Trigonometry Review

Find one positive and one negative coterminal angle of each of the following. There is no need to graph the angles.

1) $30°$ — $30° + 360° = 390°$; $30° - 360° = -330°$

2) $-\frac{2\pi}{3}$ — $-\frac{2\pi}{3} + \frac{6\pi}{3} = \frac{4\pi}{3}$; $-\frac{2\pi}{3} - \frac{6\pi}{3} = -\frac{8\pi}{3}$

3) $\frac{5\pi}{2}$ — $\frac{5\pi}{2} - \frac{4\pi}{2} = \frac{\pi}{2}$; $\frac{\pi}{2} - \frac{4\pi}{2} = -\frac{3\pi}{2}$

4) $\frac{\pi}{3}$ — $\frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3}$; $\frac{\pi}{3} - \frac{6\pi}{3} = -\frac{5\pi}{3}$

Use the unit circle to find the exact value of each of the following. (Exact value means no decimal approximations.)

5) $\tan\left(-\frac{10\pi}{3}\right) = -\sqrt{3}$ — actually at $\frac{2\pi}{3}$; $-\frac{10\pi}{3} + \frac{6\pi}{3} = -\frac{4\pi}{3}$

6) $\csc\frac{7\pi}{3} = \frac{2\sqrt{3}}{3}$ — actually at $\frac{\pi}{6}$; $\frac{7\pi}{3} - \frac{6\pi}{3} = \frac{\pi}{3}$; $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

7) $\sec\frac{4\pi}{3} = -2$ — $\cos\frac{4\pi}{3} = -\frac{1}{2}$

8) $\cos\left(-\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$

9) $\sin\frac{13\pi}{4} = -\frac{\sqrt{2}}{2}$ — $\frac{13\pi}{4} = \frac{5\pi}{4} + \frac{8\pi}{4}$; $\frac{5\pi}{4} = \frac{8\pi}{4}$

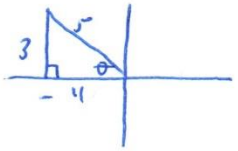
10) $\csc\left(-\frac{5\pi}{6}\right) = -2$ — at $\frac{7\pi}{6}$; $\sin\frac{7\pi}{6} = -\frac{1}{2}$

11) $\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$ — at $\frac{11\pi}{6}$

12) $\cot\frac{2\pi}{3} = -\frac{\sqrt{3}}{3}$ — $\tan\frac{2\pi}{3} = -\sqrt{3}$

Given the following information, find the exact value of the trigonometric function of angle θ.

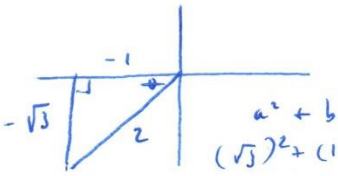
13) Given $\sin\theta = \frac{3}{5}$ and angle θ lies in quadrant II, find $\cot\theta$.



$\sin\theta = \frac{opp}{hyp}$; $\tan\theta = \frac{opp}{adj}$; $\tan\theta = -\frac{3}{4}$ so $\cot\theta = -\frac{4}{3}$

14) Given $\tan\theta = \sqrt{3}$ and $\cos\theta < 0$, find $\sin\theta$.

$\sin\theta = \frac{opp}{hyp}$; $\sin\theta = \frac{-\sqrt{3}}{2}$

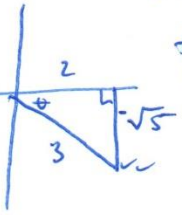


$\tan\theta = +$ Quad III; $\cos\theta = -$

$a^2 + b^2 = c^2$; $(\sqrt{3})^2 + (1)^2 = c^2$; $3 + 1 = c^2$; $4 = c^2$; $c = 2$

15) Given $\tan\theta = -\frac{\sqrt{5}}{2}$ and $\sin\theta < 0$, find $\sec\theta$.

$\cos\theta = \frac{adj}{hyp}$; $\cos\theta = \frac{2}{3}$ so $\sec\theta = \frac{3}{2}$

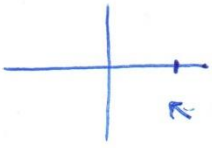


$a^2 + b^2 = c^2$; $(2)^2 + (\sqrt{5})^2 = c^2$; $4 + 5 = c^2$; $9 = c^2$; $c = 3$

$\tan\theta = -$ Quad IV; $\sin\theta = -$

16) Given $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta < 0$, find $\csc\theta$.

