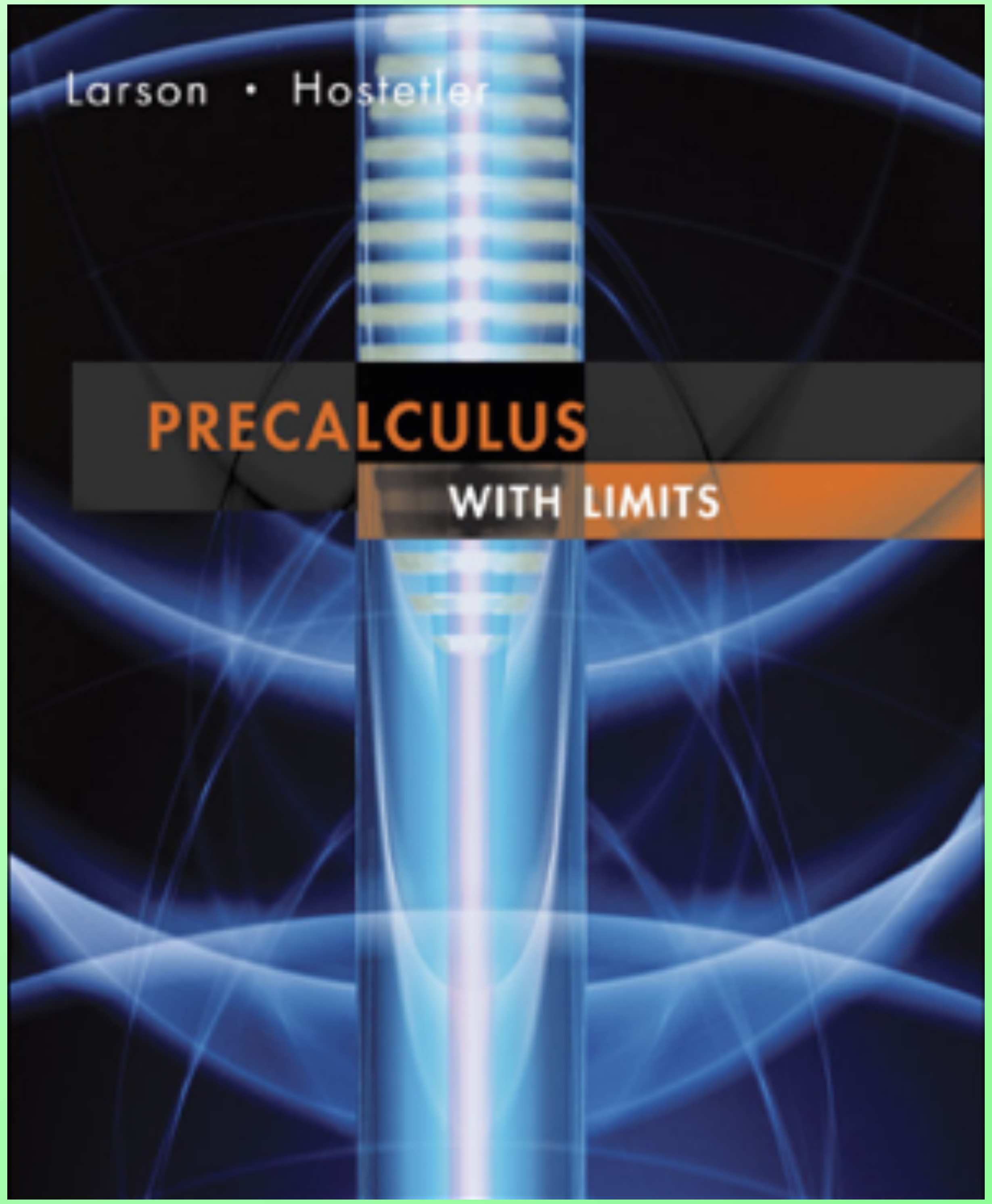
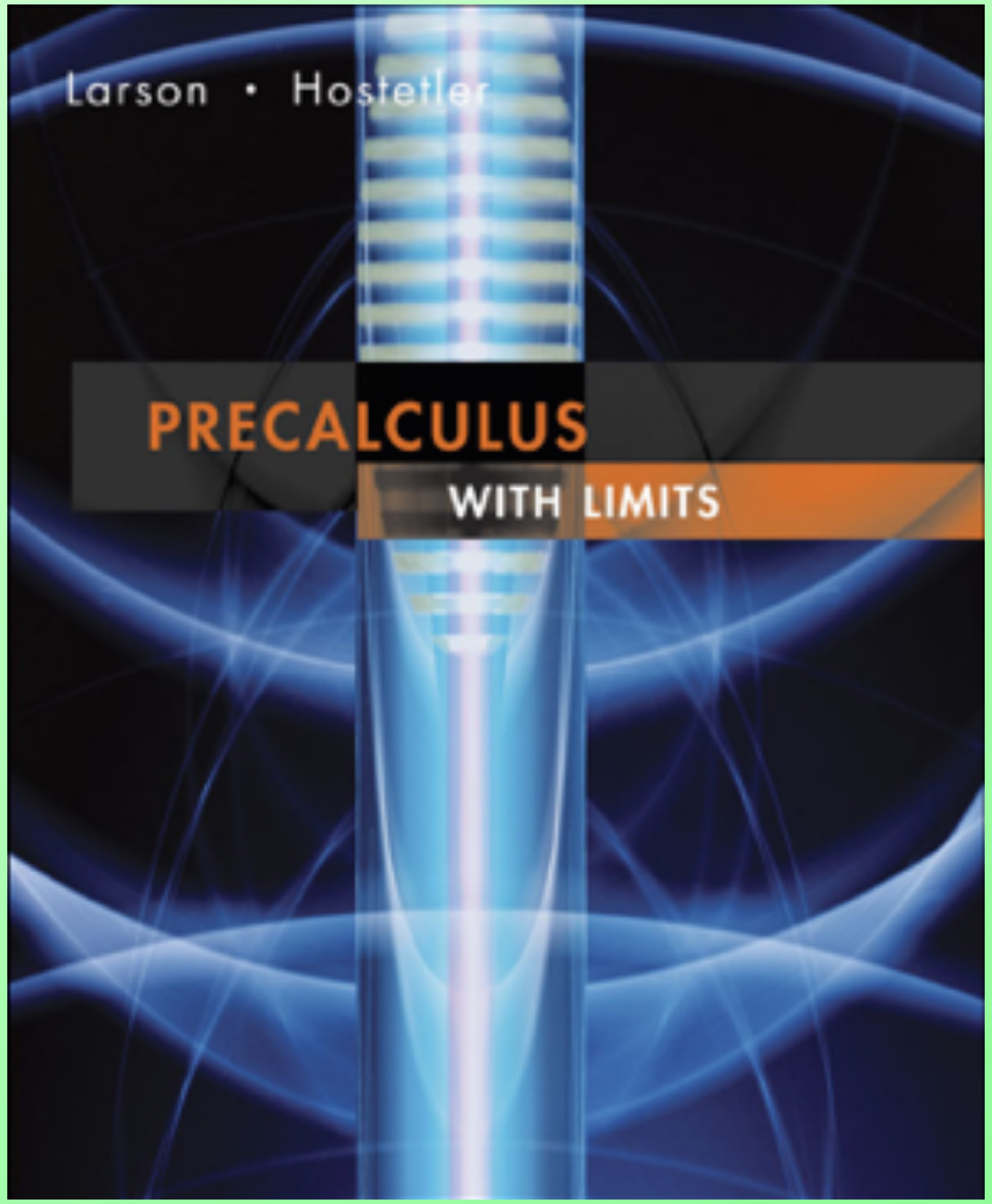
Chapter 4

Trigonometric Functions



∡ 4.5 Graphs of Sine and Cosine Functions

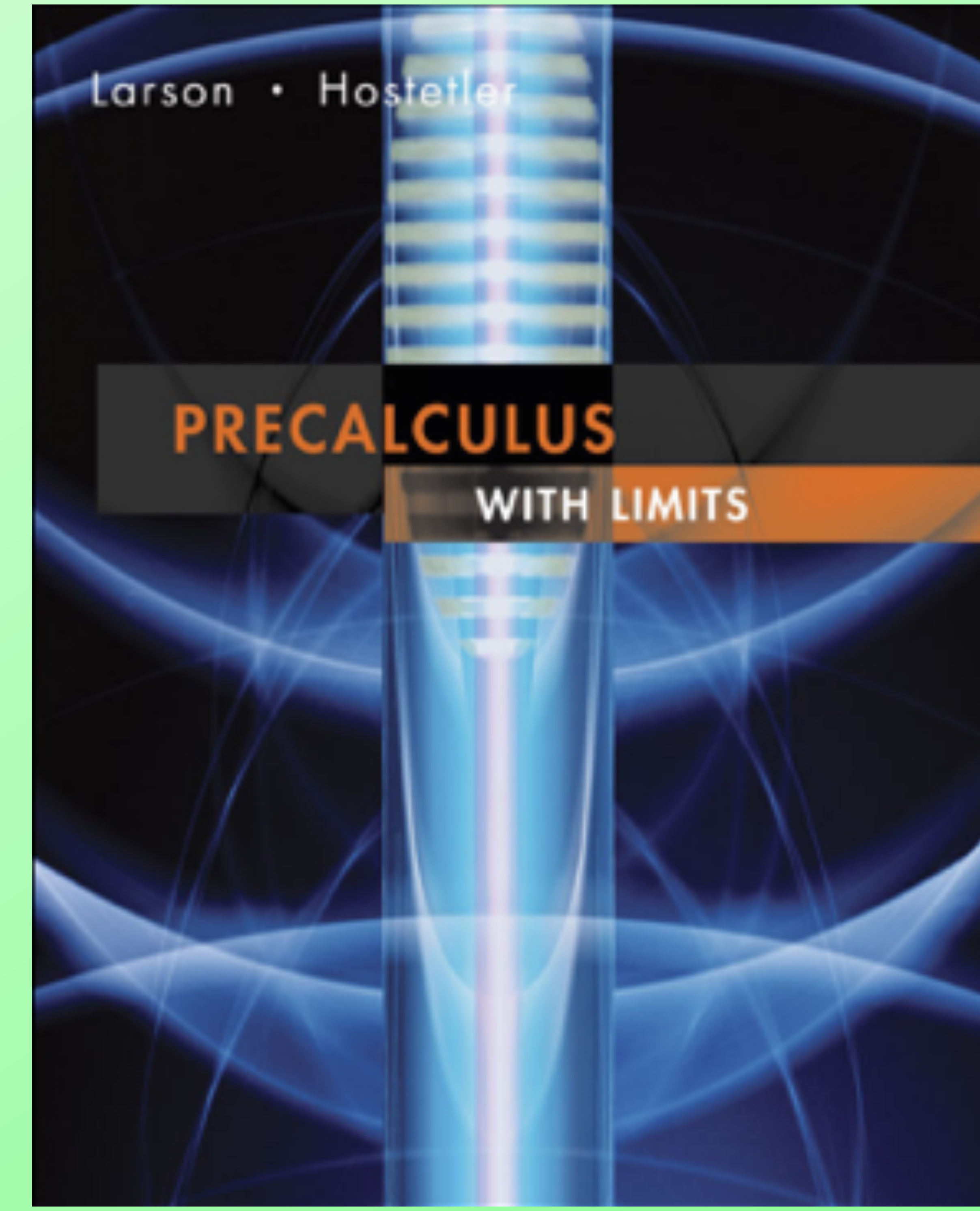
Chapter 4



Homework

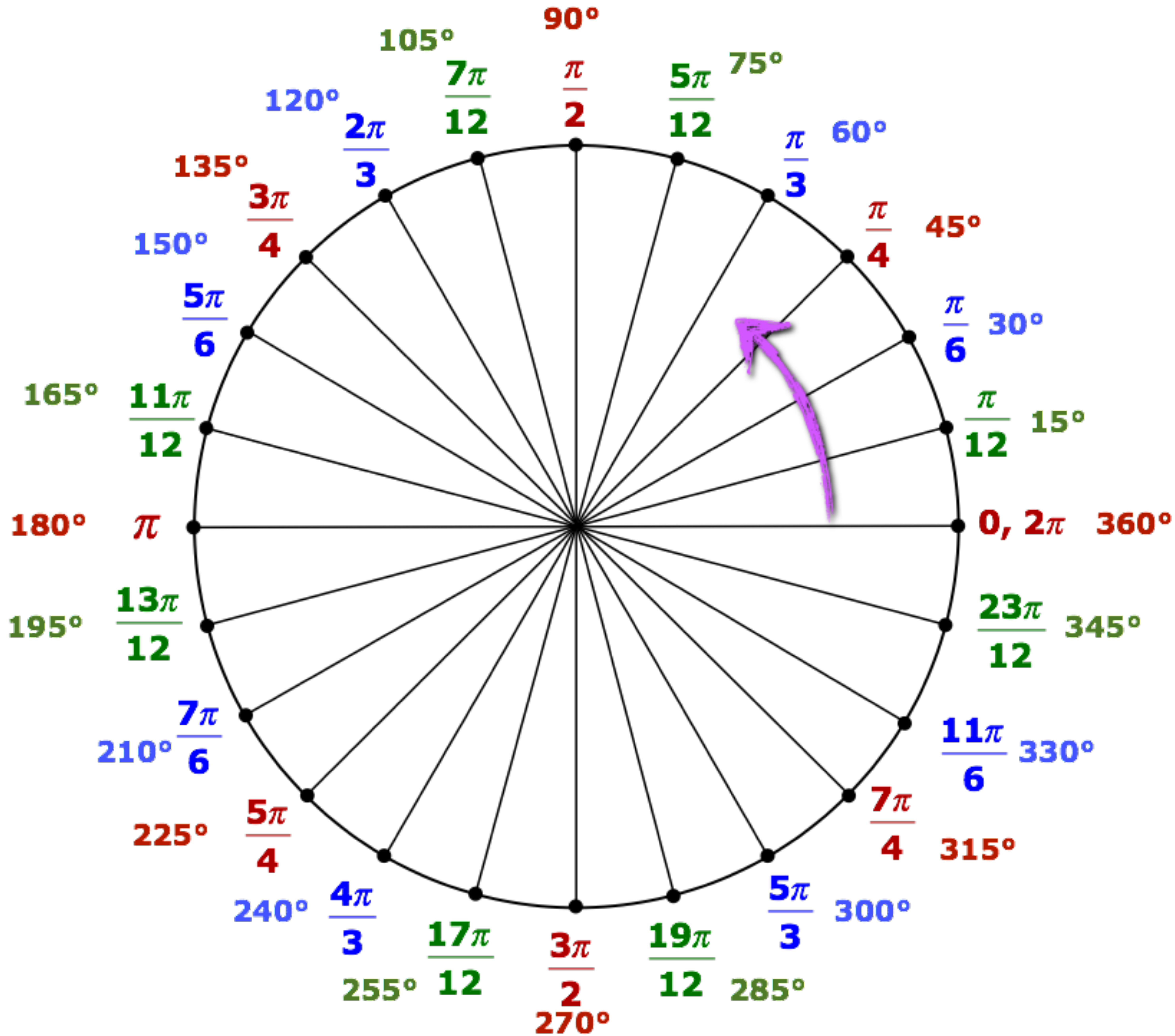
- 4.4 p328 1, 3, 5, 9, 13, 17, 29, 41, 49, 63, 69

Chapter 4



Objectives

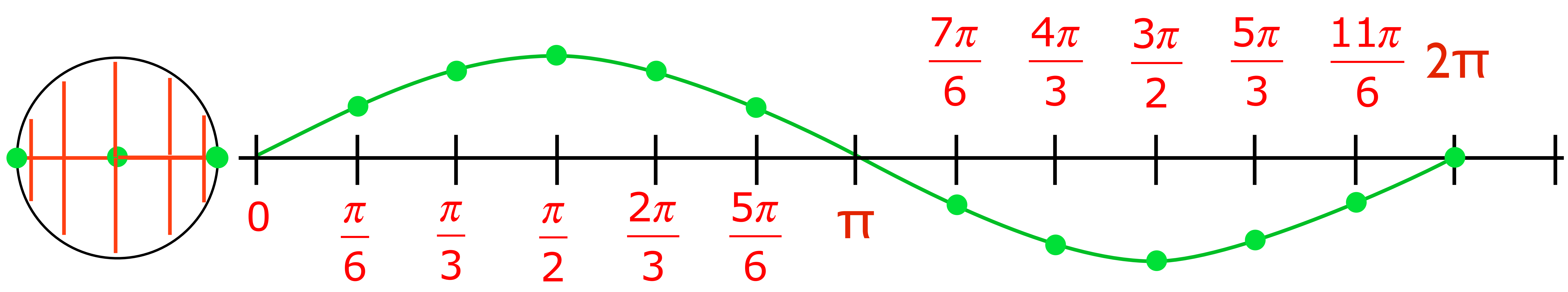
- Sketch the graph of $y = \sin x$.
- Sketch the graph of $y = \cos x$.
- Graph transformations of $y = \cos x$.
- Find Amplitude and Period of sine and cosine graphs.
- Graph vertical shifts of sine and cosine curves.
- Model periodic behavior.



