Trigonometry/Pre-Calculus
End of Course Review

1) Sum and Difference: P604 #13, 25, 35

sin (x+y) = sin x cos y + cos x sin y

sin (x-y) = sin x cos y – cos x sin y

cos (x+y) = cos x cos y – sin x sin y

cos (x – y) = cos x cos y + sin x sin y

2) Law of Sines: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ P651 #1

3) Law of Cosines: $a^2 = b^2 + c^2 - 2bc\cos A$ P661 #1

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

4) Absolute Value of a Complex Number: $|a+bi| = \sqrt{a^2+b^2}$ P696 #5

5) Rectangular Form to Polar Form

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

6) Polar to Rectangular Form

$r = \sqrt{x^2+y^2}$

$\tan\theta = \dfrac{y}{x} \rightarrow \theta = \tan^{-1}\left(\dfrac{y}{x}\right)$

7) Rectangular Form of a Complex Number to Polar Form: P696 #23

$$z = a + bi \quad \rightarrow \quad z = r(\cos\theta + \sin\theta)$$

where $r = \sqrt{a^2+b^2}$

8) Product of Two Complex Numbers in Polar Form: P696 #37

Given: $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$$z_1 z_2 = r_1 r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$$

9) Quotient of Two Complex Numbers in Polar Form: P696 #45

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

10) DeMoivre's Theorem of Powers of Complex Numbers in Polar Form: P696 #53

$$z^n = \left[r(\cos\theta + i\sin\theta)\right]^n$$

11) DeMoivre's Theorem for Finding Complex Roots: P696 #65

$$z_k = \sqrt[n]{r}\left[\cos(\frac{\theta + 2\pi k}{n}) + i\sin(\frac{\theta + 2\pi k}{n})\right] \text{ in radians}$$

Or $$z_k = \sqrt[n]{r}\left[\cos(\frac{\theta + 360^\circ k}{n}) + i\sin(\frac{\theta + 360^\circ k}{n})\right] \text{ in degrees.}$$

12) Write a Vector in Terms of $v = ai + bj$: P709 #15

a = $x_2 - x_1$, b = $y_2 - y_1$

13) Magnitude of Vector $v = ai + bj$: P709 #5

$$\|v\| = \sqrt{a^2 + b^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

14) Magnitude of a two forces acting on an object. Chapter 6 Test Problem

15) Standard Form

Line: y= m x + b

Parabola: $(y-k)^2 = 4p(x-h)$ (horizontal), focus = (h + p, k) directrix: x = h – p
vertex = (h, k), If p > 0, opens right. If p < 0, opens left.

$(x-h)^2 = 4p(y-k)$ (vertical), focus = (h, k + p) directrix: y = k - p
vertex = (h, k), If p > 0, opens up. If p < 0, opens down.

Circle: $(x-h)^2 + (y-k)^2 = r^2$

Ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ (horizontal) a^2 is the larger than b^2

$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ (vertical) P884 #51

Foci: $c^2=a^2-b^2$ Worksheet #25

Hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ (horizontal) first term tells orientation

Asymptote = $\frac{rise}{run}=\pm\frac{b}{a}$ P898 #45

$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$ (vertical)

Asymptote = $\frac{rise}{run}=\pm\frac{a}{b}$

Foci: $c^2=a^2+b^2$

16) SOH CAH TOA: P575 #45, P574 #1

$\sin\theta=\frac{opp}{hyp}$ $\cos\theta=\frac{adj}{hyp}$ $\tan\theta=\frac{adj}{hyp}$

17) Trig Identities (These will be used within the other sections to solve problems)

$\sin 2\theta=2\sin\theta\cos\theta$ $\cos 2\theta=\cos^2\theta-\sin^2\theta$

$\cos^2\theta+\sin^2\theta=1$ $\cos^2\theta=1-\sin^2\theta$ $\sin^2\theta=1-\cos^2\theta$

$\sin^2\theta=1-\cos^2\theta$ $\cos(-\theta)=\cos\theta$ $\sec(-\theta)=\sec\theta$

$\cot^2\theta+1=\csc^2\theta$

$1+\tan^2\theta=\sec^2\theta$ $\sin(-\theta)=-\sin\theta$ $\csc(-\theta)=-\csc\theta$

$\tan(-\theta)=-\tan\theta$ $\cot(-\theta)=-\cot\theta$

$\sin\theta=\frac{1}{\csc\theta}$ $\cos\theta=\frac{1}{\sec\theta}$ $\tan\theta=\frac{\sin\theta}{\cos\theta}$ $\cot\theta=\frac{\cos\theta}{\sin\theta}$

18) Simplify trigonometric expressions: P 581 #63, 67, P594 #31, P595 #71

19) Solve trigonometric equations using SADMEP: P636 #21

20) Solve trigonometric equations by factoring or quadratic formula: P636 #59, 69

21) Matrix Operations (Add, Subtract, Scalar Multiplication): P838 #9a, 9b, 9c, 9d

22) Matrix Multiplication P839 #27a, 31a

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \qquad B = \begin{bmatrix} d_1 & e_1 & f_1 \\ d_2 & e_2 & f_2 \\ d_3 & e_3 & f_3 \end{bmatrix}$$

$$A \bullet B = \begin{bmatrix} a_1d_1 + b_1d_2 + c_1d_3 & a_1e_1 + b_1e_2 + c_1e_3 & a_1f_1 + b_1f_2 + c_1f_3 \\ a_2d_1 + b_2d_2 + c_2d_3 & a_1e_1 + b_1e_2 + c_1e_3 & a_1f_1 + b_1f_2 + c_1f_3 \\ a_3d_1 + b_3d_2 + c_3d_3 & a_3e_1 + b_3e_2 + c_3e_3 & a_3f_1 + b_3f_2 + c_3f_3 \end{bmatrix}$$

$$A \bullet B = \begin{bmatrix} \sum Row1xColumn1 & \sum Row1xColumn2 & \sum Row1xColumn3 \\ \sum Row2xColumn1 & \sum Row2xColumn2 & \sum Row2xColumn3 \\ \sum Row3xColumn1 & \sum Row3xColumn2 & \sum Row3xColumn3 \end{bmatrix}$$

23) Multiplicative Inverse of a Matrix P853 #13

Matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Multiplicative Inverse: $A^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

24) Determinant of a Matrix : P866 #1

Matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Determinant: $|A| = ad - bc$

25) Solving Systems of Equations Using Multiplicative Inverses of Matrices: P854 #37

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$AX = B$

$A^{-1}AX = A^{-1}B$ Solve for X by multiplying by the inverse of matrix A

$X = A^{-1}B$