From the unit circle



Quad IV has an x value of $\frac{\sqrt{3}}{2}$?

check angle $\frac{11\pi}{6}$; $\sin\frac{11\pi}{6} = -\frac{1}{2}$

$\csc\frac{11\pi}{6} = -2$

17) Change 270° to radian measure.

$270° \cdot \frac{\pi}{180°} = \frac{27\pi}{18}$

$\frac{3\pi}{2}$

Conversions

R → D

Radian · $\frac{180°}{\pi}$

D → R

Degree · $\frac{\pi}{180°}$

18) Change 210° to radian measure.

$210° \cdot \frac{\pi}{180°} = \frac{21\pi}{18}$

$\frac{7\pi}{6}$

19) Change $\frac{7\pi}{4}$ to degree measure.

$\frac{7\pi}{4} \cdot \frac{180°}{\pi} = 315°$

20) Change $\frac{2\pi}{3}$ to degree measure.

$\frac{2\pi}{3} \cdot \frac{180°}{\pi} = 120°$

Use the arc length formula for numbers 21 thru 24.

Arc-Length $s = \theta r$ where θ is measured in radians.

21) If $r = 12.5$ and $s = 25$ find θ.

$\frac{25}{12.5} = \frac{\theta(12.5)}{12.5}$

$\theta = 2$ radians

22) If $r = 22$ and the measure of the central angle is 180° find s.

$180° = \pi$ radians

$s = \theta r$

$s = 22\pi$

23) A bicycle wheel with a 20 in diameter rotates 120°. What distance has the bicycle traveled?

radius = 10 in

$120° \cdot \frac{\pi}{180°} = \frac{2\pi}{3}$ radians

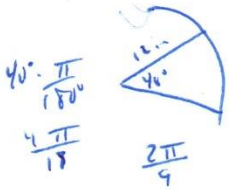
$\theta = \frac{2\pi}{3}$ $r = 10$ in

$s = \theta r$

$s = \left(\frac{2\pi}{3}\right)(10)$ in

$s = \frac{20\pi}{3}$ inches

24) Find the measure of the arc subtended by an angle of 40° if the radius of the circle is 12 inches.



$40° \cdot \frac{\pi}{180°} = \frac{4\pi}{18} = \frac{2\pi}{9}$

$s = \theta r$

$s = \left(\frac{2\pi}{9}\right)(12\text{ in})$

$s = \frac{8\pi}{3}$ inches

25) Find all angles θ in the interval $[0, 2\pi)$ that satisfy the expression:

$\sin\theta = -\frac{\sqrt{3}}{2}$ $\theta =$ $\frac{4\pi}{3}, \frac{5\pi}{3}$

26) Find all angles θ in the interval $[0, 2\pi)$ that satisfy the expression:

$\sin\theta = \frac{1}{\sqrt{2}}$ $\sin\theta = \frac{\sqrt{2}}{2}$

$\csc\theta = \sqrt{2}$ $\theta =$ $\frac{\pi}{4}, \frac{3\pi}{4}$

27) Find all angles θ in the interval $[0, 2\pi)$ that satisfy the expression:

$\tan\theta = \sqrt{3}$ $\theta =$ $\frac{\pi}{3}, \frac{4\pi}{3}$

28) Find all angles θ in the interval $[0, 2\pi)$ that satisfy the expression:

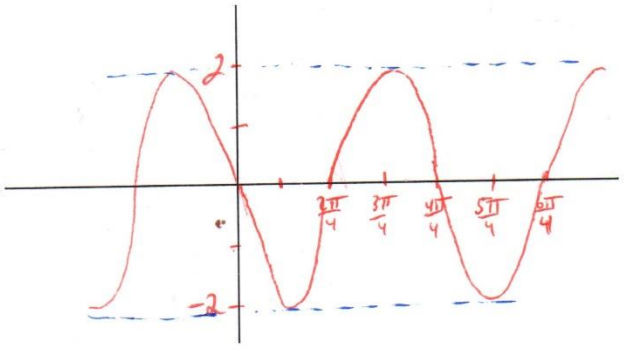
$\frac{1}{\cos\theta} = \frac{1}{0}$

$\cos\theta = 0$

$\sec\theta = \textit{undefined}$ $\theta =$ $\frac{\pi}{2}, \frac{3\pi}{2}$

GRAPH EACH OF THE FOLLOWING FUNCTIONS
Be sure to find the amplitude, period and initial interval of each function.