90°
π/2
105°
7π/12
75°
5π/12
120°
2π/3
60°
π/3
135°
3π/4
45°
π/4
150°
5π/6
30°
π/6
165°
11π/12
15°
π/12
180°
π
0, 2π
360°
195°
13π/12
345°
23π/12
210°
7π/6
330°
11π/6
225°
5π/4
315°
7π/4
240°
4π/3
300°
5π/3
255°
17π/12
285°
19π/12
270°
3π/2

Graph of Sine Function

- The sine function can be graphed by plotting the points (x, y) from the unit circle onto the coordinate plane.



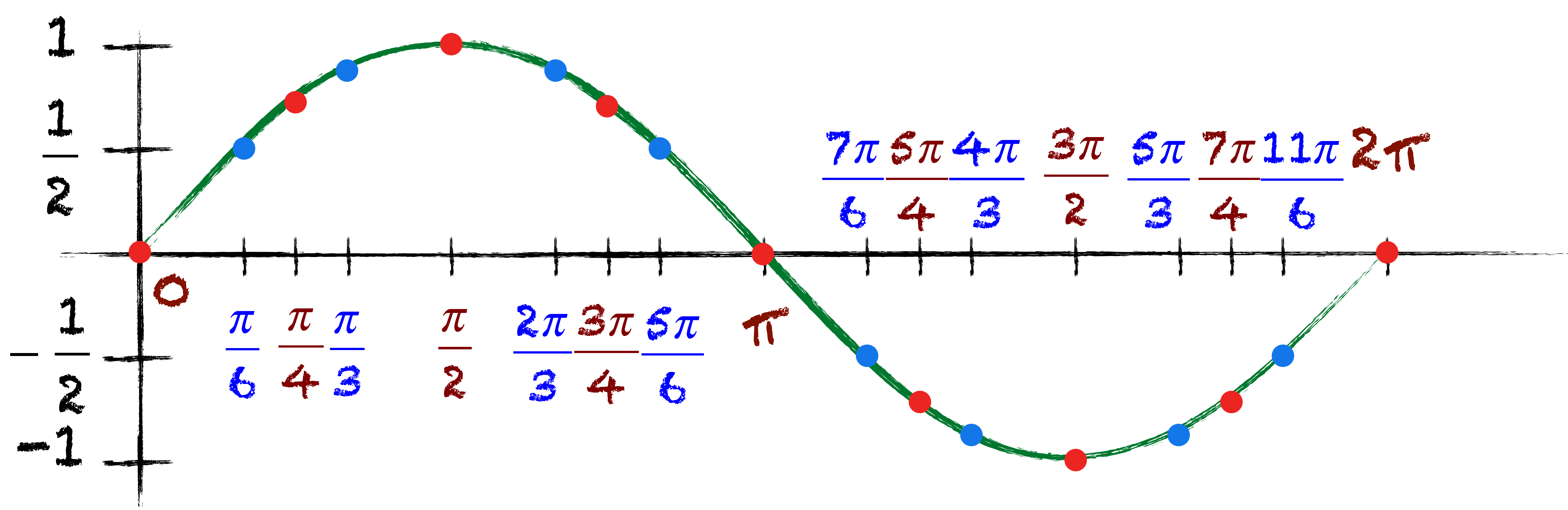
- Slides

The Graph of y = sinx

- Complete the table:

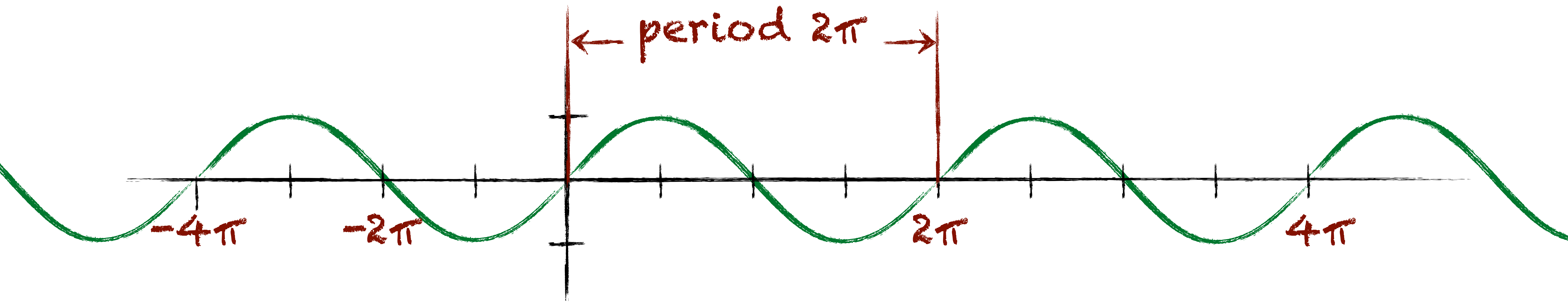
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
sinx	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0

- Graph the results:



The Graph of $y = \sin x$

- The sine function is periodic, with a period 2π. That means the graph continues forever in both directions, repeating the pattern every 2π.



- The sine function is an odd function, $\sin(-x) = -\sin x$.
- The domain is $(-\infty, \infty)$; the range is $[-1, 1]$.

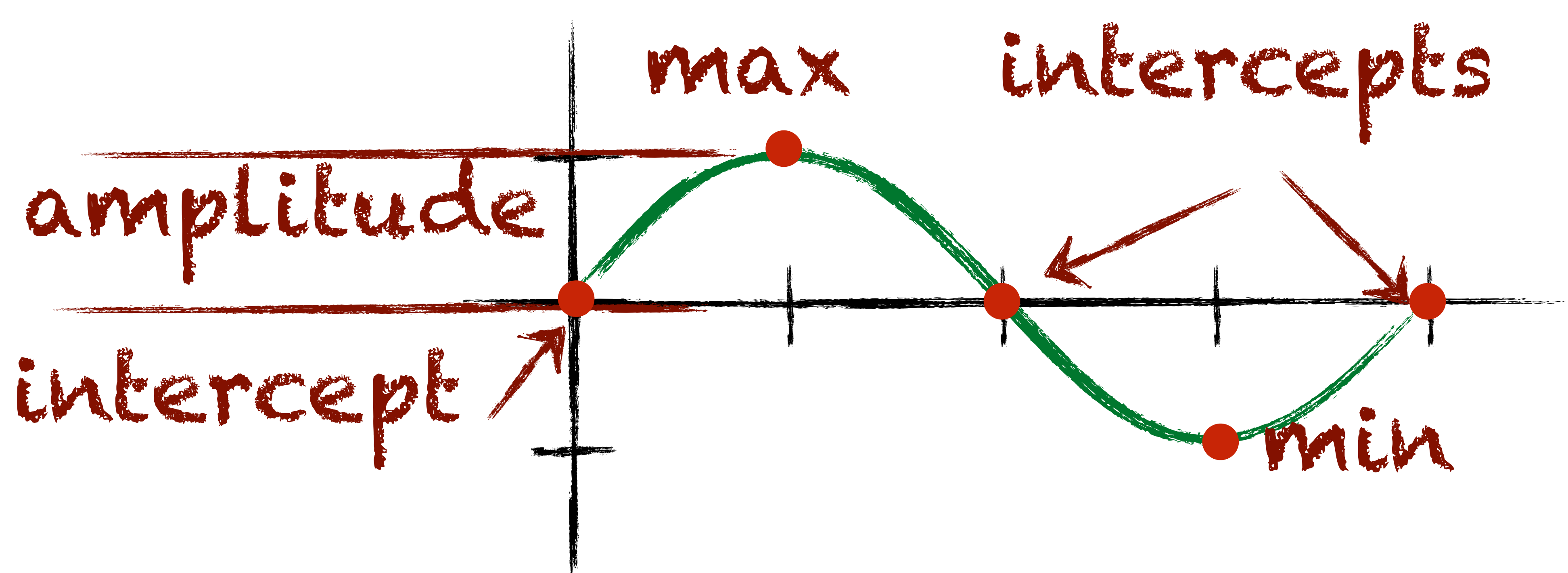
Graphing Variations of $y = \sin x$

- The function $f(x) = \sin x$ is the parent function. The graph of $g(x) = a\sin(bx-c)+d$ transforms like any other function. The rules for transformations (shift, stretch or compress) apply.

- To graph using values it is necessary to find the period, maximum, and minimum values.
  - The maximum and minimum values come from the amplitude of the graph. The amplitude is the distance from the extreme values of sine and cosine to the line of equilibrium.

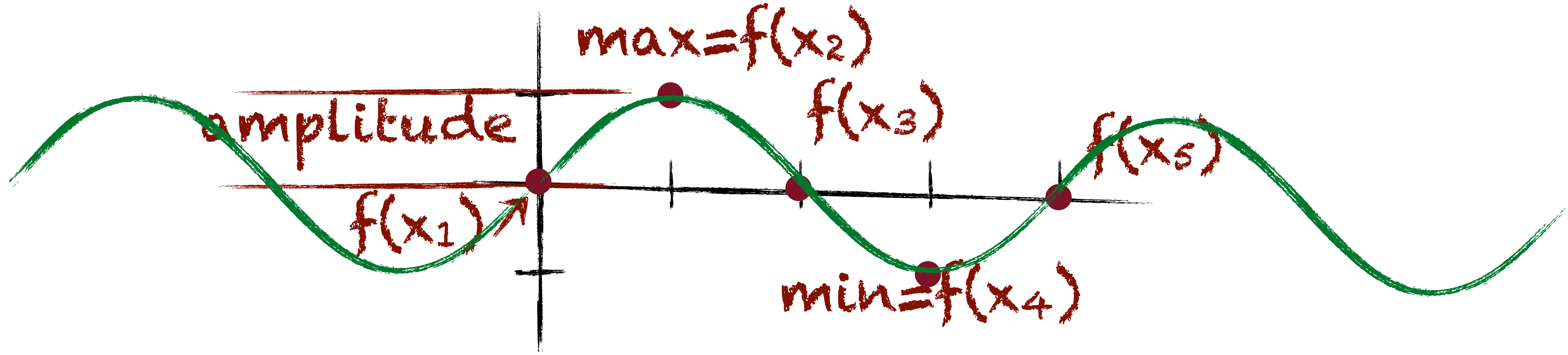
Graphing Variations of $y = \sin x$

- To graph $y = a\sin(bx-c)+d$ follow the procedure
  - 1. Identify period and amplitude
  - 2. Find 5 key x-values; the x-intercepts, x-value of maximum f(x), and x-value of minimum f(x).
    - To find the 5 x-values, divide the period into 4 sections. The first, middle, and last x are the intercepts. The 2nd x will be the maximum, the 3rd x is the minimum.



Graphing Variations of $y = \sin x$

- Once the x-values have been determined.
  - 3. Find $y = f(x)$ for each of those 5 x-values.

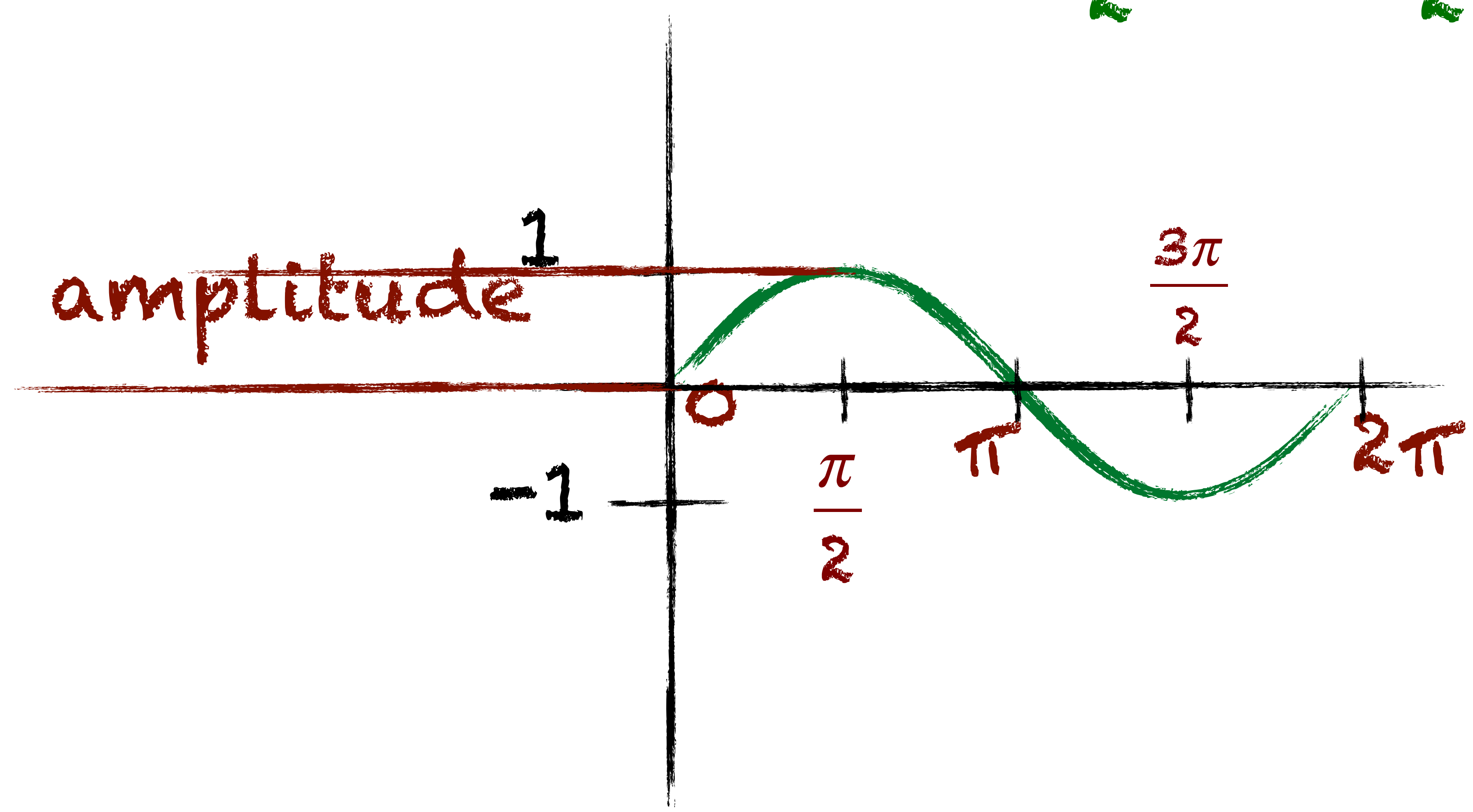


- 4. Draw the sine wave.
- 5. Repeat the sine wave over the desired domain.

