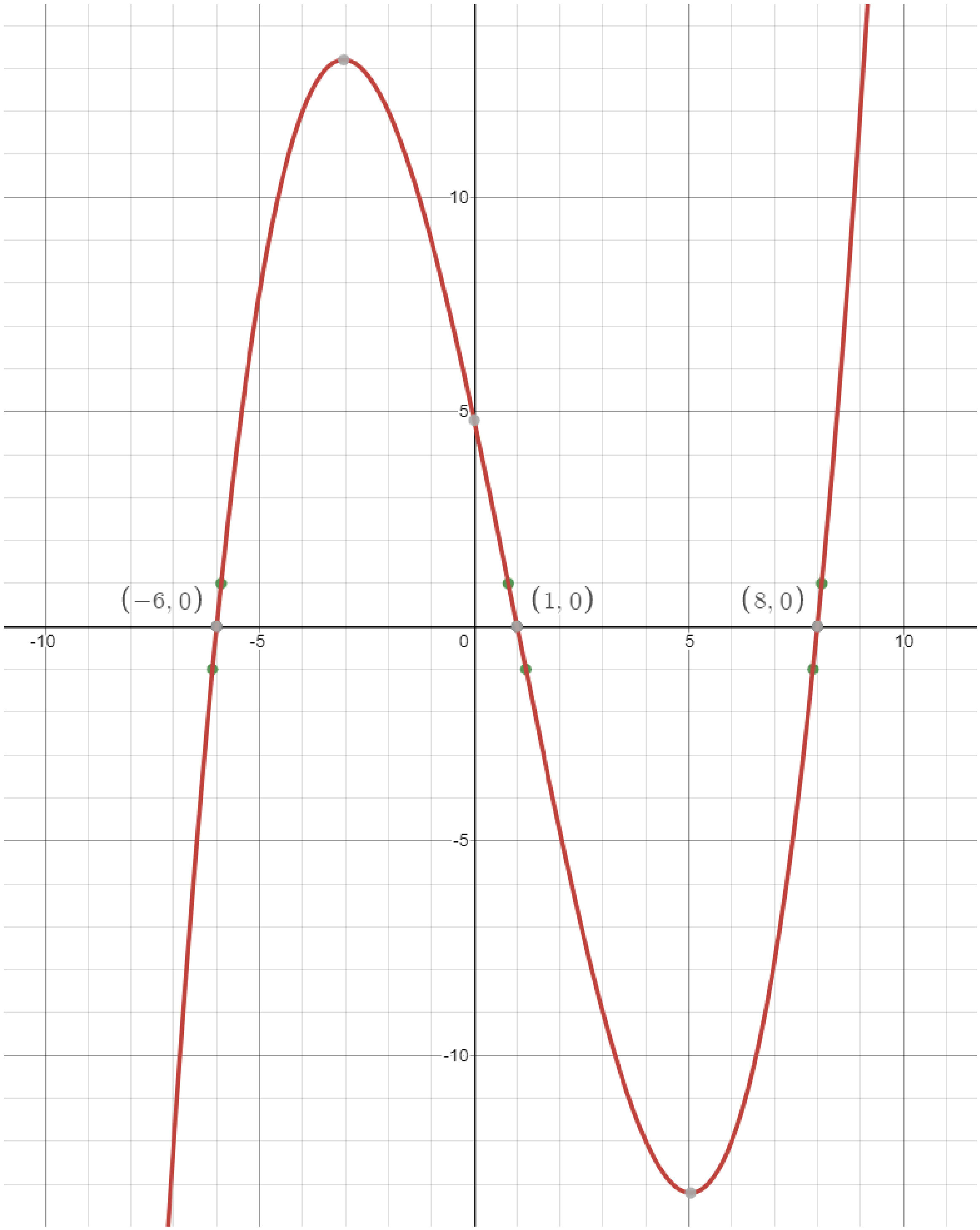
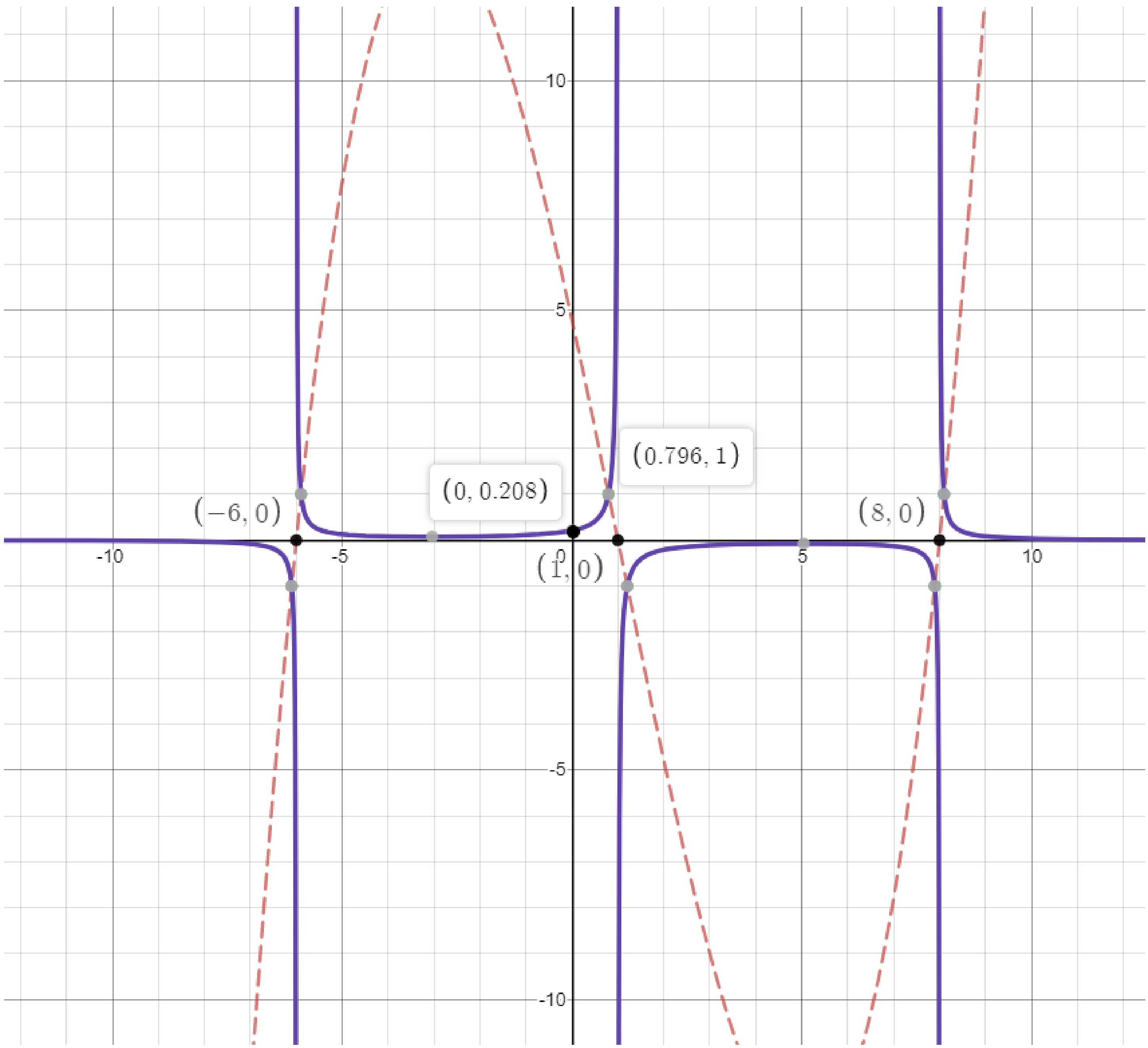
How Does a Rational Function Get It's Shape?