29) $y = 2\sin(2x - \pi)$



Amplitude: 2

Period: $\frac{2\pi}{2} = \pi$

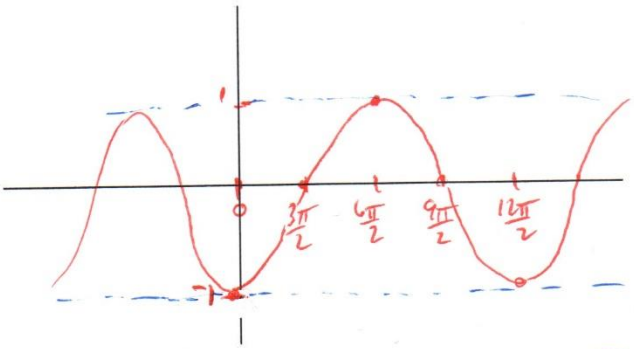
Phase Shift: $\pi/2$

Initial Interval: $\pi/2 \le x \le \frac{3\pi}{2}$

Period/4 = $\frac{\pi}{4}$ all 4 things

$\frac{\pi}{2}$ is same as $\frac{2\pi}{4}$

30) $y = -\cos\left(\frac{x}{3}\right)$



Amplitude: 1

Period: $\frac{2\pi}{\frac{1}{3}} = 6\pi$

Phase Shift: none

Initial Interval: $0 \le x \le 6\pi$

Period/4 = $\frac{6\pi}{4} = \frac{3\pi}{2}$ add

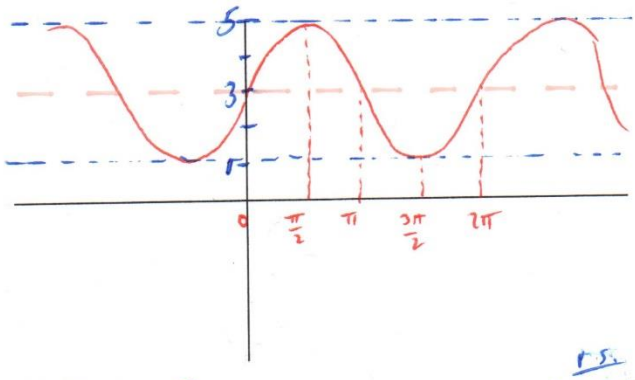
$0 + \frac{3\pi}{2} = \frac{3\pi}{2}$

$\frac{3\pi}{2} + \frac{3\pi}{2} = \frac{6\pi}{2}$

$\frac{6\pi}{2} + \frac{3\pi}{2} = \frac{9\pi}{2}$

$\frac{9\pi}{2} + \frac{3\pi}{2} = \frac{12\pi}{2}$

31) $y = 2\sin x + 3$



Amplitude: 2

Period: 2π

Phase Shift: none

Initial Interval: $0 \le x \le 2\pi$

for 5:

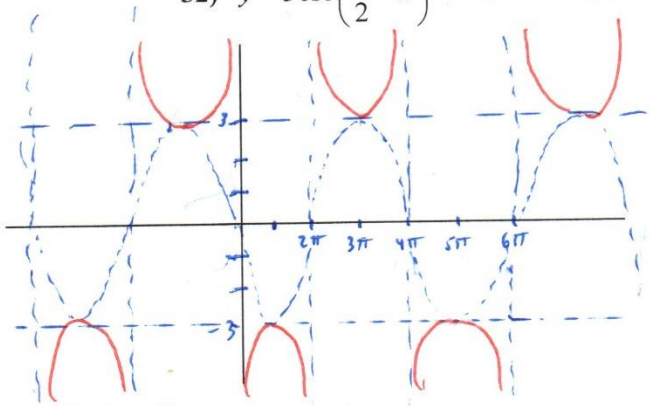
$\frac{x}{2} - \pi = 0$

$\frac{x}{2} = \pi$

$x = 2\pi$

32) $y = 3\csc\left(\frac{x}{2} - \pi\right)$

$y = 3\sin\left(\frac{x}{2} - \pi\right)$



Amplitude: 3

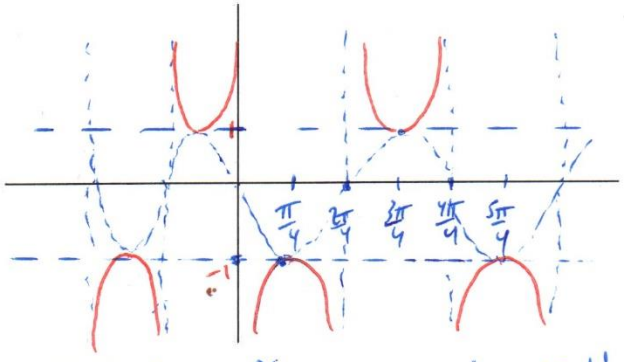
Period: $\frac{2\pi}{\frac{1}{2}} = 4\pi$