Finding Amplitude

- When we graph $y = \sin x$, the range for y is [−1 1].
  - That means the maximum value for $\sin x = 1$. The amplitude of $\sin x$ is 1.
  - To graph $y=\sin x$ we find the 5 values for x by dividing the period by 4. $\frac{2\pi}{4} = \frac{\pi}{2}$ our 5 values of x are $0 \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2} \quad 2\pi$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sinx	0	1	0	−1	0

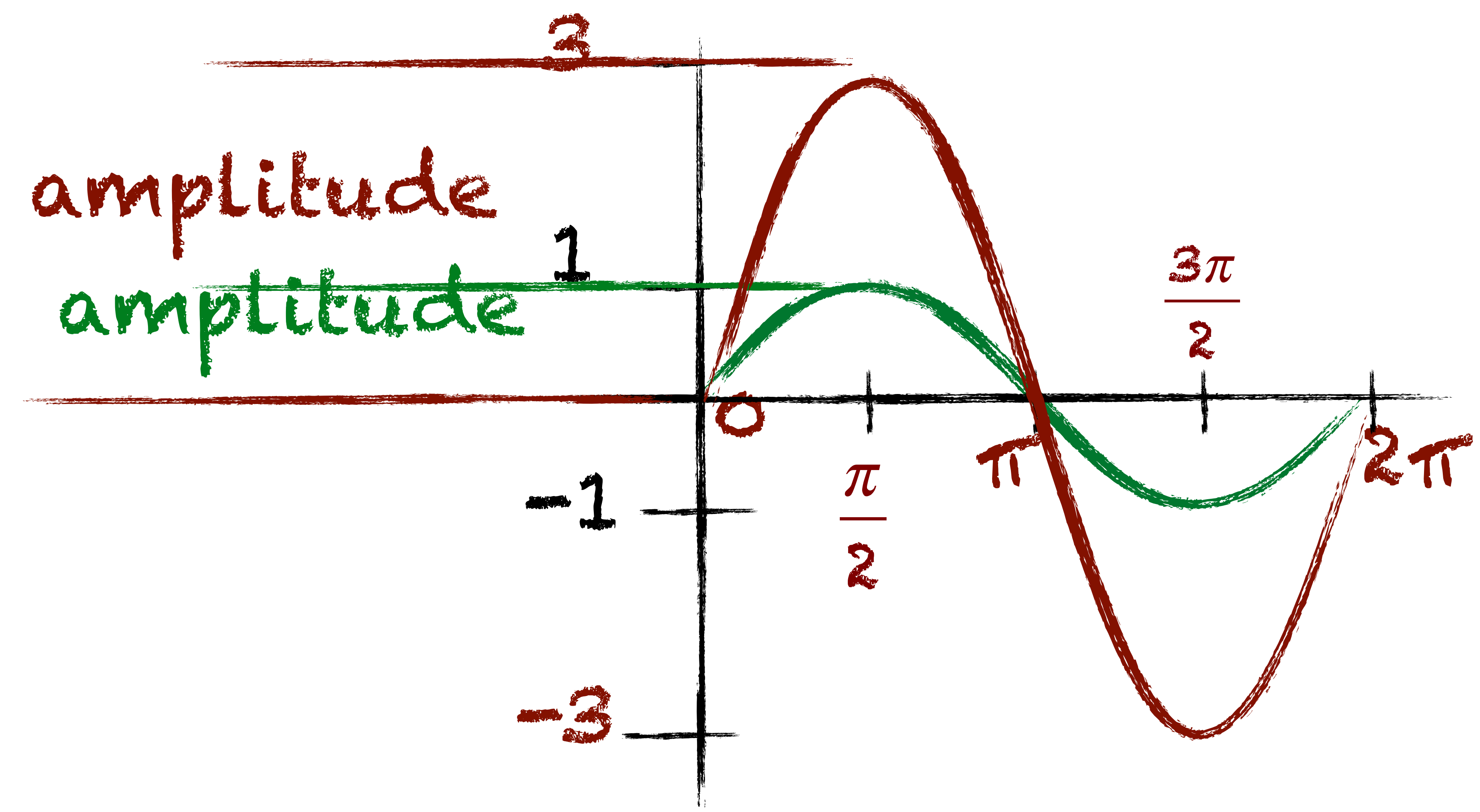


Finding Amplitude

- If we graph $y = 3\sin x$, we multiply each $f(x)$ by 3. You should remember that this is a vertical stretch of factor 3. Thus the maximum value of $3\sin x = 3(1) = 3$. Then the amplitude of $y=3\sin x$ is 3. The period remains 2π.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sinx	0	1	0	−1	0

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
3Sinx	0	3	0	−3	0

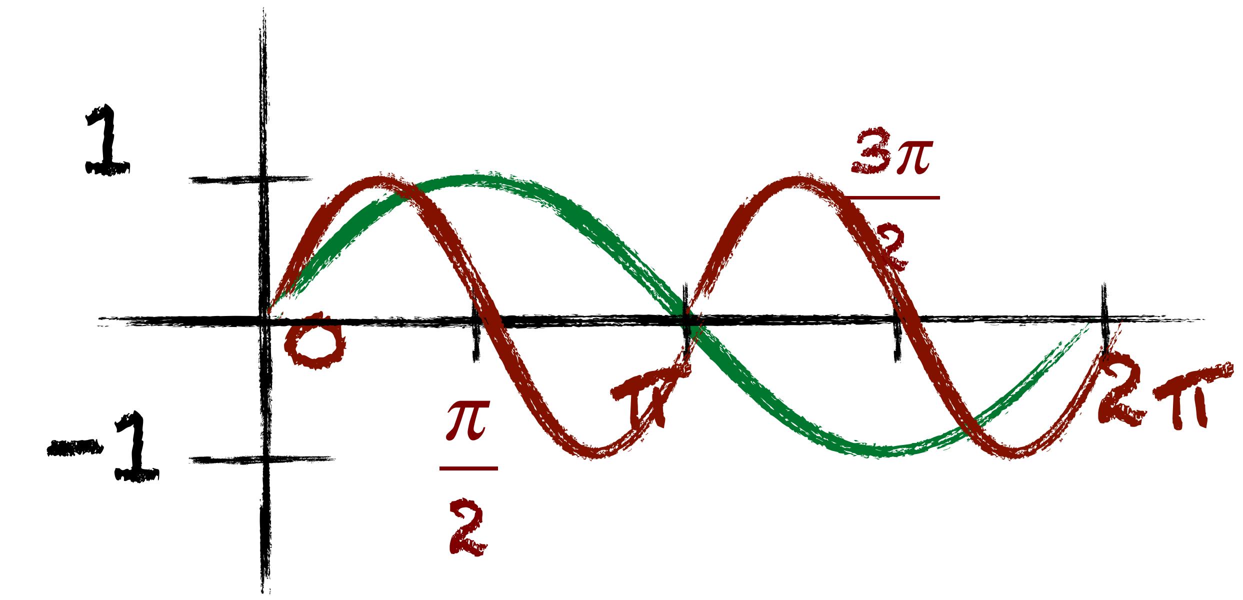


Finding Period of sin(2x)

- We know the period of sinx = 2π. But what happens with sin2x?
  - For the moment, let p=2x. We know sinp has period 2π, that means the graph begins a new period at 2π.
- p=2x, so when 2x = 2π the graph begins a new cycle.
- Thus, the cycle repeats when x = π. The period of y=sin2x is π.

Finding Period of sin(2x)

- If we graph y=sin2x we can see the period.



x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sinx	0	1	0	–1	0

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sin2x	0	0	0	0	0

Uh oh!

Our 5 values work, but we must remember we are working with 2x, $2x = 0$, $2x = \pi/2$, $2x = \pi$, $2x = 3\pi/2$, $2x = 2\pi$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
Sin2x	0	1	0	–1	0	1	0	–1	0

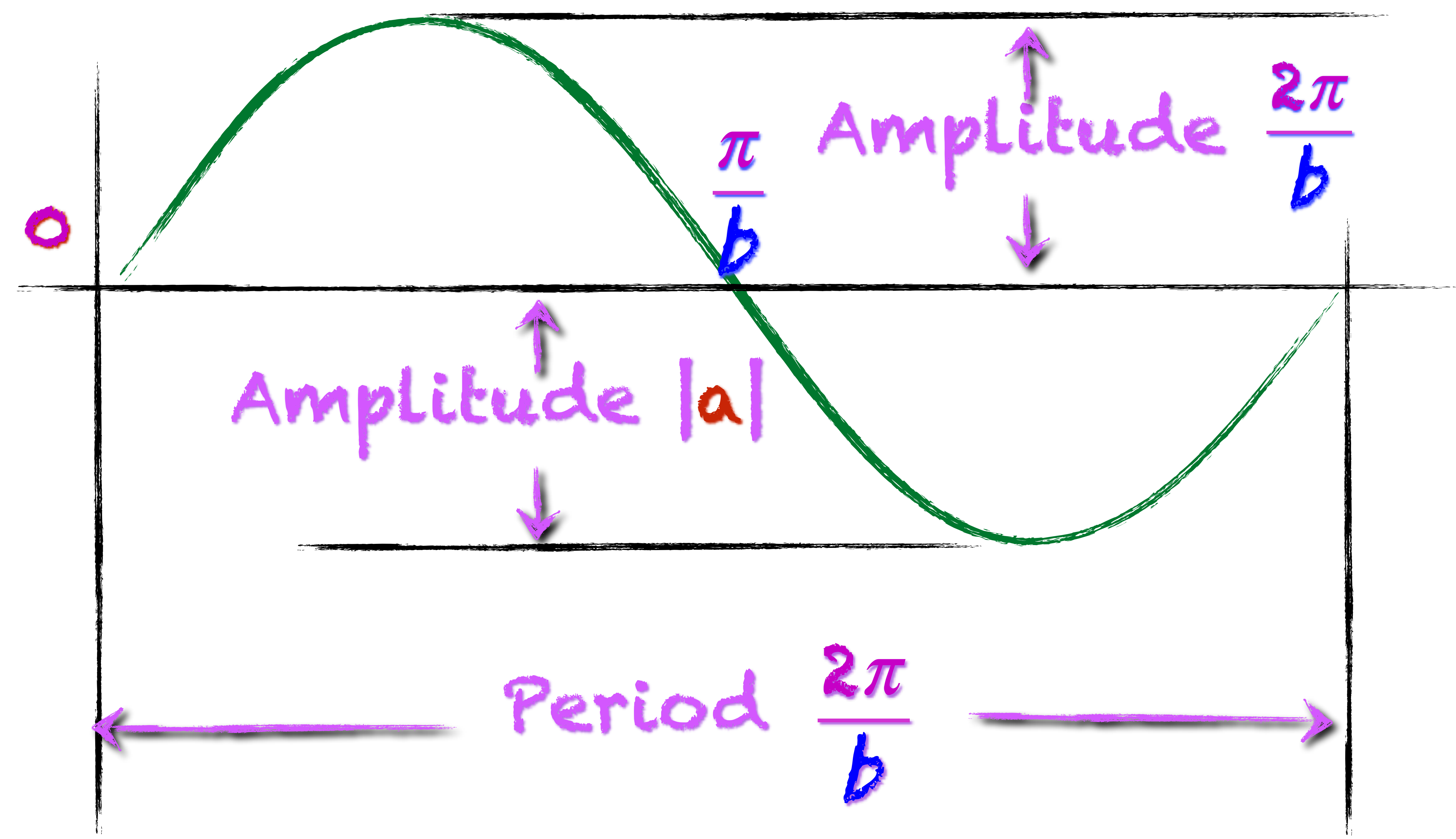
Over the domain $[0,2\pi]$ the graph of y=sin2x repeats itself. y=sin2x completes one cycle (period) over the interval $[0,\pi]$. The period is π.

Amplitudes and Periods

The graph of $f(x) = a\sin bx$, where $b>0$ has:

Amplitude = $|a|$

Period = $\frac{2\pi}{b}$



Example: Graphing a Function of the Form $y = a\sin bx$

Determine the amplitude and period of $y = 2\sin\frac{1}{2}x$

Then graph the function for $0 \le x \le 8\pi$.

Step 1 Identify the amplitude and the period.

The equation is of the form $y = a\sin bx$ $a = 2, b = \frac{1}{2}$

amplitude $= |2| = 2$ period $= \dfrac{2\pi}{\frac{1}{2}} = 4\pi$

The maximum value of y is 2, the minimum value of y is −2, the graph completes one cycle (period) in the interval $[0, 4\pi]$

Example: Graphing a Function of the Form $y = a\sin bx$

Step 2 Find the values of x for the five key points.

To generate x-values for each of the five key points, divide the period(=4π) by 4. The cycle begins at $x_1 = 0$. We add quarter periods to generate x-values for each of the key points.

$$\frac{4\pi}{4} = \pi$$

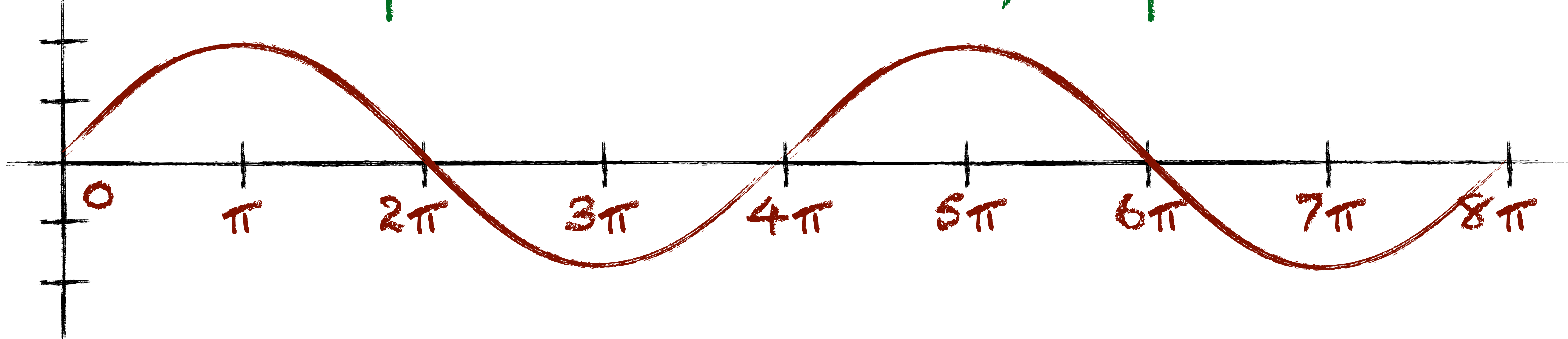
The 5 x-values are 0, $0+\pi = \pi$, $\pi+\pi=2\pi$, $2\pi+\pi=3\pi$, $3\pi+\pi=4\pi$

Example: Graphing a Function of the Form y = asinbx

Step 3 Find the values of y for the five key points.

x	0	π	2π	3π	4π
$y = 2\sin\frac{1}{2}x$	0	2	0	-2	0

Step 4 Plot the points and draw the first period.



