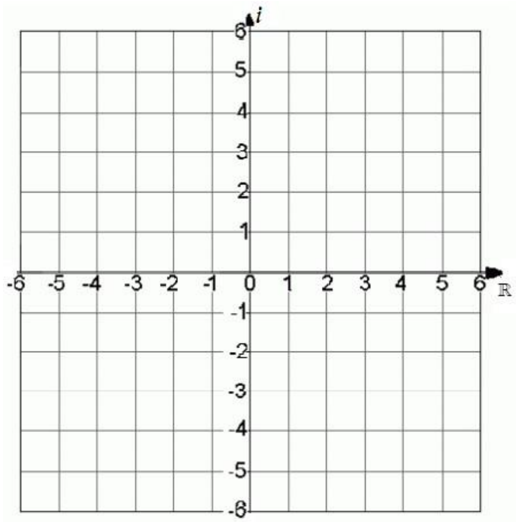
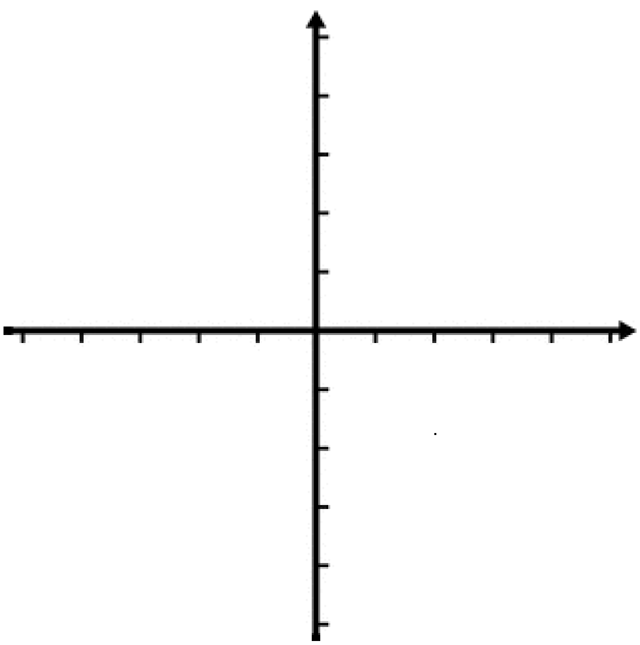
THE TRIGONOMETRIC (OR POLAR) FORM OF

A COMPLEX NUMBER

ARGAND DIAGRAM



THE ABSOLUTE VALUE OF A COMPLEX NUMBER



$$|x| = \begin{cases} x, & \text{If and only if } x \text{ is positive.} \\ -x, & \text{If and only if } x \text{ is negative.} \end{cases}$$

Given the Complex Number

$$z = a + bi$$

The absolute value of z is...

$$|a+bi| = \sqrt{a^2 + b^2}$$

THE TRIGONOMETRIC (POLAR) FORM OF A COMPLEX NUMBER

Let $z = a + bi$, be a complex number.

The trigonometric form of the number is...

$$z = r\left(\cos\theta + i\sin\theta\right)$$

$$where\ r = \sqrt{a^2 + b^2},\ a = r\cos\theta,\ b = r\sin\theta$$

r is called the modulus of z.

θ is an argument of z.

Notice the similarity to our work with vectors

$$\text{vector: } \|v\|\langle\cos\theta, \sin\theta\rangle$$

$$= \|v\|\cos\theta i + \|v\|\sin\theta j$$

Example

Write the complex number $z = 6 - 2i$ in polar form.

2nd find direction

1st find $|6 - 2i|$

$a + bi \quad \tan\theta = \frac{b}{a}$

Now, rewrite using $z = r\left(\cos\theta + i\sin\theta\right)$

Write the following complex numbers in polar form.

1) $z = -8 - 5i$

2) $z = -4 + 4i$

3) $z = 2i$

Convert the following complex numbers from polar form to standard form.

4) $z = 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

5) $z = 2\sqrt{2}\left(\cos 315^\circ + i\sin 315^\circ\right)$

PRODUCTS AND QUOTIENTS OF COMPLEX NUMBERS

Let
$$z_1 = r_1\left(\cos\theta_1 + i\sin\theta_1\right)$$
$$z_2 = r_2\left(\cos\theta_2 + i\sin\theta_2\right)$$

Then,

$$z_1 \cdot z_2 = r_1 \cdot r_2\left[\cos\left(\theta_1 + \theta_2\right) + i\sin\left(\theta_1 + \theta_2\right)\right]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos\left(\theta_1 - \theta_2\right) + i\sin\left(\theta_1 - \theta_2\right)\right]$$

Example

Given $z_1 = 2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$ and $z_2 = 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Find: A) $z_1 \cdot z_2$ B) $\frac{z_1}{z_2}$