RADICALS

Identify the parts of a radical expression.

$$\sqrt[n]{\#}$$

If you do not see the index of a radical expression, it is always two.

Question	Meaning
$\sqrt{36}$	What number times itself is 36?
$\sqrt[3]{27}$	What number times itself 3 times is 27?

The index how many like factors are needed to pull a number out of the radical.

Examples

Simplify the following.

$\sqrt[3]{32}$ $\qquad$ $\sqrt[4]{64x^8y^4}$

Simplify:

$7\sqrt[3]{40}$ $6\sqrt{500}$

If a radical already has a coefficient and it is simplified, multiply the coefficients together.

Evaluate: $\sqrt{25}$

a) 5 b) -5 c) 25 d) both a and b

PROPERTIES USED IN SIMPIFYING RADICAL EXPRESSIONS

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

Properties for simplification continued:

$$a^{\frac{m}{n}} = \left(a^m\right)^{\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^m$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}$$

$$\left(\sqrt[n]{a}\right)^n = a$$

$\sqrt[n]{a^n} = a$ if and only if n is odd.

$\sqrt[n]{a^n} = |a|$ if and only if n is even.

PROPERTIES FOR OPERATIONS WITH RADICALS

Addition: $$a\sqrt[n]{c} + b\sqrt[n]{c} = (a+b)\sqrt[n]{c}$$

To add any two radical expressions, both the index and radicand Must match. If the index and radicand match, only add the Coefficients of the expression.

Note, it will sometimes be necessary to simplify the radical first.

Examples of adding radical expressions

$6\sqrt{7} + 2\sqrt{7} - 5\sqrt{7}$

$3\sqrt{12} + 5\sqrt{27}$

$2\sqrt[3]{16} - 4\sqrt[3]{54}$

$5\sqrt[3]{32x^6y^4} - 3xy\sqrt[3]{500x^3y}$

$12\sqrt{32} + 6\sqrt{12} - 10\sqrt{50}$

Multiplication: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

Generally in order to multiply any two radical expressions, they must have the same index. There is however, a very rare exception to this.

Examples of multiplying radical expressions:

$3\sqrt{5} \cdot 4\sqrt{30}$

$\sqrt[3]{16} \cdot 5\sqrt[3]{32}$

$\sqrt[3]{4x} \cdot \sqrt[3]{18x^2}$

$3\sqrt{6}\left(4\sqrt{2} - 5\sqrt{3}\right)$

$\sqrt[4]{8}\left(5\sqrt[4]{32} + 6\sqrt[4]{18}\right)$

Multiplication Continued:

$\left(3\sqrt{5}\right)^2$ $\left(6\sqrt{3}+\sqrt{2}\right)\left(6\sqrt{3}-\sqrt{2}\right)$ $\left(3\sqrt{2}+4\right)^2$

$$\sqrt[3]{5x^5y^2z^3}\cdot\sqrt[3]{4xy^5z}\cdot\sqrt[3]{50x^2y^4z^2}$$

Division: $\frac{\sqrt{35}}{\sqrt{7}}$

This is a simple division problem of a monomial divided by a monomial
This can be done one of three ways.

Using the Power Rule	**Using Factors**	**Rationalize the Denominator**
$\frac{\sqrt{35}}{\sqrt{7}}$	$\frac{\sqrt{35}}{\sqrt{7}}$	$\frac{\sqrt{35}}{\sqrt{7}}$

When dividing radical expressions, most often we rationalize the denominator.

What does "Rationalize" mean.

If a term has already been simplified and the radical is still present, then it is an Irrational Number (decimal values that neither nor repeat or terminate). Rationalization is the process of changing a number from Irrational to Rational. Typically the denominator is Rationalized, however, there are times were the numerator will be Rationalized in the future.

Rationalize the following:

$$\frac{\sqrt{15}}{2\sqrt{3}}$$

$$\frac{16}{\sqrt[3]{5}}$$

$$\frac{\sqrt[4]{3x}}{\sqrt[4]{9x^2y^3}}$$

Multiply the numerator and denominator by whatever is needed to pull the radicands out of the radical in the denominator.

What do I need in order to pull the 3 x and y out of the denominator?

Divide: $$\frac{4}{2\sqrt{6}-\sqrt{5}}$$

$$\frac{\sqrt{30}-\sqrt{15}}{2\sqrt{6}}$$

$$\frac{12}{\sqrt{5}+3}$$

$$\frac{3\sqrt{2}-5}{\sqrt{3}+2}$$