Step 5 Repeat to cover the interval $[0, 8\pi]$.

Another approach

- Determine the amplitude and period of $y = 2\sin\frac{1}{2}x$. Then graph the function for $0 \le x \le 8\pi$.

- Let us start with the 5 y-values we know are the critical 5 points for the parent function $y = \sin x$.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sinx	0	1	0	−1	0

- We find the x values for those 5 critical points.

$\frac{1}{2}x = 0, x = 0$ $\frac{1}{2}x = \frac{\pi}{2}, x = \pi$ $\frac{1}{2}x = \pi, x = 2\pi$

$\frac{1}{2}x = \frac{3\pi}{2}, x = 3\pi$ $\frac{1}{2}x = 2\pi, x = 4\pi$

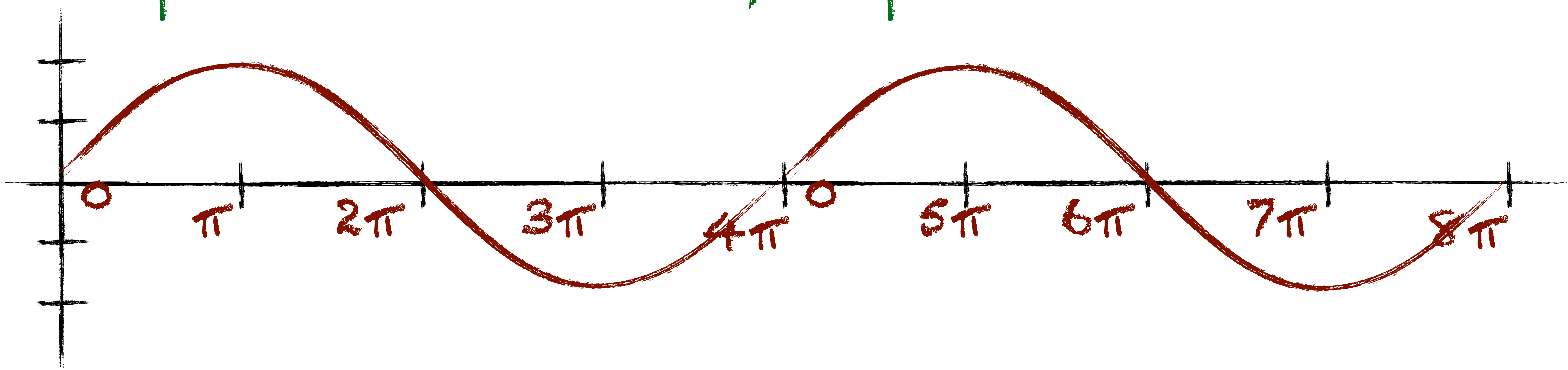
$\frac{1}{2}x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	0	π	2π	3π	4π
$y = \sin\frac{1}{2}x$	0	1	0	−1	0
$y = 2\sin\frac{1}{2}x$	0	2	0	−2	0

Another approach

- Now we have the same table of values

x	0	π	2π	3π	4π
$y = 2\sin\frac{1}{2}x$	0	2	0	-2	0

Plot the points and draw the first period.



Repeat to cover the interval $[0,8\pi]$.

The Graph of $y = a\sin(bx - c)$

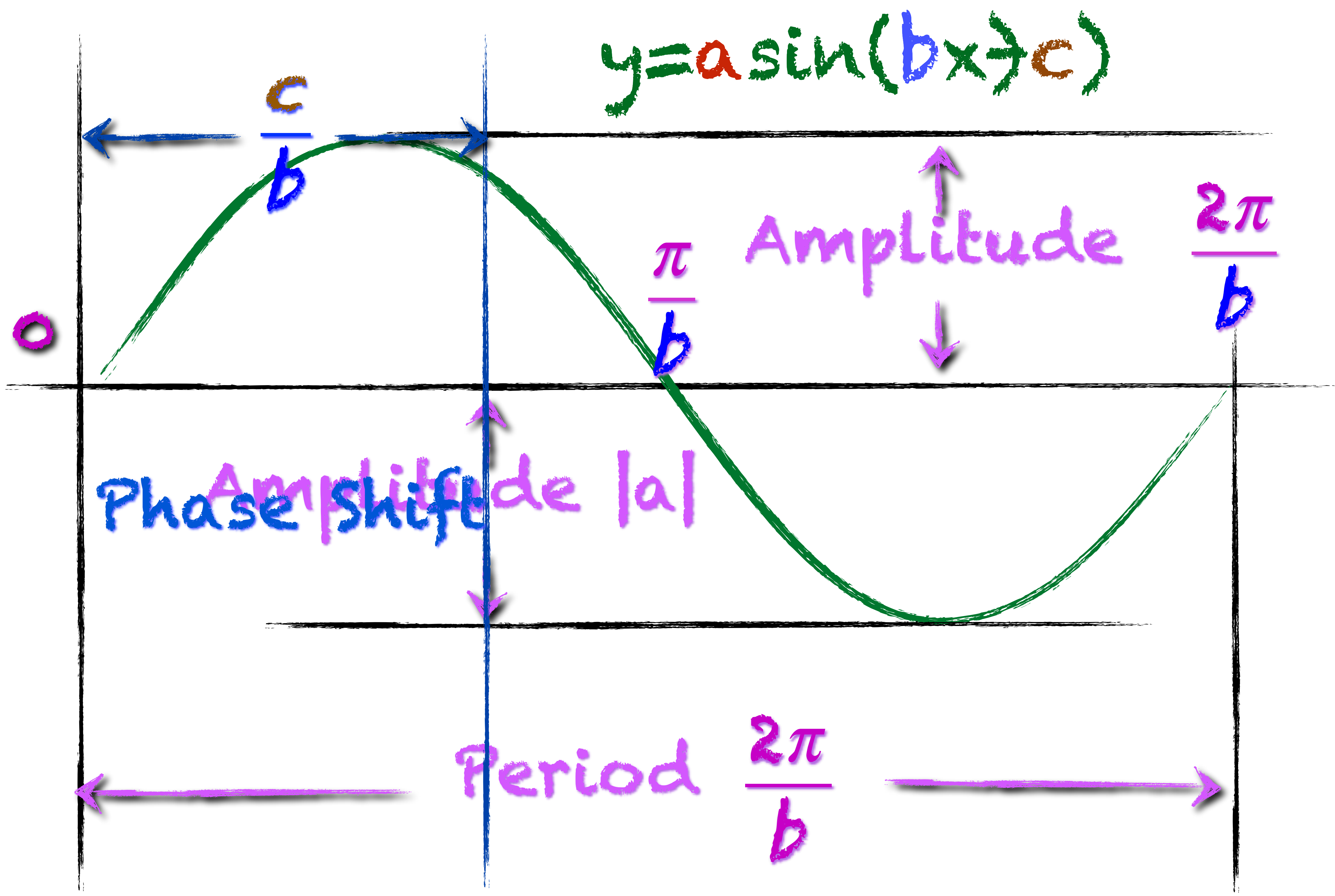
- The graph of $y=a\sin(bx-c)$ is identical to the graph of $y=a\sin bx$, shifted right. (Just like any other function shift.) The amount of shift is c/b.

- Think of $y=a\sin(bx-c)$ as $y = a\sin\left(b\left(x-\frac{c}{b}\right)\right)$.
- If $c/b>0$ shift right (remember $x-c/b$), if $c/b<0$ shift left.
  - With a periodic function, this is known as a "phase shift" of c/b.

- The amplitude remains $|a|$, and the period remains $2\pi/b$.

The Graph of y = asin(bx − c)

∡ y=asin(bx−c)



Example: Graphing a Function of the Form $y = a\sin(bx - c)$

Determine the amplitude, period, and phase shift of $y = 3\sin\left(2x - \frac{\pi}{3}\right)$ then graph one period.

Step 1 amplitude, period, and phase shift.

$$a = 3, b = 2, c = \frac{\pi}{3}$$

amplitude: $|a| = |3| = 3$

period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

phase shift: $\frac{c}{b} = \frac{\frac{\pi}{3}}{2} = \frac{\pi}{6}$

Example: Graphing a Function of the Form $y = a\sin(bx - c)$

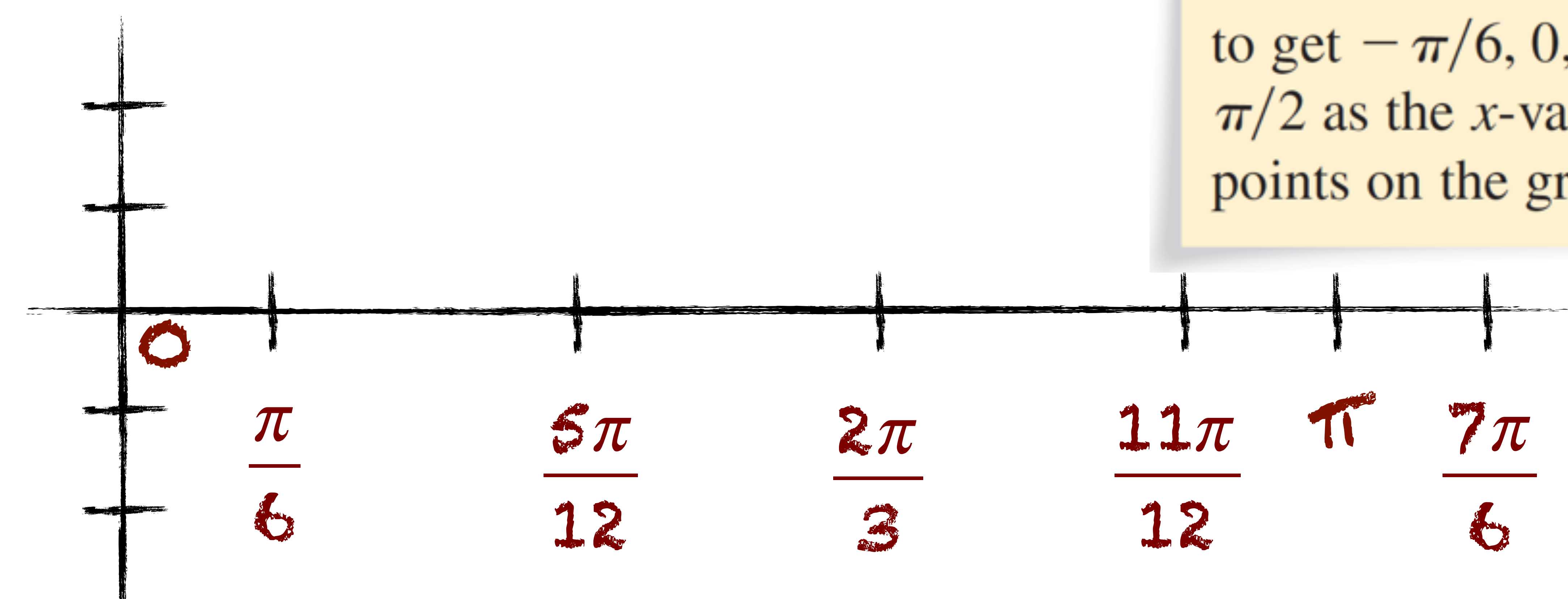
Step 2 5 key values of x.

period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$ phase shift: $\frac{c}{b} = \frac{\frac{\pi}{3}}{2} = \frac{\pi}{6}$

$x_1 = 0 + \frac{\pi}{6}$ $x_2 = \frac{\pi}{6} + \frac{\pi}{4} = \frac{5\pi}{12}$ $x_3 = \frac{5\pi}{12} + \frac{\pi}{4} = \frac{2\pi}{3}$

$x_4 = \frac{2\pi}{3} + \frac{\pi}{4} = \frac{11\pi}{12}$

$x_5 = \frac{11\pi}{12} + \frac{\pi}{4} = \frac{7\pi}{6}$



STUDY TIP

In general, to divide a period-interval into four equal parts, successively add "period/4," starting with the left endpoint of the interval. For instance, for the period-interval $[-\pi/6, \pi/2]$ of length $2\pi/3$, you would successively add

$$\frac{2\pi/3}{4} = \frac{\pi}{6}$$

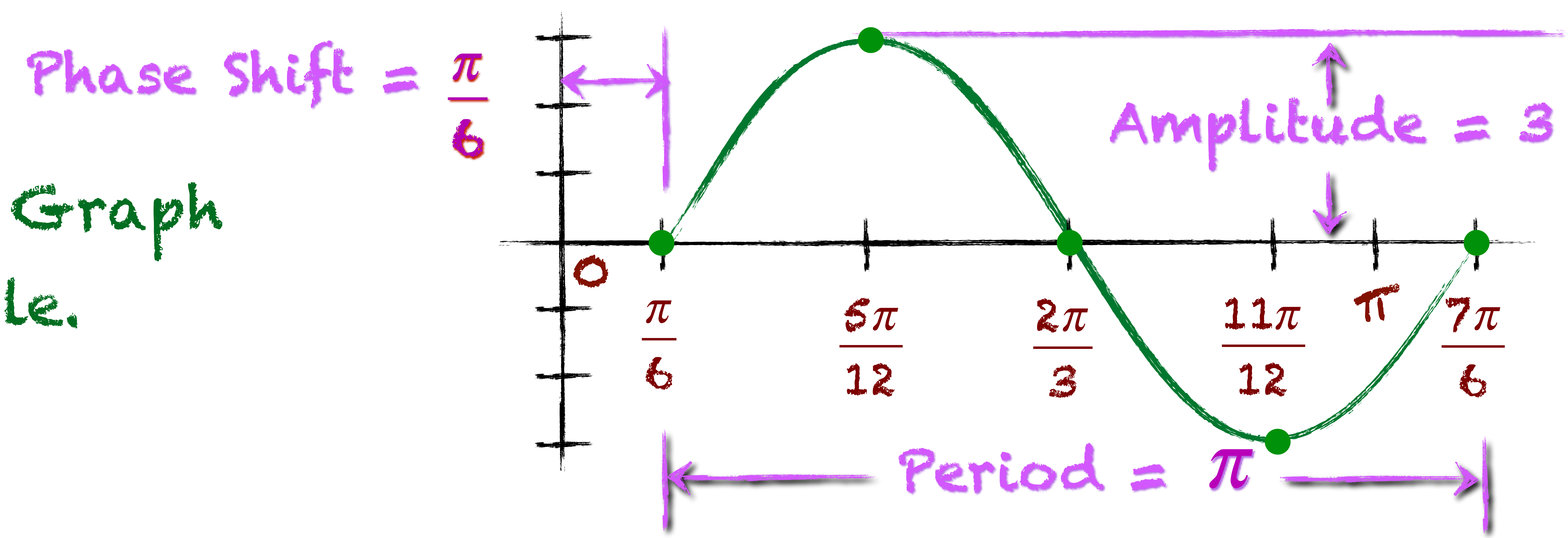
to get $-\pi/6, 0, \pi/6, \pi/3$, and $\pi/2$ as the x-values for the key points on the graph.

