The Domain of a Rational Function

The domain of a rational function is found using only the vertical asymptotes. As previously noted, rational functions are undefined at vertical asymptotes. The rational function will be defined at all other x values of the domain.

$$f_{(x)} = \frac{x}{(x+2)(x-3)}$$

Here is a rational function in completely factored form.

$$x = -2 \quad and \quad x = 3$$

Since the zeros of the denominator are -2 and 3, these are the vertical asymptotes of the function.

Therefore, the domain of this function is $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$. ***Notice there are two vertical asymptotes, and the domain is split into three parts. This pattern will repeat. If there are 4 vertical asymptotes, the domain of that function will be split into 5 parts.***

Find the domain of each of the following rational functions.

A) $f_{(x)} = \frac{x-7}{x+5}$

B) $f_{(x)} = \frac{3}{x^2-4}$

C) $f_{(x)} = \frac{x^2}{x-5}$

D) $f_{(x)} = \frac{2x^2-5x+3}{x-1}$

E) $f_{(x)} = \frac{x-8}{x^3-x^2-12x}$

F) $f_{(x)} = \frac{x^3}{x^2-7x+12}$

G) $f_{(x)} = \frac{1}{3-x}$

H) $f_{(x)} = \frac{x^2-4}{x^4-81}$

I) $f_{(x)} = \frac{x^3-2x^2+5}{x^2}$

<u>Finding Intercepts of Rational Functions</u>

We have found that the zeros of the denominator of a rational function are the vertical asymptotes of the function. The zeros of the numerator on the other hand, are the x intercepts of the function.

Find all x and y intercepts of the function $f_{(x)} = \frac{x^2-9}{x-1}$.

$$f_{(x)} = \frac{(x+3)(x-3)}{x-1}$$

Write out the function in completely factored form.

Now, find the zeros of the numerator

$$x = -3 \quad and \quad x = 3$$

These are the x intercepts of the function.

Look at the original function.

$$f_{(x)} = \frac{x^2-9}{x-1}$$

From here, substitute zero for x, and find the y intercept, which in this case will be the ratio of the two constants.

$$y = 9$$

This is the y intercept of the function. In this case, it is the ratio of the two remaining constants once zero is substituted in for x. If there is no constant in the denominator, then there will be no y intercept as x=0 is a vertical asymptote and the graph is undefined at the y axis.

The x intercepts are $(-3,0)$ *and* $(3,0)$

The y intercept is $(0,9)$

As demonstrated above, the y intercept of a rational function is the ratio of the two constants. Like always, substitute zero for x, and solve for y to find the y intercept.

Find the x and y intercepts of each rational function.

A) $f_{(x)} = \frac{x-7}{x+5}$

B) $f_{(x)} = \frac{3}{x^2-4}$

C) $f_{(x)} = \frac{x^2}{x-5}$

D) $f_{(x)} = \frac{2x^2-5x+3}{x-1}$

E) $f_{(x)} = \frac{x-8}{x^3-x^2-12x}$

F) $f_{(x)} = \frac{x^3}{x^2-7x+12}$