, & 1 < x \le 9 \end{cases}$$



Which if any of the piece-wise functions you just graphed are discontinuous?