Example: Graphing a Function of the Form $y = a\sin(bx - c)$

Step 3 Find the points for the 5 key values of x.

x	$\frac{\pi}{6}$	$\frac{5\pi}{12}$	$\frac{2\pi}{3}$	$\frac{11\pi}{12}$	$\frac{7\pi}{6}$
$y = 3\sin\left(2x - \frac{\pi}{3}\right)$	0	3	0	–3	0

Step 4 Graph one cycle.



Another approach

Determine the amplitude, period, phase shift, and graph one period of $y = 3\sin\left(2x - \frac{\pi}{3}\right)$.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0

We find the x values for the 5 critical points.

$2x - \frac{\pi}{3} = 0,\ x = \frac{\pi}{6}$ $\quad$ $2x - \frac{\pi}{3} = \frac{\pi}{2},\ x = \frac{5\pi}{12}$

$2x - \frac{\pi}{3} = \pi,\ x = \frac{2\pi}{3}$ $\quad$ $2x - \frac{\pi}{3} = \frac{3\pi}{2},\ x = \frac{11\pi}{12}$

$2x - \frac{\pi}{3} = 2\pi,\ x = \frac{7\pi}{6}$

$2x - \frac{\pi}{3}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	$\frac{\pi}{6}$	$\frac{5\pi}{12}$	$\frac{2\pi}{3}$	$\frac{11\pi}{12}$	$\frac{7\pi}{6}$
$y = \sin\left(2x - \frac{\pi}{3}\right)$	0	1	0	-1	0
$y = 3\sin\left(2x - \frac{\pi}{3}\right)$	0	3	0	-3	0

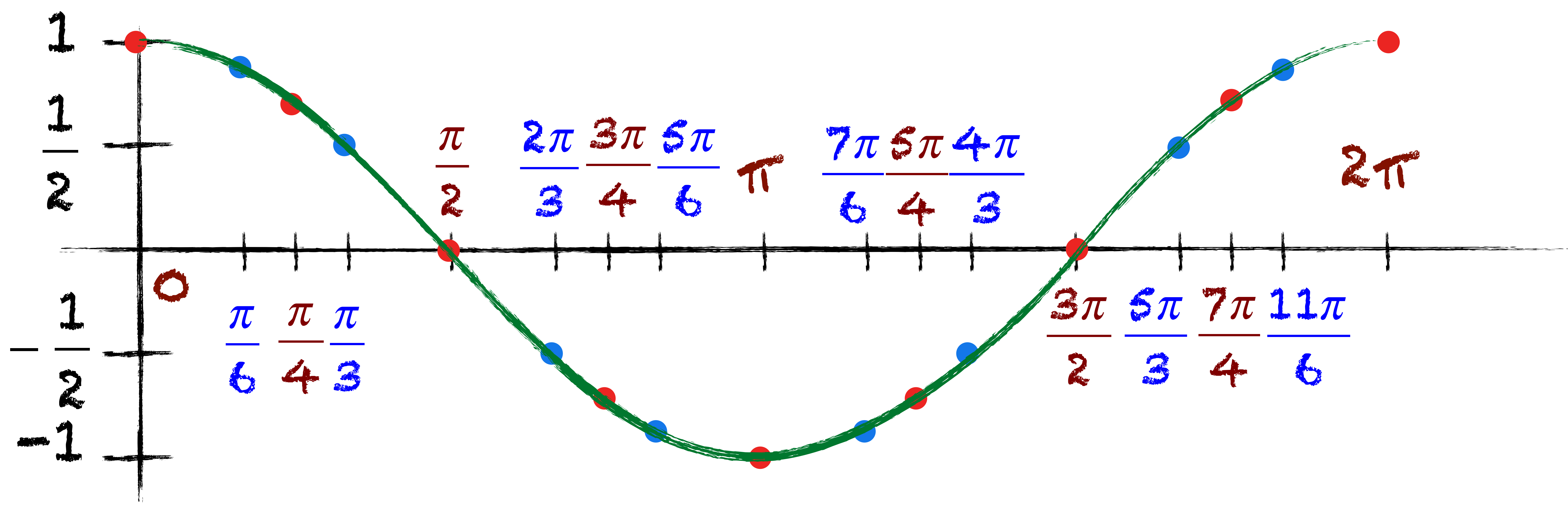
We see the phase shift $= \frac{\pi}{6}$, the period is $\frac{7\pi}{6} - \frac{\pi}{6} = \pi$, and the amplitude is 3.

The Graph of $y = \cos x$

∡Complete the table:

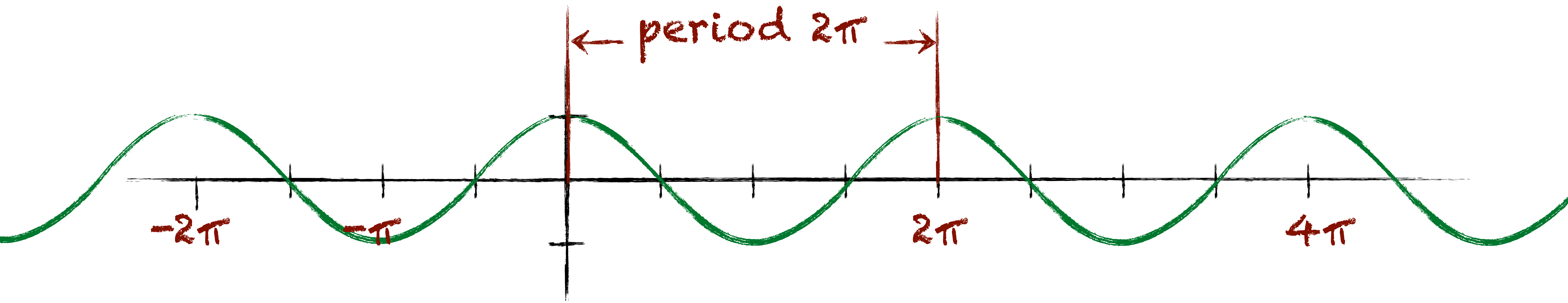
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

∡Graph the results:



The Graph of $y = \cos x$

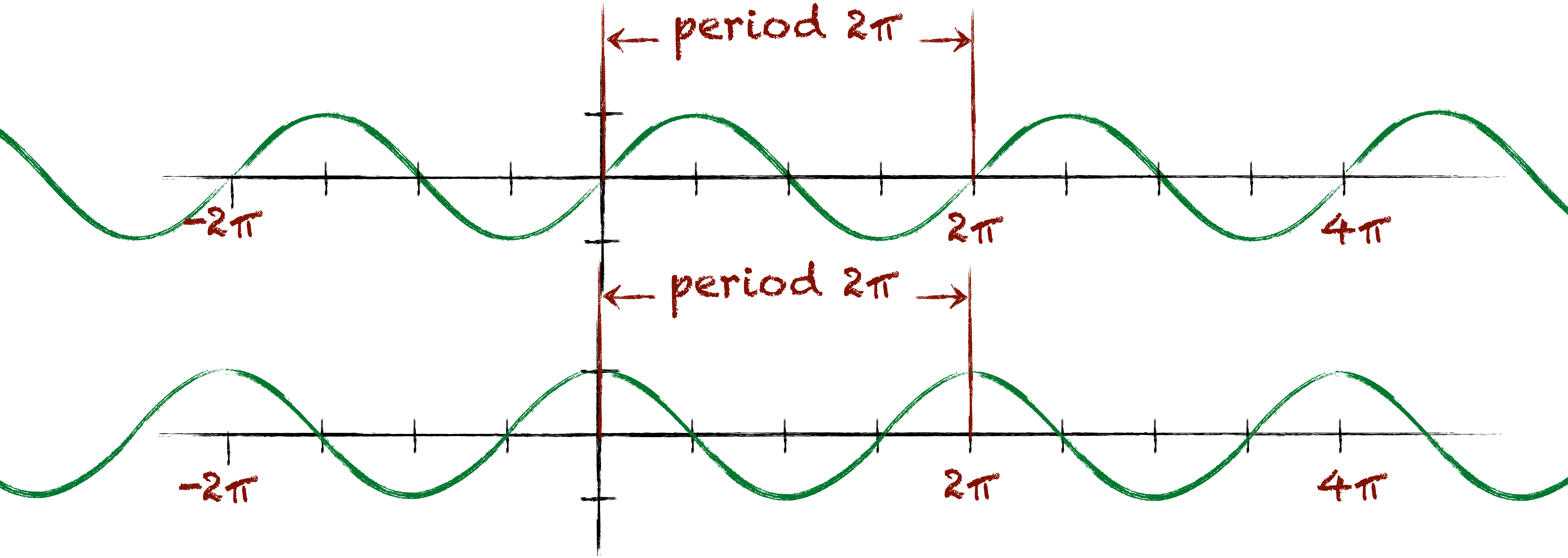
- The cosine function is periodic, with a period 2π. That means the graph continues forever in both directions, repeating the pattern every 2π.



- The cosine function is an even function, $\cos(-x) = \cos x$.
- The domain is $(-\infty, \infty)$; the range is $[-1, 1]$.

Sinusoidal Graphs

- The graphs of sine and cosine functions are called sinusoidal graphs.

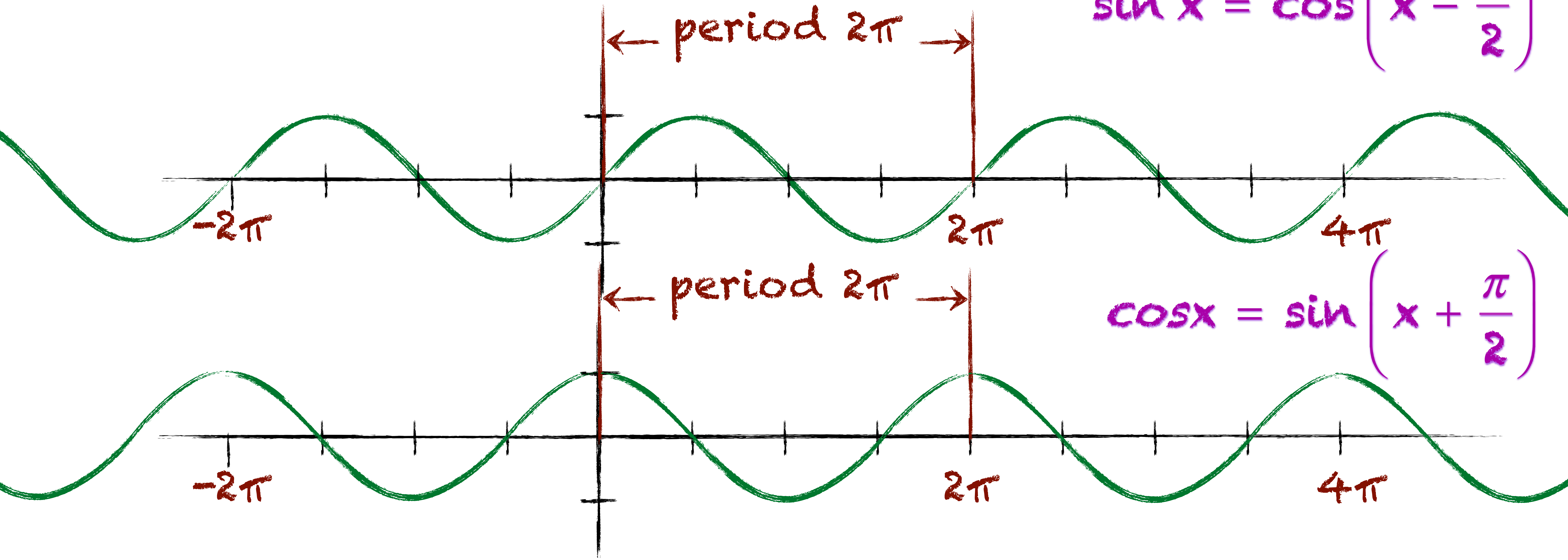


Sinusoidal Graphs

- The graph of $y = \cos x$ is the graph of $y = \sin x$ with a phase shift of $\frac{\pi}{2}$.

$$\sin x = \cos\left(x - \frac{\pi}{2}\right)$$

$$\cos x = \sin\left(x + \frac{\pi}{2}\right)$$

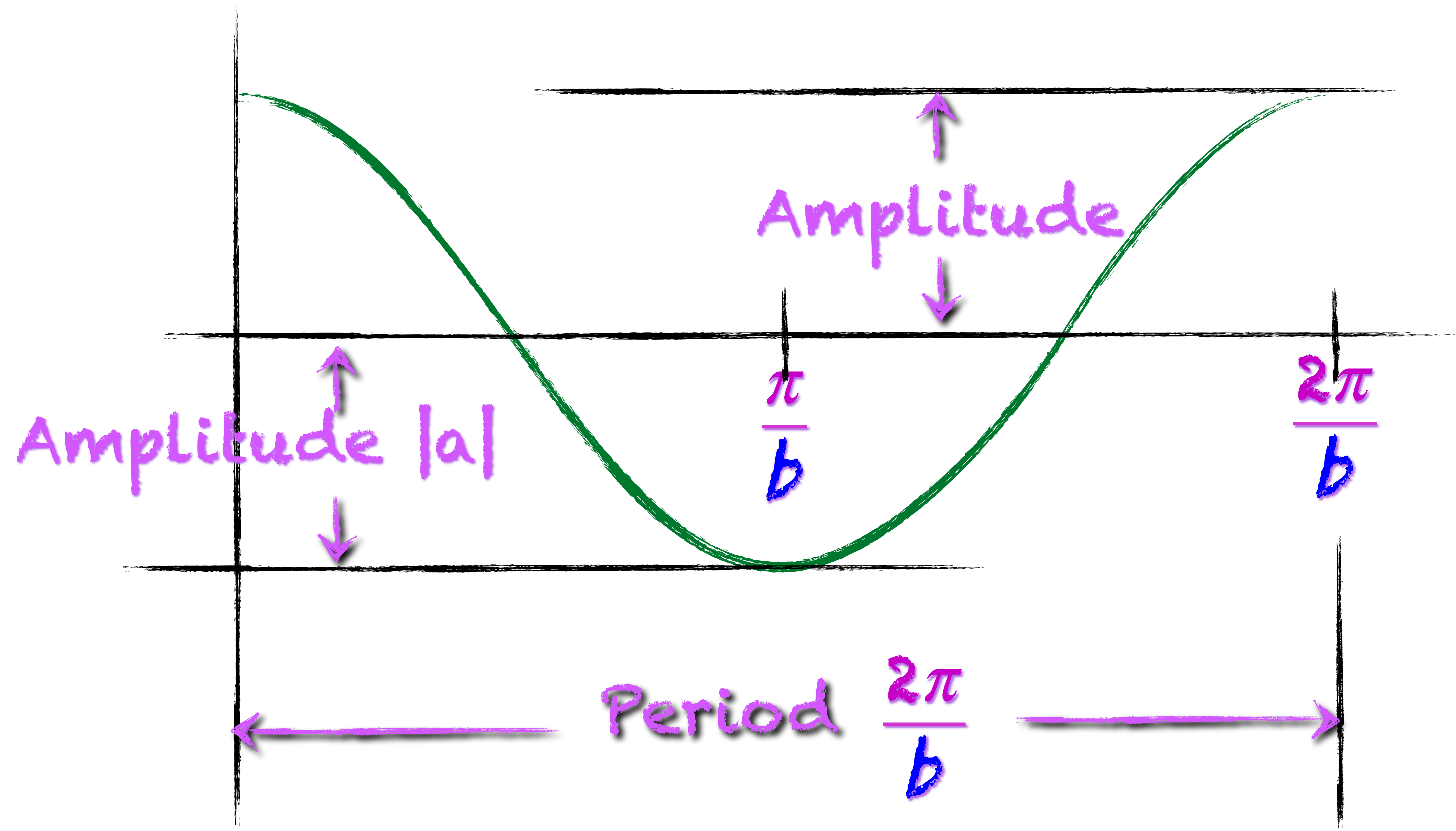


Amplitudes and Periods

- The graph of $f(x) = a\cos bx$, where $b > 0$ has:

Amplitude $= |a|$

Period $= \dfrac{2\pi}{b}$



Example: Graphing a Function of the Form $y = a\cos bx$

Determine the amplitude and period of $y=-4\cos\pi x$, then graph the function for $-2 \le x \le 2$.