10
5
-5
-10
-10
-5
0
5
10
(−6, 0)
(0, 0.208)
(0.796, 1)
(1, 0)
(8, 0)

RATIONAL FUNCTIONS 2

$$f(x)=\frac{ax^{n}+bx^{n-1}+cx^{n-2}\ldots+k}{hx^{d}+ix^{d-1}+jx^{d-2}\ldots+l}$$

Vertical Asymptotes (V.A.): The vertical asymptotes are the zeros of the denominator.

For notes: $N \rightarrow$ Degree of the numerator
$D \rightarrow$ Degree of the denominator

Horizontal Asymptotes (H.A.):

- If N > D by more than one, there is no Horizontal Asymptote.
- If N = D, the H.A. is the ratio of the two leading coefficients.

Ex.

$$f(x)=\frac{3x^{2}+4x-5}{2x^{2}-8}$$

- If N < D the Horizontal Asymptote is at zero (y = 0).
- If N > D by 1 the function has an Oblique (Slant) Asymptote.
  To find the Oblique Asymptote divide the numerator by the Denominator. If there is a remainder, discard it.

Ex.

$$f(x)=\frac{x^{2}-4x-18}{x+2}$$

The y Intercept: To find the y intercept of the function substitute zero for x, and solve for y.
(This gives you the ratio of the two constants.)

The x intercept(s): To find the x intercept(s) of the function, set the numerator equal to zero and solve for x. (**The x intercepts are the zeros of the numerator**.)

The Range: The range of the function is completely dependent on the graph. (Keep the Horizontal Asymptote in mind.)

The Domain: The domain of a rational function is **all real numbers except for the zeros of the denominator**. (The zeros of the denominator are Vertical Asymptotes.)

Find all Asymptotes of the following functions.

1) $f(x) = \dfrac{x-6}{x^2-9}$ 2) $g(x) = \dfrac{3x-9}{x-2}$ $h(x) = \dfrac{2x^2-3x+1}{x+2}$

Find the domain of each function.

1) $f(x) = \dfrac{x-6}{x^2-9}$ 2) $g(x) = \dfrac{3x-9}{x-2}$ $h(x) = \dfrac{2x^2-3x+1}{x+2}$

This is the graph of a function f, describe the behavior of the function.

As x approaches ___, the value of the function approaches ____.

As $x \rightarrow -\infty,\ f(x) \rightarrow$ _____

As $x \rightarrow -3^{-},\ f(x) \rightarrow$ _____

As $x \rightarrow -3^{+},\ f(x) \rightarrow$ _____

As $x \rightarrow 5^{-},\ f(x) \rightarrow$ _____

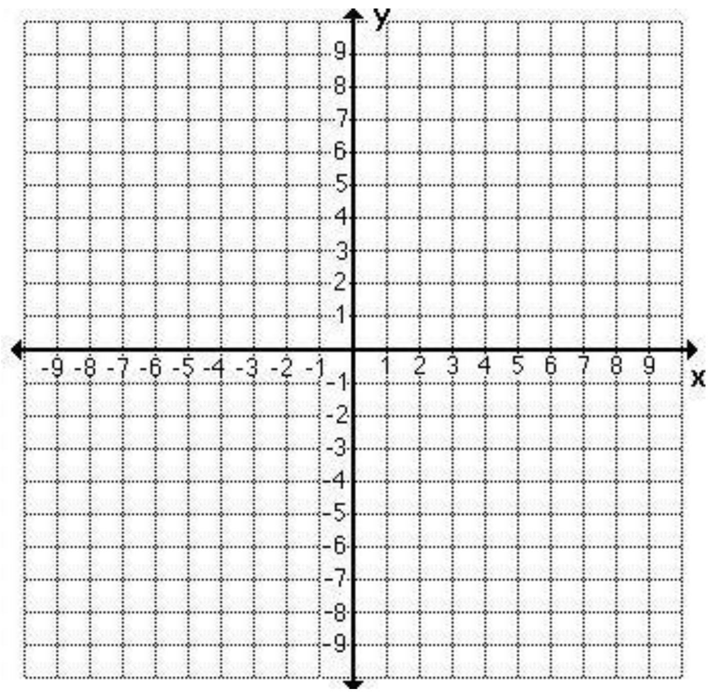
As $x \rightarrow 5^{-},\ f(x) \rightarrow$ _____

As $x \rightarrow \infty,\ f(x) \rightarrow$ _____

GRAPHING RATIONAL FUNCTIONS

The Parent Function

1. Graph: $f(x)=\frac{1}{x}$ on the right.



a) V. A.:

b) H.A. or O.A.:

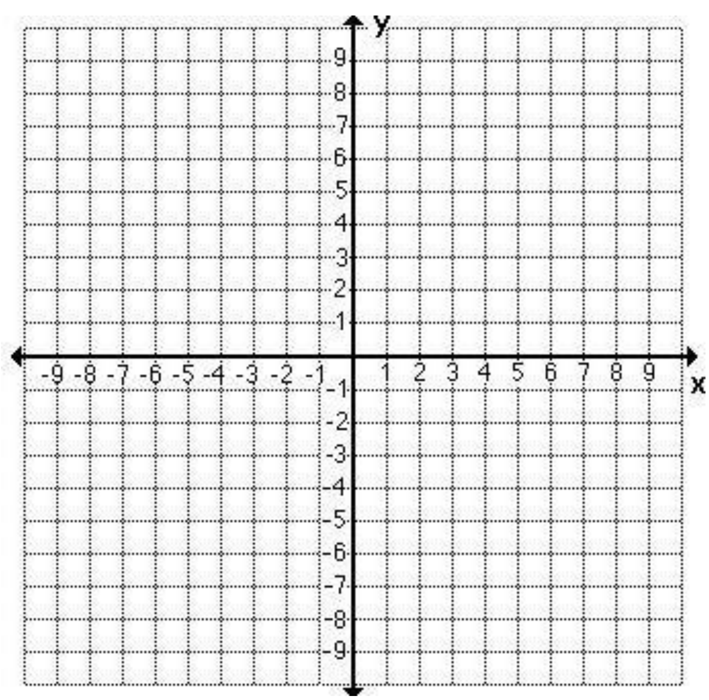
c) y-intercept:

d) x-intercepts:

e) Range:

f) Domain:

2. Graph: $f(x)=\frac{1}{x-3}$



a) V. A.:

b) H.A. or O.A.:

c) y-intercept:

d) x-intercepts:

e) Range:

f) Domain:

3. Graph: $f(x) = \frac{1}{x+2} - 3$

a) V. A.:

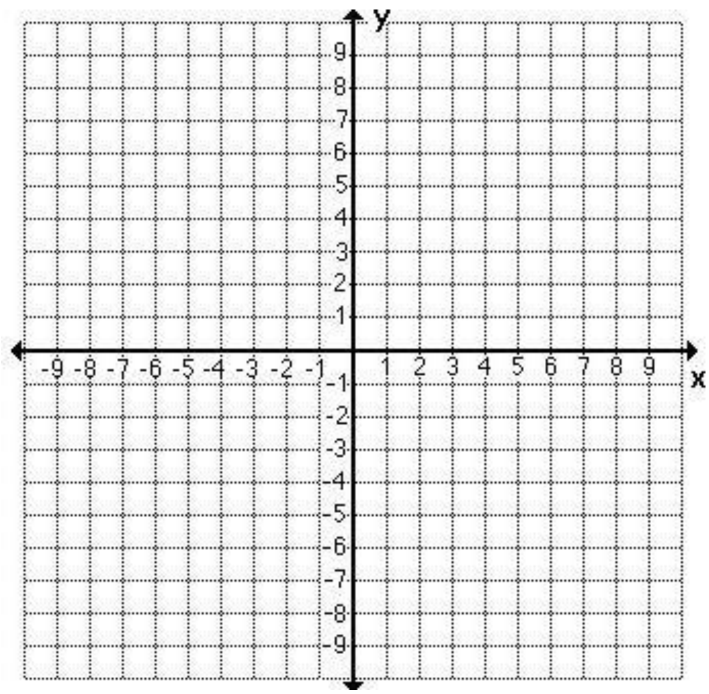
b) H.A. or O.A.:

c) y-intercept:

d) x-intercepts:

e) Range:

f) Domain:



4. Graph: $f(x) = \frac{2x}{x^2 - 4}$

a) V. A.:

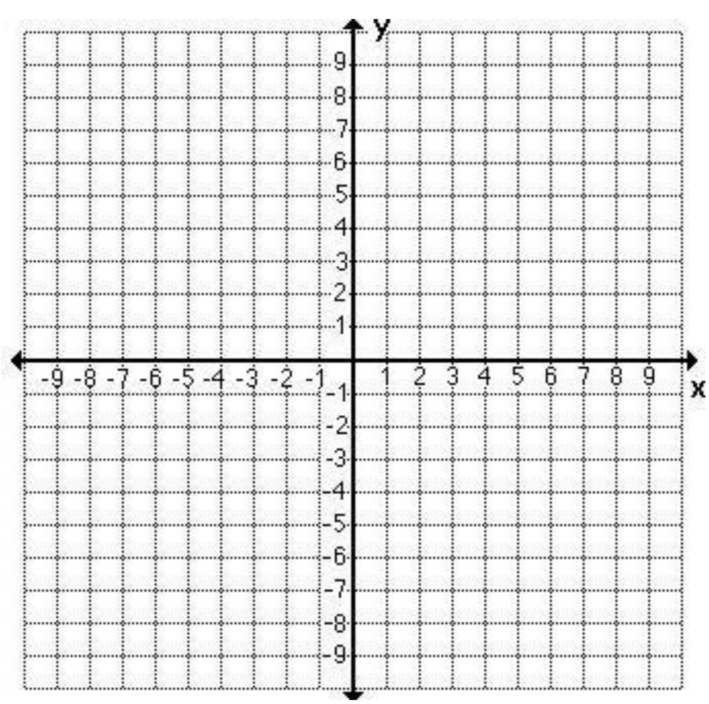
b) H.A. or O.A.:

c) y-intercept:

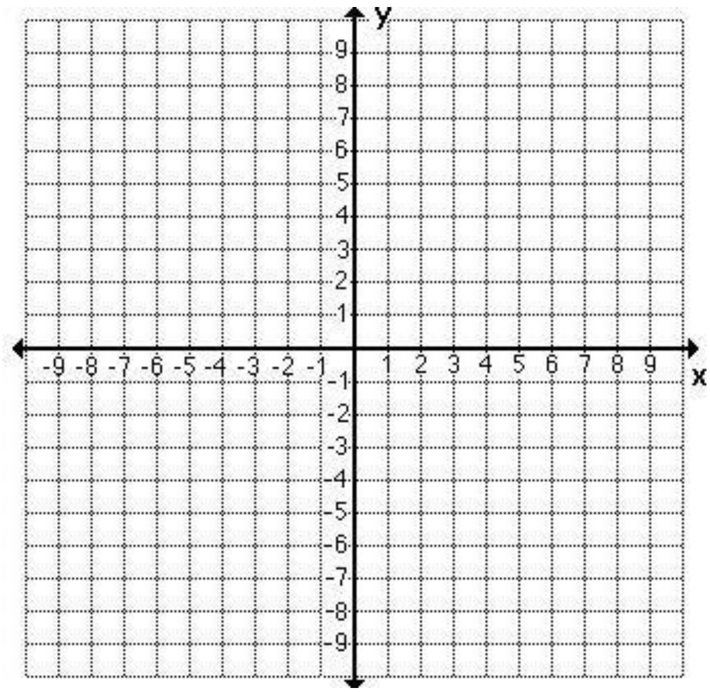
d) x-intercepts:

e) Range:

f) Domain:



5. Graph: $f_{(x)} = \dfrac{x^2 - x - 2}{x - 1}$



a) V. A.:

b) H.A. or O.A.:

c) y-intercept:

d) x-intercepts:

e) Range:

f) Domain:

SPECIAL CASE

6. Graph: $f(x) = \dfrac{x^2 + x - 6}{x^2 + 3x - 10}$

a) V. A.:

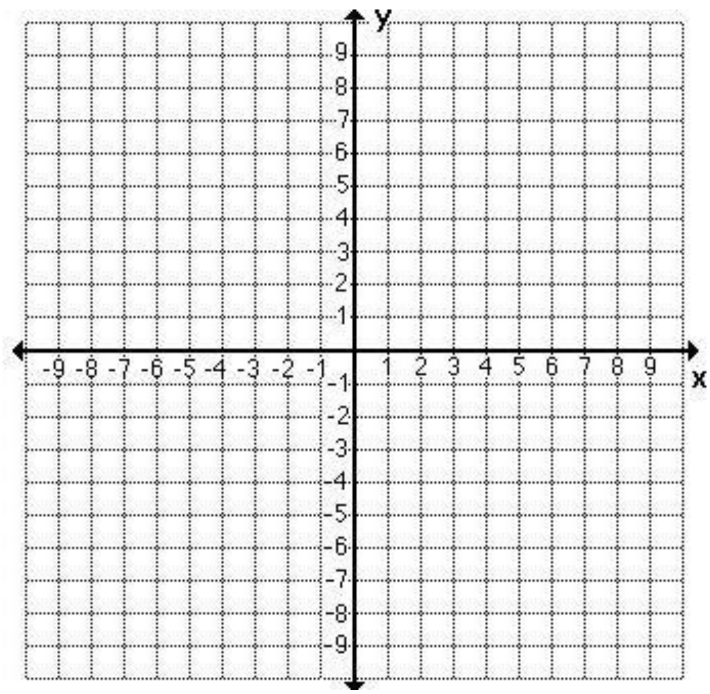
b) H.A. or O.A.:

c) y-intercept:

d) x-intercepts:

e) Range:

f) Domain:



FUN EXERCISE

Just for fun, try to work backwards and create a rational function that satisfies the following conditions.
(Your function may be left in factored form.)

a) The function has zeros of -3 and 3.

The function has a Horizontal Asymptote of $y = 5$.

The function has a Vertical Asymptote at $x = 0$.

$f(x) =$

b) The function has zeros of -5 and 2.

The function has a Horizontal Asymptote of $y = 0$.

The function has a Vertical Asymptote at $x = -2$.

$f(x) =$

c) The function has zeros of -3 and 5.

The function has a Horizontal Asymptote of $y = 2$.

The function is continuous.

$f(x) =$