Phase Shift: 2π

Initial Interval: $2\pi \le x \le 6\pi$

Period/4 = $\frac{4\pi}{4} = \pi$ Add

33) $y = -\sec\left(2x - \frac{\pi}{2}\right)$ $y = -\cos\left(2x - \frac{\pi}{2}\right)$



Amplitude: 1

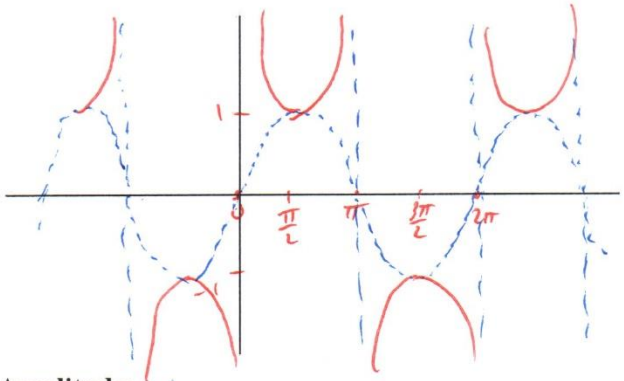
Period: $\frac{2\pi}{2} = \pi$ $\frac{1}{2}2x = \frac{\pi}{2}\cdot\frac{1}{2}$ $\frac{\text{period}}{4} = \frac{\pi}{4}$ all

Phase Shift: $\frac{\pi}{4}$ $x = \frac{\pi}{4}$

Initial Interval: $\frac{\pi}{4} \le x \le \frac{5\pi}{4}$

$\frac{\pi}{4}$ to $\frac{4\pi}{4}$

34) $y = \csc x$ $y = \sin x$



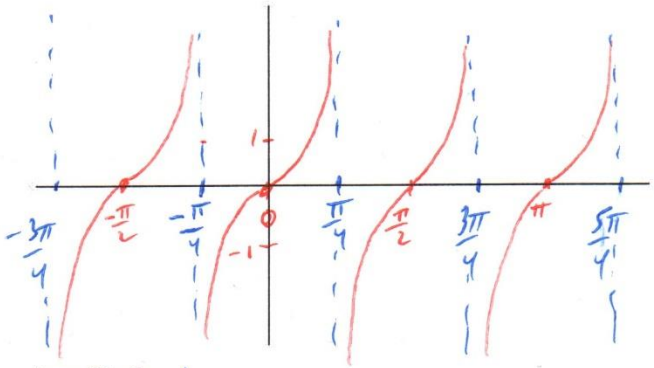
Amplitude: 1

Period: 2π

Phase Shift: none

Initial Interval: $0 \le x \le 2\pi$

35) $y = \tan(2x)$



Amplitude: 1

Period: $\frac{\pi}{2}$

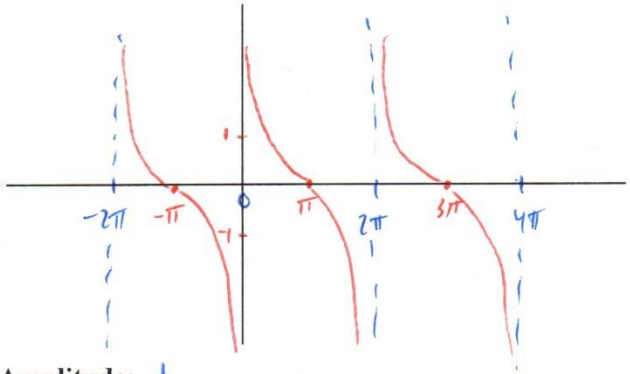
~~**Phase Shift:**~~

Initial Interval: $-\frac{\pi}{4} < x < \frac{\pi}{4}$

$\left(\frac{1}{2}\right) -\frac{\pi}{2} < \frac{1}{2}(2x) < \frac{\pi}{2}\left(\frac{1}{2}\right)$

$-\frac{1}{4} < x < \frac{\pi}{4}$

36) $y = \cot\left(\frac{x}{2}\right)$



Amplitude: 1

Period: $\frac{\pi}{\frac{1}{2}} = 2\pi$

~~**Phase Shift:**~~

Initial Interval: $0 < x < 2\pi$