Step 1 Identify the amplitude and the period.

The equation is of the form $y = a\cos bx$ $a = -4$, $b = \pi$

amplitude $= |-4| = 4$ period $= \dfrac{2\pi}{\pi} = 2$

The maximum value of y is 4, the minimum value of y is -4, the graph completes one cycle (period) in the interval [0,2]

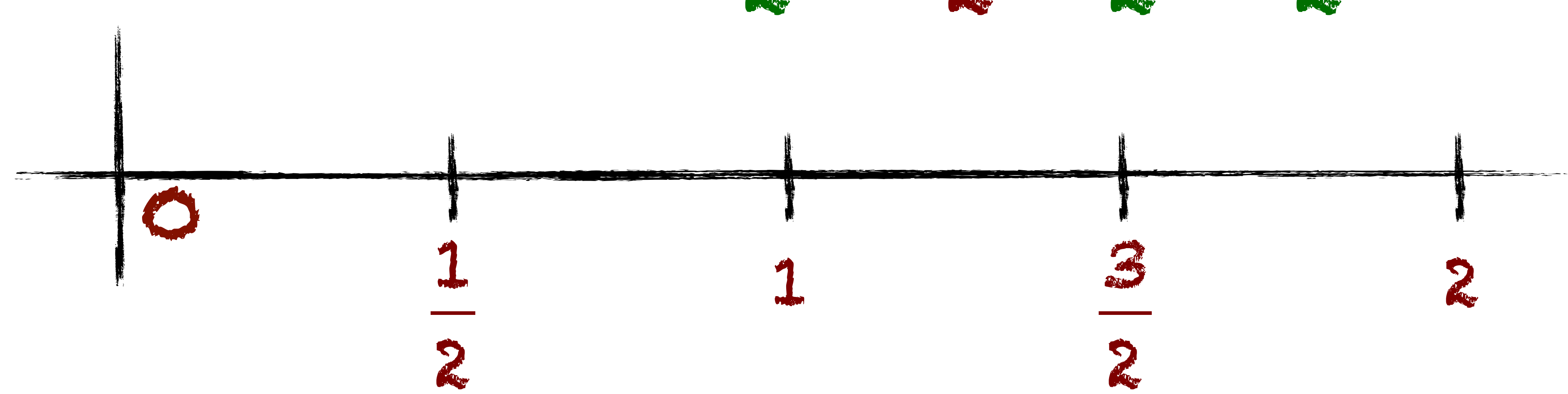
Example: Graphing a Function of the Form $y = -4\cos\pi x$

Step 2 Find the values of x for the five key points.

To generate x-values for each of the five key points, divide the period (=2) by 4. The cycle begins at $x_1 = 0$. We add quarter periods to generate x-values for each of the key points.

$$\frac{2}{4} = \frac{1}{2}$$

The 5 x-values are 0, $0 + \frac{1}{2} = \frac{1}{2}$, $\frac{1}{2} + \frac{1}{2} = 1$, $\frac{1}{2} + 1 = \frac{3}{2}$, $\frac{1}{2} + \frac{3}{2} = 2$.



Example: Graphing a Function of the Form $y = -4\cos \pi x$

Step 3 Find the values of y for the five key points.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$y = -4\cos \pi x$	–4	0	4	0	–4

$$y = -4\cos \pi(0) = -4\cos 0 = -4(1) = -4$$

$$y = -4\cos \pi\left(\frac{1}{2}\right) = -4\cos \frac{\pi}{2} = -4(0) = 0$$

$$y = -4\cos \pi(1) = -4\cos \pi = -4(-1) = 4$$

$$y = -4\cos \pi\left(\frac{3}{2}\right) = -4\cos \frac{3\pi}{2} = -4(0) = 0$$

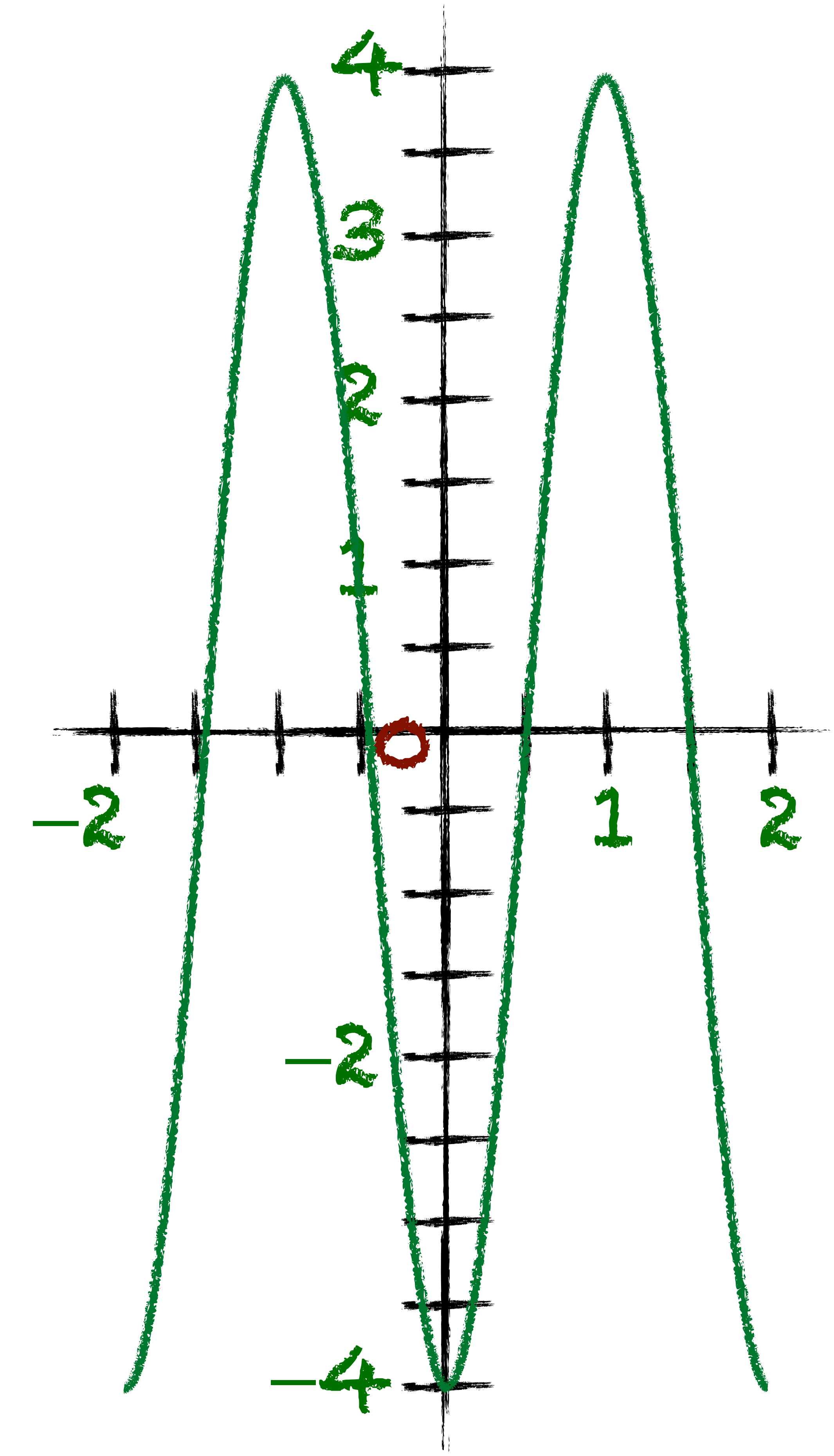
$$y = -4\cos \pi(2) = -4\cos 2\pi = -4(1) = -4$$

Example: Graphing a Function of the Form $y = -4\cos\pi x$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$y = -4\cos\pi x$	-4	0	4	0	-4

Step 4 Plot the points and draw the first cycle.

Step 5 Repeat to cover the interval $[-2,2]$.



Another approach

- Determine the amplitude and period of $y = -4\cos \pi x$. Then graph the function for $-2 \le x \le 2$.
- Let us start with the 5 y-values we know are the critical 5 points for the parent function $y = \cos A$.

A	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
cosA	1	0	−1	0	1

- We find the x values for those 5 critical points.

$\pi x = 0, x = 0$ $\quad \pi x = \frac{\pi}{2}, x = \frac{1}{2}$ $\quad \pi x = \pi, x = 1$

$\pi x = \frac{3\pi}{2}, x = \frac{3}{2}$ $\quad \pi x = 2\pi, x = 2$

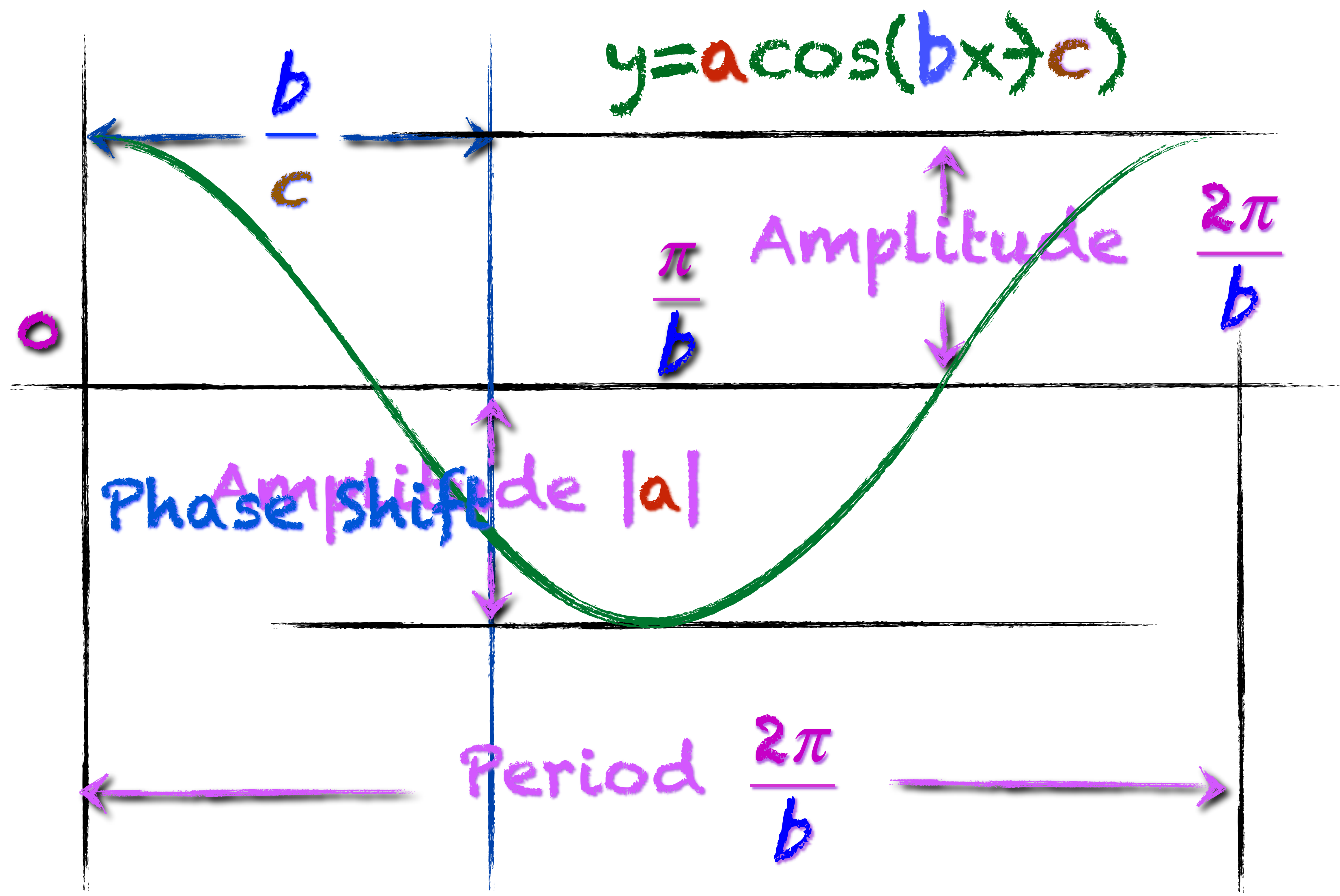
πx	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$y = \cos \pi x$	1	0	−1	0	1
$y = -4\cos \pi x$	−4	0	4	0	−4

The Graph of $y = a\cos(bx - c)$

- The graph of $y=a\cos(bx-c)$ is identical to the graph of $y=a\cos bx$, shifted right. (Just like any other function shift.) The amount of shift is c/b.
  - Think of $y=a\cos(bx-c)$ as $y=a\cos[b(x-c/b)]$.
    - If $c/b>0$ shift right $(bx-c)$, if $c/b<0$ shift left.
    - This is also a "phase shift" of c/b.
- The amplitude remains $|a|$, and the period remains $2\pi/b$.

The Graph of $y = a\cos(bx - c)$

∡ $y = a\cos(bx - c)$



Example: Graphing a Function of the Form $y = a\cos(bx - c)$

Determine the amplitude, period, and phase shift of $y = \frac{3}{2}\cos\left(2x + \pi\right)$ then graph one period.

Step 1 amplitude, period, and phase shift.

$$a = \frac{3}{2}, b = 2, c = \pi$$

amplitude: $\left|\frac{3}{2}\right| = \frac{3}{2}$

period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

phase shift: $\frac{c}{b} = \frac{-\pi}{2} = -\frac{\pi}{2}$

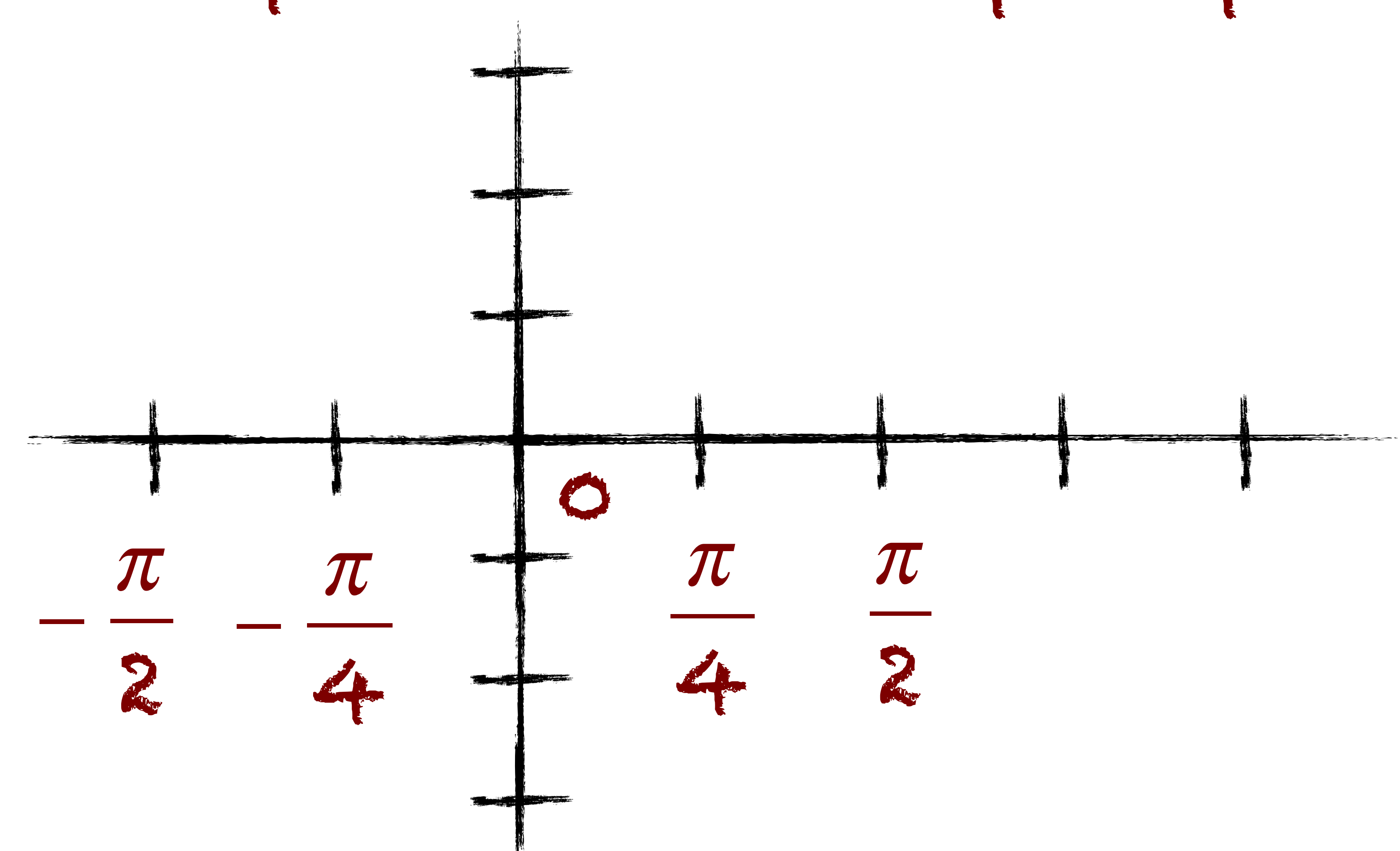
Example: Graphing a Function of the Form $y = \frac{3}{2}\cos(2x + \pi)$

Step 2 5 key values of x. amplitude: $\left|\frac{3}{2}\right| = \frac{3}{2}$

phase shift: $\frac{c}{b} = \frac{-\pi}{2} = -\frac{\pi}{2}$ period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

$x_1 = 0 + -\frac{\pi}{2} = -\frac{\pi}{2}$ $x_2 = -\frac{\pi}{2} + \frac{\pi}{4} = -\frac{\pi}{4}$ $x_3 = -\frac{\pi}{4} + \frac{\pi}{4} = 0$

$x_4 = 0 + \frac{\pi}{4} = \frac{\pi}{4}$ $x_5 = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

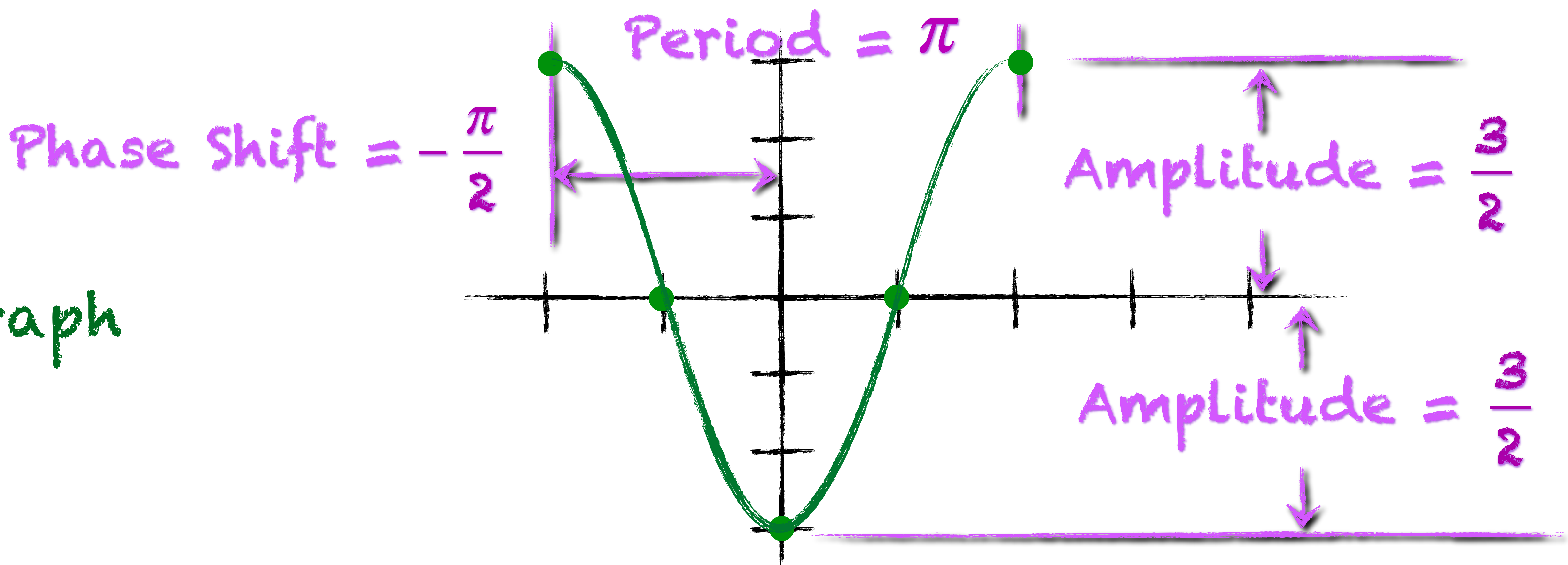


Example: Graphing a Function of the Form $y = \frac{3}{2}\cos(2x + \pi)$

Step 3 Find the points for the 5 key values of x.

x	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$y = \frac{3}{2}\cos(2x + \pi)$	$\frac{3}{2}$	0	$-\frac{3}{2}$	0	$\frac{3}{2}$

Step 4 Graph one cycle.



Another approach

Determine the amplitude, period, phase shift, and graph one period of $y = \frac{3}{2}\cos\left(2x + \pi\right)$.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
cosx	1	0	-1	0	1

∡ We find the x values for the 5 critical points.

$2x + \pi = 0, x = -\frac{\pi}{2}$ $\quad 2x + \pi = \frac{\pi}{2}, x = -\frac{\pi}{4}$

$2x + \pi = \pi, x = 0$ $\quad 2x + \pi = \frac{3\pi}{2}, x = \frac{\pi}{4}$

$2x + \pi = 2\pi, x = \frac{\pi}{2}$

$2x + \pi$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$y = \cos\left(2x + \pi\right)$	1	0	-1	0	1
$y = \frac{3}{2}\cos\left(2x + \pi\right)$	$\frac{3}{2}$	0	$-\frac{3}{2}$	0	$\frac{3}{2}$

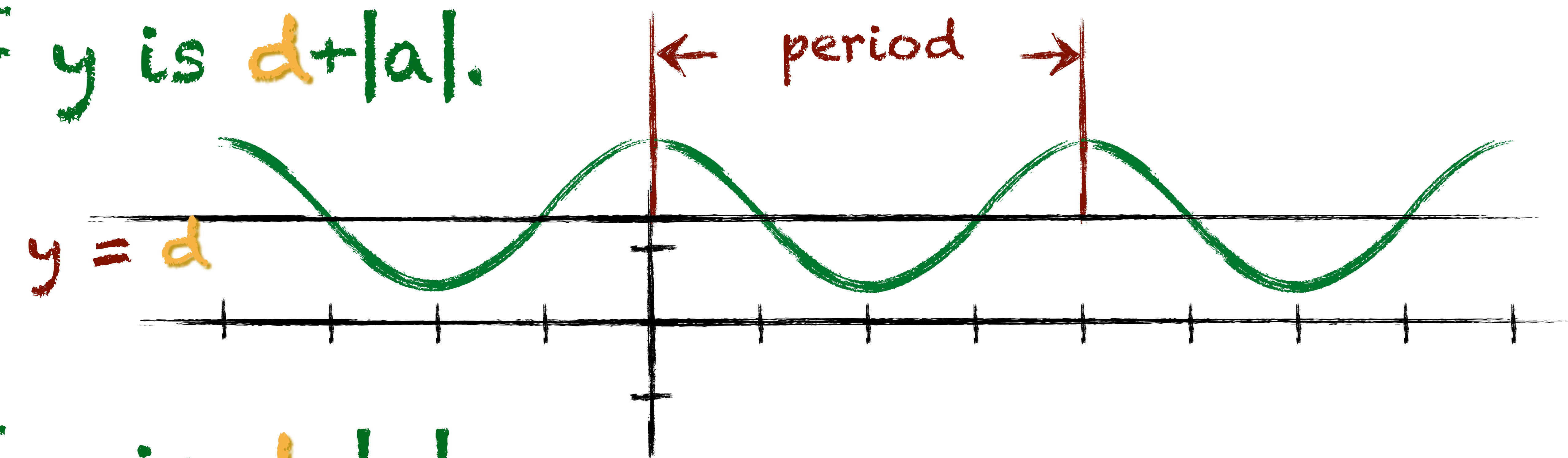
∡ We see the phase shift $= -\frac{\pi}{2}$, the period is $\frac{\pi}{2} - -\frac{\pi}{2} = \pi$, and the amplitude is $\frac{3}{2}$.

Vertical Shifts of Sinusoidal Graphs $y=a\sin(bx-c)+d$

For sinusoidal graphs of the form $y=a\sin(bx-c)+d$ and $y=a\cos(bx-c)+d$ the constant d causes a vertical shift in the graph.

These vertical shifts result in sinusoidals oscillating about the horizontal line $y = d$ (equilibrium) rather than about the x-axis.

The maximum value of y is $d+|a|$.



The minimum value of y is $d-|a|$.

Example: A Vertical Shift $y=2\cos x+1$

Graph one period of the function $y=2\cos x+1$.

Step 1 amplitude, period, and phase shift.

$$a = 2, b = 1, c = 0, d = 1$$

amplitude: $|2| = 2$

period: $\frac{2\pi}{1} = 2\pi$

phase shift: $\frac{c}{b} = \frac{0}{1} = 0$

vertical shift: $d = +1$

Example: A Vertical Shift $y=2\cos x+1$

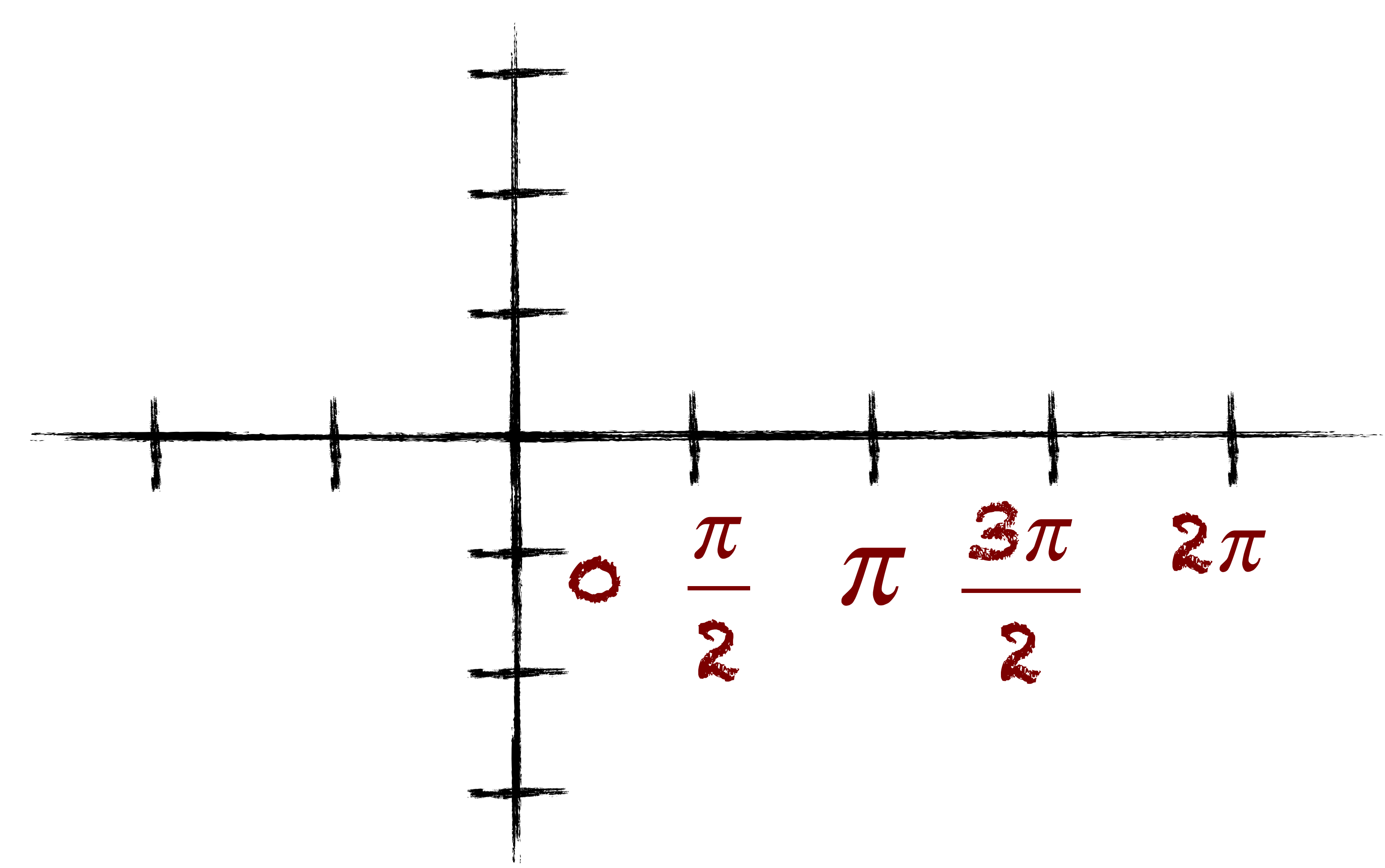
Step 2 5 key values of x.

period: $\frac{2\pi}{1} = 2\pi$ phase shift: $\frac{c}{b} = \frac{0}{1} = 0$

$\frac{2\pi}{4} = \frac{\pi}{2}$ $x_1 = 0$

$x_2 = 0 + \frac{\pi}{2} = \frac{\pi}{2}$ $x_3 = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$x_4 = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$ $x_5 = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$



Example: A Vertical Shift $y=2\cos x+1$

Step 3 Find the points for the 5 key values of x.

$$y = 2\cos 0 + 1 = 3$$

$$y = 2\cos \frac{\pi}{2} + 1 = 1$$

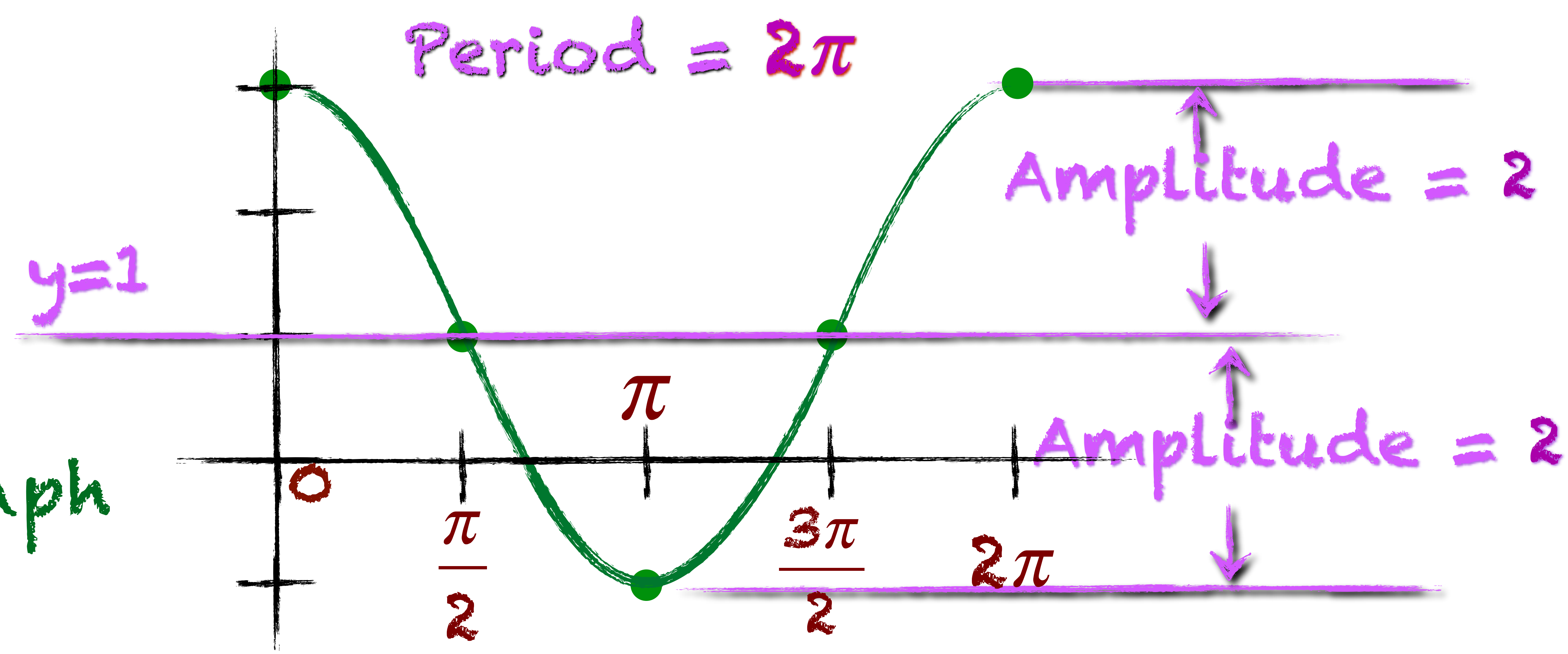
$$y = 2\cos \pi + 1 = -1$$

$$y = 2\cos \frac{3\pi}{2} + 1 = 1$$

$$y = 2\cos 2\pi + 1 = 3$$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = 2\cos x + 1$	3	1	−1	1	3

Step 4 Graph one cycle.



Example: Modeling Periodic Behavior

A region that is 30° north of the Equator averages a minimum of 10 hours of daylight in December. Hours of daylight are at a maximum of 14 hours in June. Let x represent the month of the year, with 1 for January, 2 for February, 3 for March, and 12 for December. If y represents the number of hours of daylight in month x, use a sine function of the form $y = a\sin(bx - c) + d$ to model the hours of daylight.

Because the hours of daylight range from a minimum of 10 to a maximum of 14, the curve oscillates about the middle value, 12 hours. Thus, $d = 12$.

Example: Modeling Periodic Behavior

A region that is 30° north of the Equator averages a minimum of 10 hours of daylight in December. Hours of daylight are at a maximum of 14 hours in June. Let x represent the month of the year, with 1 for January, 2 for February, 3 for March, and 12 for December. If y represents the number of hours of daylight in month x, use a sine function of the form $y = a\sin(bx - c) + d$ to model the hours of daylight.

The maximum number of hours of daylight is 14, which is 2 hours more than 12 hours. Thus, a, the amplitude, is 2; $a = 2$.

Example: Modeling Periodic Behavior

A region that is 30° north of the Equator averages a minimum of 10 hours of daylight in December. Hours of daylight are at a maximum of 14 hours in June. Let x represent the month of the year, with 1 for January, 2 for February, 3 for March, and 12 for December. If y represents the number of hours of daylight in month x, use a sine function of the form $y = a\sin(bx - c) + d$ to model the hours of daylight.

One complete cycle occurs over a period of 12 months.

$$\text{period} = 12\text{mo} = \frac{2\pi}{b} \qquad b = \frac{\pi}{6}$$

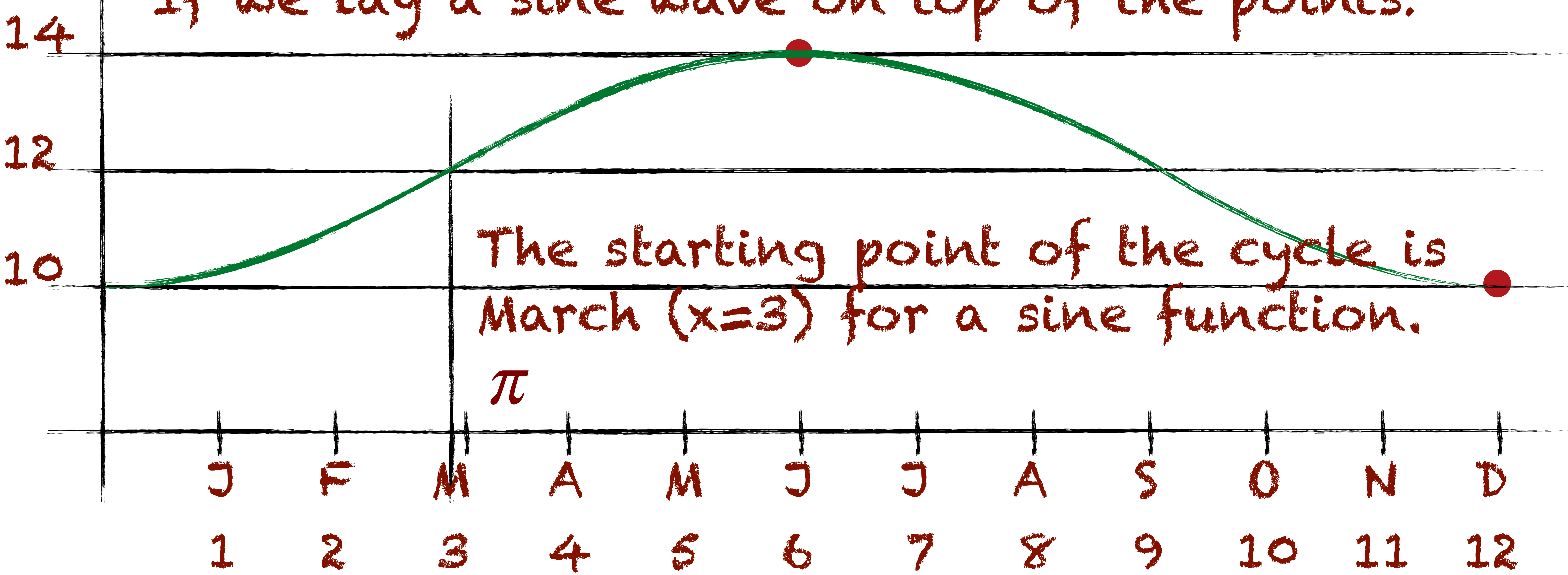
Example: Modeling Periodic Behavior

A region that is 30° north of the Equator averages a minimum of 10 hours of daylight in December. Hours of daylight are at a maximum of 14 hours in June. Let x represent the month of the year, with 1 for January, 2 for February, 3 for March, and 12 for December. If y represents the number of hours of daylight in month x, use a sine function of the form $y = a\sin(bx - c) + d$ to model the hours of daylight.

The maximum number of hours of daylight occur in June, the minimum occurs in December.

Example: Modeling Periodic Behavior

If we lay a sine wave on top of the points:



Phase shift $= 3 = \dfrac{c}{b}$ $\qquad 3 = \dfrac{c}{\frac{\pi}{6}}$ $\qquad c = \dfrac{\pi}{2}$

Example: Modeling Periodic Behavior

Because the hours of daylight range from a minimum of 10 to a maximum of 14, the curve oscillates about the middle value, 12 hours. Thus, $d = 12$.

The maximum hours is 14, minimum 12 hours. Thus, a, the amplitude, is 2; $a = 2$.

One complete cycle occurs over a period of 12 months.

$$\text{period} = 12\text{mo} = \frac{2\pi}{b} \qquad b = \frac{\pi}{6}$$

$$\text{Phase shift} = 3 = \frac{c}{b} \qquad c = \frac{\pi}{2}$$

Example: Modeling Periodic Behavior

A region that is 30° north of the Equator averages a minimum of 10 hours of daylight in December. Hours of daylight are at a maximum of 14 hours in June. Let x represent the month of the year, with 1 for January, 2 for February, 3 for March, and 12 for December. If y represents the number of hours of daylight in month x, use a sine function of the form $y = a\sin(bx - c) + d$ to model the hours of daylight.

$a = 2.$ $\quad b = \frac{\pi}{6}$

$c = \frac{\pi}{2}$ $\quad d = 12$

$$y = 2\sin\left(\frac{\pi}{6}x - \frac{\pi}{2}\right) + 12$$

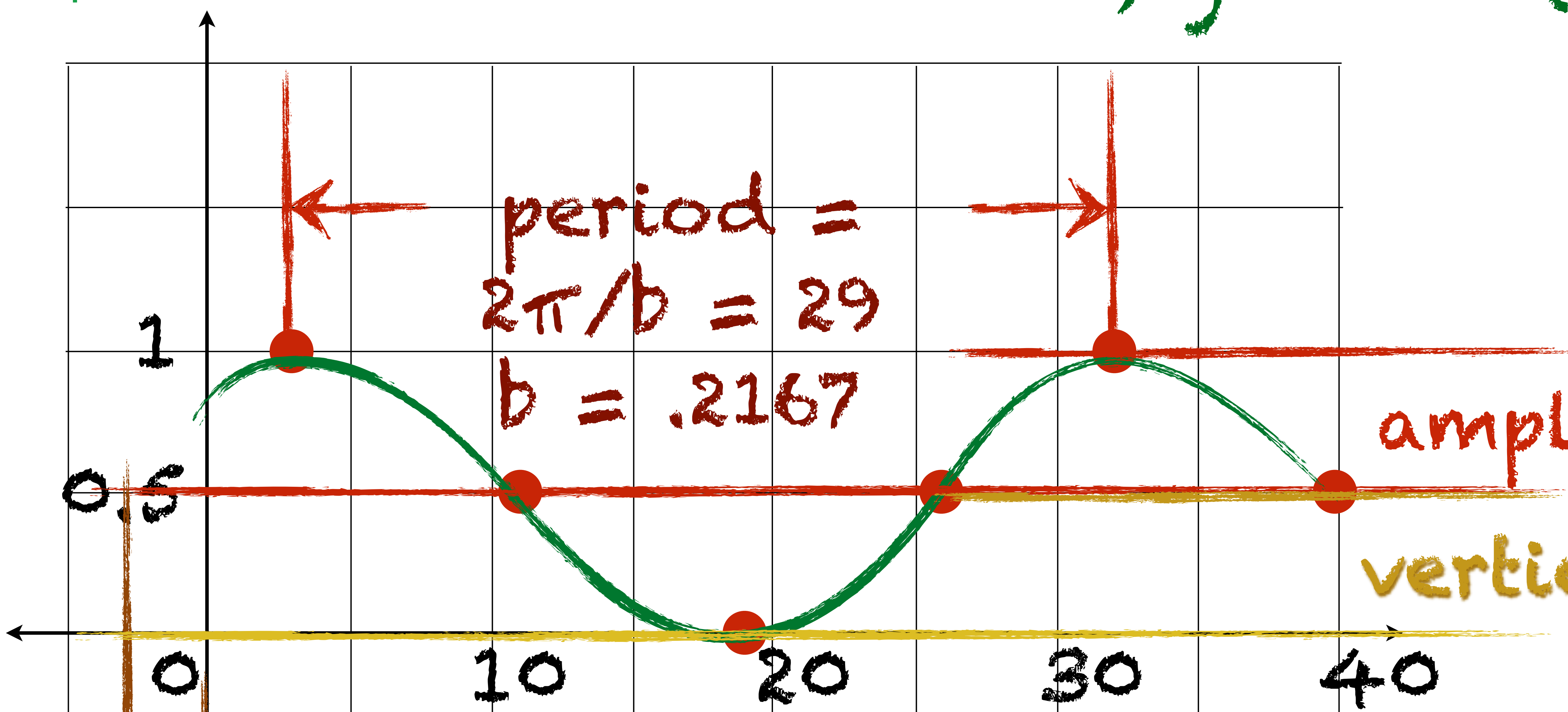
Modeling Sinusoidal Behavior

- **Data Analysis: Astronomy** The percent of the moon's face that is illuminated on day of the year 2007, where x = 1 represents January 1, is shown in the table.

x	y
3	1.0
11	0.5
19	0.0
26	0.5
32	1.0
40	0.5

  - (a) Create a scatter plot of the data.
  - (b) Find a trigonometric model that fits the data.
  - (c) Add the graph of your model in part (b) to the scatter plot. How well does the model fit the data?
  - (d) What is the period of the model?
  - (e) Estimate the moon's percent illumination for March 12, 2007.

Looks like a sine wave, $y = a\sin(bx - c) + d$



amplitude $a = .5$

vertical shift $d = .5$

phase shift =

$1 = .5\sin(.2167(3) - c) + .5$

$1 = \sin(.2167(3) - c)$

$.2167(3) - c = \pi/2$

$c = -.9207$

x	y
3	1.0
11	0.5
19	0.0
26	0.5
32	1.0
40	0.5

$y = a\sin(bx - c) + d$

$y = .5\sin(.2167x + .92) + .5$

Mar 3 = 71

$y = .5\sin(.2167(71) + .921) + .5$

$y = .2182$

We could also estimate the point at which the curve comes back to equilibrium $(3 - 29/4) = -4.25$

$-4.25 - c/.2167 = 0 \quad (\sin(0) = 0)$

Modeling Sinusoidal Behavior with TI-84

- Let us see if TI agrees with us.
- Enter the data into two lists
- Now we will do a sine regression

y=a*sin(bx+c)=d

a= .5111434882

b= .2163933129

c= .7258071718

d= .488303521

STAT ➢ **CALC** ⩓ **C:SinReg** ENTER

Iterations: 3

Xlist: L_1 2nd 1

Ylist: L_2 2nd 2

Period: 29

Store RegEQ: Y_1 **VARS** ➢ **Y-VARS** 1 1:Function ENTER ENTER

Calculate ENTER

$y = .5111\sin(.2164x+.7258)+.4883$

$y = .5\sin(.2167x+.92)+.5$